THE KEY TO THE TUTOR's GUIDE: OR, THE ARITHMETICIANS REPOSITORY.

CONTAINING

The SOLUTIONS of the QUESTIONS, &c. in the TUTOR's GUIDE.

With the REFERENCES as they ſtand in the Second Edition.

To which is added (where neceſſary)

Some USEFUL RULES, &c.

As thoſe for the attaining a thorough Knowledge of CIRCULATING NUMBERS.

Likewiſe

AN APPENDIX,

Shewing the Combination of Quantities; the different Ways they may be varied; with the Method of filling the Magic Squares, &c.

The whole being principally deſigned for the Eaſe of School-maſters, and, with the GUIDE, furniſhes a more Complete and Extenſive SYSTEM of ARITHMETIC, than any extant; and will enable all thoſe who are acquainted with the firſt Principles, to attain a competent Knowledge of the ſeveral Rules, with Eaſe and Preciſion.

The SECOND EDITION, Corrected.

By CHARLES VYSE, K

Teacher of the Mathematics, and Maſter of Weſtham-Abbey Boarding-School.

LONDON,

Printed for G. ROBINSON, Pater-noſter Row. 1775.

THE

PREFACE.

WHAT gave rise to the following Sheets, was my receiving Letters from feveral eminent Mathematicians, and School-mafters; wherein they expreffed great Defire for fuch a Performance, mentioning the Utility fuch a Work would be to School-mafters in general; as the TUTOR'S GUIDE contained fuch a Variety of Queftions, fuitable to all Capacities, and adapted for the Ufe of the Gentleman and Scholar as well as for

THE PREFACE.

WHAT gave riſe to the following Sheets, was my receiving Letters from ſeveral eminent Mathematicians, and School-maſters, wherein they expreſſed great Deſire for ſuch a Performance, mentioning the Utility ſuch a Work would be to School-maſters in general; as the TUTOR'S GUIDE contained ſuch a Variety of Queſtions, ſuitable to all Capacities, and adapted for the Uſe of the Gentleman and Scholar as well as

for the Man of Buſineſs—But as many of the Queſtions are long and difficult, made it impoſſible for Schools in general to make uſe of the GUIDE, without the KEY; the Time in Schools not admitting: therefore they gave me great Hopes of Encouragement, by promiſing to make Uſe of the TUTOR's GUIDE in their own Schools; beſide recommending it to all School-maſters in Great Britain, as the moſt complete Epitome of Arithmetic, or Queſtion Book, extant; and with the KEY, would enable the Tutor to inſtruct ten Pupils with greater Eaſe, more correctly, and with leſs Perplexity both to himſelf and Scholars, than one by any other Book. Thus encouraged, and at the ſame Time being ſenſible of the favourable Reception the GUIDE has met with, gives me great Hopes that the following Pages will meet with that Encouragement, due to ſo uſeful and laborious a Work.

The

The Title Page gives a ſhort Account of what the following Pages contain, which I think needleſs to enlarge upon, therefore ſhall leave the Book to ſpeak for itſelf; and if it does not give Satisfaction to the Reader, I am ſure all I can ſay in its Behalf will never recommend it: but this may be juſtly ſaid, whoever reads it over, will find the Solutions of a greater Variety of Queſtions than in any other Treatiſe on the ſame Subject; and are performed in as intelligible, and comprehenſive a Manner, as my Bounds would admit of, or even as is neceſſary.

How well I have performed this Work, muſt be left to proper Judges; and as I am not ſenſible of any fundamental Error in the following Pages, yet I cannot pretend to ſay it is without Imperfections; which I hope the good-natured Reader will excuſe and paſs over with the like Candour and Goodwill.

will, with which it was compoſed for his Uſe, by

Waltham-Abbey,
Oct. 24, 1774.

CHARLES VYSE.

N. B. As I am anxious of making this Book as perfect and complete as poſſible, I moſt earneſtly requeſt, that if any of my Readers ſhould diſcover any Defect in it, that they will be ſo obliging as to favour me with a Line, which ſhall be carefully attended to in the next Impreſſion.

To Mr. CHARLES VYSE.

SIR,

YOUR valuable Treatiſe on Arithmetic, as ſoon as it came from the Preſs, was immediately introduced into my School, where I have continued to uſe it ever ſince; and look upon it as exceedingly well calculated to facilitate the Buſineſs of a School, in thoſe Branches of Learning of which it treats.

A ſecond Edition falling into my Hands, it gave me great Pleaſure to find by your Preface thereto, that you had ſome Thoughts of giving us (in a ſeparate Publication) the Solutions to all the Queſtions at large; which would be in my Opinion of great Uſe to the Maſter, and what I wiſh of all things to hear you have done. I immediately wrote to my Bookſeller, for the Key to your Book, but received for anſwer, it was not in Print; which has induced me to give you this Trouble, to know whether that be really the Caſe; and whether I may expect ever to ſee ſo excellent a ſyſtem of School Inſtructions rendered complete, by the Publication of the Key, &c.

I am, Sir,

Your moſt obedient humble Servant,

Sept. 16, 1773.

CHRISTOPHER CAVE.

Maſter of the Free-School at Caiſter, in Lincolnſhire.

To Mr. CHARLES VYSE,

SIR,

I Purchaſed your ſecond Edition of the Tutor's Guide ſome Time ago, with a view of being furniſhed with a complete Syſtem of Arithmetic, when the Key ſhould be publiſhed; and have been very impatient for ſo uſeful a Performance, being ſenſible of the utility ſuch a Work muſt be to the maſters in general, particularly to all thoſe who have the Care of

a nu-

a numerous School; I beg you will inform me whether the Key is (already, or intended to be) publifhed, and you will highly oblige

Your moft humble Servant,

and well wifher,

Plumton,
June 12, 1773.

ISAAC SLEE.

School-mafter at Plumton in Cornwall.

To Mr. CHARLES VYSE.

SIR,

I Cannot help thanking you for the pains you have fo happily employed for the facilitating of the Science of Numbers.

Solutions of Mr. Clare's very ingenious Queftions have been much wanted; and it muft certainly afford great fatisfaction to every Lover of Figures to find that Tafk fo well executed.

I am, Sir,

Your obliged humble Servant,

July 14, 1774.

SAMPSON WRIGHT.

Mafter of the Free-Grammar-School of Bradley, near Stafford.

WE have alfo received Letters to the fame purport from the following gentlemen, viz. Mr. Rigge, Writing-mafter and Land-furveyor, at Cambridge; Mr. Rofs, Author of the Inftructor's Affiftant, at Portfmouth; Mr. Boome, School-mafter, at Putney; Mr. ——, Mafter of the Boarding-fchool, at St. Edmund's Bury; Mr. Lloyd, Mafter of the Boarding-fchool, in Kennington-lane, near Vauxhall; Mr. Applegarth, Mafter of the Free Grammar-fchool, in Caftle-ftreet, St. Martins in the Fields; the Rev. Mr. Holiday, of Lincolnfhire; likewife from feveral others, whofe Letters being miflaid, their Names are forgot.

THE

THE

K E Y

TO THE

TUTOR's GUIDE:

OR, THE

ARITHMETICIAN'S REPOSITORY.

BOOK I. PART I.

1. NUMERATION.

NUMBERS expressed in WORDS.

NINETY-FOUR; seven hundred and sixty-two; thirty-thousand and twenty-four; thirty-seven thousand four hundred and sixty; one hundred forty-two thousand, six hundred and thirteen; six million, forty thousand, three hundred and ninety; forty-seven million, six hundred thirty-nine thousand, one hundred and twenty-one; seven hundred ninety million, four hundred one thousand, nine hundred and fifty; seventy-nine millon, forty-one thousand, nine hundred and fifty-five.

WORDS expressed in FIGURES.

77, 490, 6055, 17709, 800002, 7044074, 694400060.

Roman Numerical LETTERS expressed in FIGURES.

19, 200, 600, 560, 1001, 1750, 70000, 110000, 1500000, 1600000.

Common FIGURES expressed in Numerical LETTERS.

$\overline{\text{XXIX}}$. CIV. CCCCXIX. $\overline{\text{MDCCXLI}}$. MMVII.
$\overline{\text{XVIII}}$DCLXXVIII, $\overline{\text{XIV}}$. $\overline{\text{DCLXXIV}}$LXXXIV.

 INTE-

INTEGERS.

2. ADDITION.

To add these Examples I begin with the first, and say 6 and 7 is 13, and 6 is 19, and 4 is 23, and 2 is 25, and 9 is 34; this is the Amount of the first Row, or Unit's Place, which contains 3 Tens, and 4 over; this 4 I set under the Row, and carry 3 to the next Row or Tens Place, saying 3 that I carry and 4 is 7, and 1 is 8, and 9 is 17, and 1 is 18, and 4 is 22, and 7 is 29, this being 2 Tens and 9 over, therefore I set down 9 and carry 2 to the next Row or Hundred's Place, saying 2 and 7 is 9, and 4 is 13, and 1 is 14, and 9 is 23, and 2 is 25, which is 5 to set down, and I carry 2 to the fourth Row, or Place of Thousands, saying 2 and 4 is 6, and 7 is 13, and 2 is 15, and 6 is 21, and 4 is 25, and 7 is 32, which is 2 to set down, and I carry 3 to the fifth Row, or Place of Tens of Thousands, and find that it also amounts to 32, then I set down the odd 2, and carry 3 to the sixth and last Row, or Place of Hundreds of Thousands, and find that it amounts to 22, and as this is the last Row, I set down the whole 22, that is, the 2 under the Row and the 2 Tens I carry to the Left Hand, and in the same Manner proceed with the rest of the Examples.

(1) 2222594	(2) 696237	(3) 433968	(4) 225879
(5) 2181162	(6) 326034	(7) 341780	(8) 88622054
(9) 1148103			

3. SUBTRACTION.

When the upper Figure is greater than the lower, as in the first Example, I say 0 from 2 there remains 2; then 2 from 4 there remains 2, and 6 from 7 there remains 1; in this manner proceed to the end of this Example. But when the lower Figure is larger than the top one, as in the second Example, I say 7 out of 1 I cannot have, therefore I take 7 out of 10, (which I borrow) and there remains 3, and the top Figure 1 makes 4, which I place under the Unit's Place, and carry 1 to the next lower Figure, saying that 1 I carry to 6 makes 7, then 7 from 4 I cannot, but 7 from 10, there remains

remains 3, and the top Figure 4 makes 7 to set down, and 1 to carry to the next lower Figure, which is 4, makes 5; then say 5 from 9 there remains 4; but now I do not carry any to the next Figure, because I did not borrow, but only say 7 from 4 I cannot, but 7 from 10 there remains 3, and the top Figure 4 makes 7 to set down, and 1 I carry to 0 is 1, from 0 I cannot, but 1 from 10 there remains 9, and carry 1 to 8 is 9, from 7 I cannot, but 9 from 10 there remains 1, and the top figure 7 makes 8; lastly, 1 I carry from 1, and there remains 0, which being the last Figure, I do not set it down; and in this manner proceed with the other Examples.

(1) 221132 (2) 3160990 (3) 897474 (4) 313992

(5) 689796 (6) 2708970 (7) 8960066 (8) 33208011

(9) 1069897 (10) 31981794 (11) 106799280

4. MULTIPLICATION.

CASE I.

I begin with the first Example, and say, twice 3, or 2 Times 3, is 6 to set down; then twice 5 is 10, that is 0 to set down, and carry 1; then twice 8 is 16, and 1 I carry is 17, that is 7 to set down, and I carry 1; then twice 9 is 18, and 1 I carry is 19, that is 9 to set down, and 1 to carry to the next Figure; and so I proceed to the End of this Example, and likewise with all the rest, always remembering to carry 1 for every 10 to the next Figure on the Left Hand, as in Addition.

(1) 835259706 (2) 1028158758 (3) 1030735656

† (4) 871478415 (5) 1649792286 (6) 5244299361

(7) 2102067152 (8) 3386626605

† This is the Product of 5.

CASE II.

(9) 856424376
142737396

2283798336

(10) 172859776
43214944

605009216

(11) 84537943
108691641

1171454353

(12) 1237835
1732969
990268

117594325

(13) 2858841
2541192
1905894

218860161

(14) 16241460
10827640
18948370
8120730

101400084860

(15) 295356
1181424
886068
738390

839106396

(16) 47299
378392
236495
141897
331093

3480307719

(17) 662229
441486
147162
515067
294324

3478100289

(18) 4585548
6878322
2292774
5349806
764258
3057032

318998232168

(19) 3339168
2086980
834792
1669584
2504376
2921772

318998232168

(20) 16314384
8157192
19033448
21752512
10876256
16314384
2719064

4483018703104

CASE III.

(21) 3365222
2884476
4326714
432963212822

(22) 85968376
10746047
10746047
42984188
430917645273076

(23) 7242305625
6437605000
5632904375
1609401250
166579474222305625

CASE IV.

(24) 24714
5492
79634000

(25) 102263
116872
12709830000

(26) 16608
11072
5536
68092800000

CASE V.

(27) 24674
$4 \times 4 = 16$
98696
4
394784

(28) 340764
$7 \times 4 = 28$
2385348
4
9541392

(29) 142395
$8 \times 7 = 56$
1139160
7
7974120

(30) 176848
$9 \times 7 = 63$
1591632
7
11141424

(31) 420746
          9×8=72

3786714
      8

30293712

(32) 17093
          9×9=81

153837
     9

1384533

(33) 43074
          12×12=144

516888
    12

6202656

(34) 14068
          12×11=132

168816
    11

1856976

CASE VI.

To perform the Examples in this Case, I begin with the last, and say, 9 times 8 is 72, put down 2 and carry 7; then 9 times 6 is 54, and 7 I carry is 61, and the Right Hand (or back) Figure 8 is 69, put down 9 and carry 6; then 9 times 7 is 63, and 6 I carry is 69, and 6 the back Figure is 75, put down 5 and carry 7; then 9 times 2 is 18, and 7 I carry is 25, and 7 the back Figure is 32, put down 2 and carry 3; then 9 times 4 is 36, and 3 I carry is 39, and 2 the back Figure is 41, put down 1 and carry 4; then 9 times 1 is 9, and 4 I carry is 13, and 4 the back Figure is 17, put down 7 and carry 1. Now as the Multiplication by the 9 (the Unit's Figure) is ended, I add the 1 I carry to the last Figure in the Multiplicand, and it makes 2, which I put down, and the work is ended; and in this Manner proceed with all the rest of the Examples in this Case.

(35) 1569876 (36) 171312 (37) 1899222 (38) 338464

(39) 542130 (40) 2822784 (41) 248353 (42) 335286

(43) 2712592

SECT.

5. SECT. V. DIVISION.

CASE I.

In the first Example I begin and say, How oft 2 in 17? Answer, 8 times 2 is 16, and 1 over, which is 10, added to 4 the next Figure, makes 14; then I say how oft 2 in 14? Answ. 7 times 2 is 14, and 0 over; how often 2 in 2? Answer, 1 and 0 over; how oft 2 in 6? Answ. 3 times 2 is 6, and 0 over; how often 2 in 3? Ans. 1 and 1 over, which is 10, and the next Figure 6 is 16; how oft 2 in 16? Ans. 8 times 2 is 16, and nothing remains; and in this Manner proceed with all the rest of the Examples in this Case.

(1) Quotient 871318
2

Proof 1742636

(2) 921354—2

(3) 540185—2

(4) 215285—1

(5) 11903492—2

(6) 667751—6

(7) 346012

(8) 752010—4

(9) 251160—4

(10) 23040—4

CASE II.

To perform this Case I begin with the first Example, and say, How oft is 25 in 73; or, which is better, how oft 2, the first Figure in the Divisor, in 7, the first in the Dividend, and I find (after the Allowance is made for what I shall have to carry) it will only go 2 times; wherefore I place 2 in the Quotient, and multiply 25 the Divisor thereby, the Product (viz. 50) set under 73, and subtract; then to the Remainder 23 I bring down the next Figure 6 in the Dividend; then I say (as before) how oft 2 in 23? Answer, 9 times, which I place in the Quotient, and multiply (25) the Divisor thereby, the Product, viz. 225 subtracted from 236 leaves 11, to which bring the next Figure in the Dividend, viz. 4, then proceed as before, till you have brought down all the Figures in the Dividend, and the Work will finished.

Divisor.

```
Divisor   Divid. (11) Quot.
      25)736473575(29458943
         50               5
         --      ---------
         236      147294715
         225              5
         ---     ----------
         .114  Pr. 736473575
          100
          ---
          .147
           125
           ---
           .223
            200
            ---
            .235
             225
             ---
             .107
              100
              ---
              ..75
              ..75
              ----
               ..
```

```
        (12)
84)35730972(425368 60/84
   336
   ---
   .213
    168
    ---
    .450
     420
     ---
     .309
      252
      ---
      .577
       504
       ---
       .732
        672
        ---
Remains 60
```

```
         (13)
648)272357640(420305
    2592
    ----
    .1315
     1296
     ----
     ..1976
       1944
       ----
       ..3240
         3240
         ----
         ....
```

```
           (14)
759)30891829676(40700697
    3036
    ----
    ..5318
      5313
      ----
      ...5296
         4554
         ----
         .7427
          6831
          ----
          .5966
           5313
           ----
Remains .653
```

(15)

```
3065)63463902247(20706003
     6130
     ----
     .21639
      21455
      -----
     ..18402
       18390
       -----
     ...12247
         9195
         ----
Remains  3052
```

(17)

```
7489)1204530760(160839
     7489
     ----
     .45563
      44934
      -----
      62907.
      59912
      -----
      29956
      22467
      -----
      74896
      67401
      -----
Remains .7489
```

(17)

```
42163)112737328(2673
      84326
      -----
     .284113
      252978
      ------
     .311352
      295141
      ------
     .162118
      126489
      ------
Remains .35629
```

(18)

```
61745)392628787(6358
      370470
      ------
      221587
      185235
      ------
     .363528
      308725
      ------
      548037
      493960
      ------
Remains .54077
```

(19)

(19)

684573)3233238699(4723
2738292

4949466
4792011

·1574559
1369146

·2054139
2053719

Remains 420

(20)

476085)98839054780(207608
952170

3622054
3332595

·2894597
2856510

··3808780
3808680

Remains 100

(21)

4728395)27750950255(5869
23641975

·41089752
37827160

·32625925
28370370

·42555555
42555555

....

CASE III.

(22)

28,00)119282,48(4260
112

72
56

168
168

Rem. ...248

2800

3408248
852

11928248 Proof.

(23)

172,000)247004,674($1436\frac{12674}{172000}$
172

·750
688

·620
516

1044
1032

Remains ..12674

(24)

$473,000)351858,000(743\frac{419}{473}$

3311

.2075
1892

.1838
1419

Remains 419|000

473|000

(25)

$697,0000)59943,0000(86\frac{1}{697}$

5576

.4183
4182

Remains 10000

697|0000

CASE IV.

(26)

16 { 4)1206816
4) 301704

Quot. 75426

(27)

48 { 6)42768
8) 7128

891

(28)

72 { 8)74682
9) 9335—2
1037—2 } 18 R.

(29)

144 { 12)14276
12) 1189—8
99—1 } 20 Rem.

(30)

28 { 4)247684
7) 61921

8845—6=24 R.

(31)

64 { 8)14652
8) 1831—4
228—7 } 60 Rem.

(32)

81 { 9)417681
9) 46409

5156—5=45 Rem.

(33)

(33)

```
    { 11)307684
132 {    ------
    { 12) 27971—3  }
         ------    } 124 Remains.
          2330—11  }
          ----
```

CASE V.

```
(34)
17)690489(40617
   ---
   104
   ---
    28
   ---
   119
   ---
   ...

(35)
86)5343698(62136
   ---
   183
   ---
   116
   ---
   309
   ---
   518
   ---
Remains 2

(36)
467)2148686(4601
    ----
    .2806
    -----
      486
      ---
Remains 19

(37)
6074)24939844(4106
     -----
     6438
     -----
     36444
     -----
     .....
```

6. REDUCTION.

MONEY.

```
      £.
(1)   27
      20
      --
      540 Shillings.
      12
      ---
      6480 Pence.
      ----

(2)  12)6480 Pence.
        ----
    2,0) 54,0 Shillings.
         ----
         27 £.
         --
```

(3)

(3)
£. s.
40 10
20

810 Shillings.
12

9720 Pence.
4

38880 Farthings.

(4)
qrs.
4)38880

12)9720

2,0)81,0

£. 40 10

(5)
£. s. d.
104 17 $6\frac{1}{4}$
20

2097 Shillings.
12

25170 Pence.
4

100683 Farthings.

(6)
qrs.
4)100683

12)$25170\frac{3}{4}$

2,0)209,7—6

£. 104 17 $6\frac{1}{4}$

(7)
Guineas.
21
7 × 3 = 21*s.* = 1 Guin.

147
3

441 Shillings.
12

5292 Pence.
4

21168 Farthings.

(8)
qrs.
4)21168

12)5292

$21\begin{cases} 3)441 \\ 7)147 \end{cases}$

21 Guineas

(9) 42 *Moidores.*
$9 \times 3 = 27s. = 1$ Moid.

378
3

1134 Shillings.
12

13608 Pence.
4

54432 Farthings.

(10) 4)54432

12)13608 Pence.

$27 \begin{cases} 3)1134 \text{ Shill.} \\ 9)378 \end{cases}$

42 Moid.

WEIGHTS, MEASURES, &c.

TROY-WEIGHT.

(1) 24 *lb.*
12

288 oz.
20

5760 dwts.
$6 \times 4 = 24$

34560
4

138240 grs.

(2) $24 \begin{cases} 4)138240 \text{ grs.} \\ 6)34560 \end{cases}$

2,0)576,0 dwts.

12)288 oz.

24 lb.

	lb.	*oz.*	*dw.*	*grs.*
(3)	12	10	0	22

12

154
20

3080
$6 \times 4 = 24$

18480
4

73942 grs.

(4) *grs.*
$24 \begin{cases} 4)73942 \\ 6)18485-2 \end{cases}$

$\left. \begin{array}{l} 2,0)308,0-5 \end{array} \right\}$ 22 grs.

12)154

lb. 12 10 0 22

APOTHECARIES WEIGHTS.

```
      lb.
(1)   14
      12
    ------
     168 ℥
       8
    ------
    1344 ʒ
       3
    ------
    4032 ℈
      20
    ------
    80640 grs.
    ------

(2)  2,0)8064,0
        ------
        3)4032 ℈
        ------
        8)1344 ʒ
        ------
       12)168 ℥
        ------
          14 lb.
          --
```

```
     lb.  ℥   ʒ  ℈  grs.
(3)   4  11   0  2  17
     12
     --
     59 ℥
      8
    ----
    472 ℈
      3
    ----
   1418 ℈
     20
   -----
  28377 grs.
  -----

(4)  2,0)2837,7
        ------
       3)1418—17 grs.
        ------
        8)472—2 ℈
         ---
       12)59
         --
     lb. 4 11 0 2 17
         -----------
```

AVOIRDUPOISE WEIGHT.

(1)
20 C.
4
80 qrs.
28
2240 lb.
16
35840 oz.
16
573440 drs.

(2)
16 { 4)573440 *drs.*
4)143360
16 { 4)35840 oz.
4) 8960
28 { 4)2240 lb.
7) 560
4)80 qrs.
20 C.

(3)
lb. oz. drs.
27 12 11
16
444
16
7115 drs.

(4)
drs.
16 { 4)7115
4)1778—3
16 { 4)444—2 } 11 drs.
4)111
lb. 27 12 11

(5)
T. C. qrs. lb. oz. drs.
12 10 0 14 11 15
20
250 C.
4
1000 qrs.
28
28014 lb.
16
448235 oz.
16
7171775 drs.

(6)
drs.
16 { 4)7171775
4)1792943—3 } 15 drs.
16 { 4)448235—3
4)112058—3 } 11 oz.
28 { 4)28014—2
7) 7003—2 } 14 lb.
4)1000—3
2,0)25,0
Tons 12 10 0 14 11 15

CLOTH MEASURE.

Yds.
(1) 24
4
96 qrs.
4
384 Nails.

Nails.
(2) 4)384
4)96 qrs.
24 Yards.

E.E. qrs. na.
(3) 72 4 2
5
364 qrs.
4
1458 Nails.

(4) 4)1458
5)364—2
E. Ells 72 4 2

(5) 121
3
363
4
1452 Nails.

(6) 4)1452
3)363
121 E. F.

F.E. qrs.
(7) 42 5
6
257 qrs.
4
1028 Nails.

Nails.
(8) 4)1028
6)257 qrs.
Fl. E. 42 5 0

LONG MEASURE.

M. F. P.
(1) 176 0 30
8
1408 Fur.
40
56350 Poles.

Poles.
(2) 4,0)5635,0
8)1408—30 P.
M. 176 0 30 P.

(3) 200 *M.*
1760

352000 Yards.
3

1056000 Feet.
12

12672000 Inches.

(4) 12)12672000 Inches.

3)1056000 Feet.

176,0)35200,0(200 Miles.
352

...00

Lea. m. fur. p. y.
(3) 12 1 6 29 4
3

37 Miles.
8

302 Fur.
40

12109 Poles.
11 Half Yds. in a Pole.

133207 Half Yards.
18 Inches $=\frac{1}{2}$ Yd.

2397726 Inches.
3

7193178 Barley Corns.

B. C.
(6) 3)7193178

18 { 3)2397726 Inches.
6) 799242

11)133207 half Yds.

4,0)1210,9—8=4 Yds.

8)302—29 P.

3)37—6 Fur.

Lea. 12 1 6 29 4

LAND MEASURE.

A.
(1) 42
4

168 Roods.
40

6720 Poles.

P.
(2) 4,0)672,0

4)168 Roods.

42 Acres.

(3)

```
      A. r. p.                            P.
(3)   12 3 29              (4)   4,0)206,9
       4                         ---------
      --                             4)51—29 P.
      51 Roods.                        --
      40                          A. 12 3 29
    ----                             -------
    2069 Poles.
    ----
```

WINE MEASURE.

```
      A.
(1)   4                   (2)   4)160
      10                          ---
      --                     1,0)4,0 Gal.
      40 Gal.                    ----
      4                            4 Anchors.
    ----                           -
     160 Qts.
    ----

      Hbds.
(3)   4                   (4)   8)2016
      63                          ----
    ----                     63 { 7)252
     252 Gall.                     ---
       8                          { 9) 36
    ----                               --
    2016 Pints.                         4 Hhds.
    ----                                -

    Tierce gal.                 Pints.
(5)   42 24               (6)   8)14304
        7×6=42                    -----
     ---                     42 { 6)1788 Gal.
     294                           ----
       6                          { 7) 298
    ----                              ----
    1788 Gal.                    Tierce 42 24
       8                                -----
   -----
   14304 Pints.
   -----
```

(7)

(7) *T. p. hd. gal. pts.*
4 1 1 42 6
2
9 Pipes.
2
19 Hhds.
63
1239
8
9918 Pints.

(8) 8)9918
63 { 7)1239—6 Pints.
9) 177
2)19—42 Gal.
2)9—1 Hhd.
Tuns. 4 1 1 42 6

WINCHESTER MEASURE.

(1) 12 *B. A.*
32
384 Gal.
4
1536 Quarts.

(2) 4)1536 Qts.
32 { 4)384 Gal.
8) 96
12 Barrels.

(3) 42 *B. B.*
9×4=36
378
4
1512 Gal.
8
12096 Pints.

(4) 8)12096
36 { 4)1512 Gal.
9) 378
42 Barrels.

(5) *A. hds. gal. pts.*
6 27 6
48
315 Gal.
8
2526 Pints.

(6) 8)2526
48 { 6)315—6 Pints.
8) 52—3 } 27 Gal.
6—4 }
Hhds. 6 27 6

B. hhd. gal.

(7) 14 47
54
———
803 Gal.
8
———
6424 Pints.
———

Pints.

(8) 8)6424
———

$54 \left\{ \begin{array}{l} 6)803 \text{ Gal.} \\ 9)133\text{—}5 \\ \quad 14\text{—}7 \end{array} \right.$ $\left.\begin{array}{l} 5 \\ 7 \end{array}\right\} 47 \text{ Gal.}$

Answ. 14 Hhds. 47 Gal.

T. b. hd. gal.

(9) 6 1 0 42
2
———
13 B.
2
———
26 Hhds.
9×6=54
———
234
6
———
1446 Gal.
4
———
5784 Qts.
———

(10) 4)5784
———

$54 \left\{ \begin{array}{l} 6)1446 \\ 9)\ 241 \end{array} \right.$

$\left.\begin{array}{l} 2)26\text{—}7 \\ 2)13 \end{array}\right\} 42 \text{ Gal.}$

T. 6 1 0 42
———

DRY MEASURE.

Qrs.

(1) 24
8
———
192 Bush.
4
———
768 Pecks.
2
———
1536 Gal.
4
———
6144 Qts.

(2) 4)6144 Qts.
———
2)1536 Gal.
———
4)768 Pecks.
———
8)192 Bush.
———
24 Qrs.
———

Cha.

Cha. bu.

(3) 36 26
6×6=36

216
6

1322 Bush.
4

5288 Pecks.

Pecks.

(4) 4)5288

36 { 6)1322 Bush.
6)220—2 }
36—4 } 26 Bush.

Answ. 36 Ch. 26 Bush.

Lasts.

(5) 64
2

128 Weys.
5

640 Qrs.
8

5120 Bush.
4

20480 Pecks.

Pecks.

(6) 4)20480

8)5120

1,0)64,0 Qrs.

64 Lasts.

T I M E.

Days.

(1) 7
24

168 Hours.
60

10080 Min.
60

604800 Seconds.

(2) 6,0)60480,0 Seconds.

6,0)1008,0 Min.

24 { 4)168 Hours.
6) 42

7 Days.

M.

M. d.

(3) 6 4
28

172 Days.
6×4=24

1032
4

4128 H.
60

247680 M.
60

14860800 Sec.

Seconds.

(4) 6,0)1486080,0

6,0)24768,0 M.

24 { 4)4128 H.
6)1032 }

7)172 D.

4) 24—4

M. 6 4 D.

D. H.

(5) 365 6
6×4=24

2190
4

8766 H.
60

525960 M.
60

31557600

(6) 6,0)3155760,0 Sec.

6,0)52596,0 M.

24 { 4)8766
6)2190—6 }

365 D. 6 H.

D.

D. h. m. '' '''	*Thirds.*
(7) 365 5 48 57 39	(8) 6,0)189341625,9
6×4=24	6,0)3155693,7—39'''
2190	6,0)52594,8—57''
4	24 { 4)8765—48 M.
8765 H.	6)2190—5 D.
60	Days 365 5 48 57 39
525948 M.	
60	
31556937 Sec.	
60	
1893416259 Thirds.	

SQUARE or SUPERFICIAL MEASURE.

Yds.	*Inches.*
(1) 42	(2) 144 { 12)54432
9	12) 4536
378 Feet.	9)378 Feet.
12×12=144	42 Square Yards.
4536	
12	
54432 Inches.	

Sq. f. in.	*Sq. f.*
(3) 3 42 64	(4) 144 { 12)49312
100	12) 4109—4 } 64 Inch.
342 Feet.	1,00)3,42—5 }
12×12=144	3 Sq. 42 F. 64 In.
4104	
12	
49312 Inches.	

CUBIC

CUBIC or SOLID MEASURE.

Yards.

(1) 27
9×3=27

243
3

729 Feet.
1728 Inches=1Foot.

15552
3456
12096

1259712 Inches

Inches.

(2) 1728)1259712(729 Feet.
12096

··5011
3456

15552
15552

·····

Feet.

27 { 3)729
9)243

27 Solid Yards.

I. F.

(3) 4 24
50

224 Feet.
1728

6912
3456
3456

387072 Inches.

Inches. *Feet.*

(4) 1728)387072(5,0|224
3456

4T. 24 Ft.

·4147
3456

·6912
6912

·:··

END of BOOK I.

THE

KEY

TO THE

TUTOR's GUIDE:

OR, THE

ARITHMETICIAN's REPOSITORY.

BOOK II. PART I.

NUMERATION.

(1) 1000000+500000=1500000 South Sea Bonds.
(2) 60000+12000+1300=73300 Lead.
(3) 15000000000+80000000=15080000000 Stivers.
(4) 120206070707 Rials of Plate.
(5) 3033030 Pieces of Eight.
(6) £. 404000 34 $15\frac{5}{4}$=£. 404001 15 $3\frac{1}{4}$.

ADDITION.

INTEGERS.

140724	27460	867
296	176	317
42	2900	69
6740	274	1720
64167	1004	276842
20	64	49
2687	596	426074
2684	41	60
	6104	
217360	38619	705998

MONEY.

MONEY.

To add these Examples, I begin with the first, the Farthings, going up, saying 1 and 3 is 4, and 2 is 6, and 3 is 9, and 1 is 10 Farthings, that is $2\frac{1}{2}$d. or 4 in 10 is 2 Times and 2 over, which is $\frac{2}{4}$ or $\frac{1}{2}$ to put down under the Row of Farthings, and carry 2 Pence to the Units Place of Pence, saying 2 I carry and 1 is 3, and 6 is 9, and 1 is 10, and 9 is 19, and 6 is 25, and 4 is 29, and 1 is 30, then coming down the Place of Tens, saying, and 10 is 40, and 10 is 50, and 10 is 60, and 10 is 70 Pence, (which by the Pence Table are 5s. 10d.) or the 12's in 70 are 5 Times and 10 over, which is 10 Pence to set down under the Row of Pence, and carry 5 Shillings to the Unit's Place of Shillings, saying 5 I carry and 8 is 13, and 2 is 15, and 4 is 19, and 7 is 26, and 1 is 27, and 6 is 33, and 7 is 40; I set down 0, and carry 4 to the Place of Tens, saying 4 I carry and 1 is 5, and 1 is 6, and 1 is 7, and 1 is 8, and 1 is 9, Half of which (or the 2's in 9) is 4, and 1 over, which I set down under the Place of Tens, in the Row of Shillings, and carry the Half (viz. 4) to the Unit's Place of Pounds, and proceed then as in Integers.

But the common Method (tho' not so concise nor yet so easy) is to go up the Unit's Place, and come down the Tens, as in the Row of Pence, which (in this Example) comes to 90 Shillings, or the 20's in 90 are 4 times and 1 over, thus, 2,0)9,0(4l. 10s. so that it plainly appears as the Unit's Figure in the Divisor, or, what I stop at is, 0, so that the Unit's Figure in the Dividend, or what the Unit's Row of Shillings comes to above 10 will be to set down, and to carry 1 for every 10 (as in Integers) to the Place of Tens, and what that Sum is divide it by that Figure which is in the Place of Tens in the Divisor, which in this Case is by 2, consequently is the Reason of my halfing the Ten's Place of Shillings: Likewise when you stop at 40, 60, or &c. proceed as above, only, instead of dividing by 2, you must divide by 4, 6, or, &c. observing always to set down what remains under the Place of Tens.—See Case III. Sect. V.

(1) £. 36 19 $10\frac{1}{2}$ (2) £. 227 18 9 (3) £. 472 11 $3\frac{1}{4}$

	£.	s.	d.		£.	s.	d.		£.	s.	d.
(4)	270	16	6½	(5)	260	17	0	(6)	276	17	0
	60	0	10¼		67	0	10¾		16	0	10¾
	60	10	0¾		170	10	0½		269	11	11½
	96	6	10		100	10	6¼		107	19	0
	176	6	6½		4	16	6½		10	0	6
	2	2	0		0	19	0¼		0	14	11
	16	17	6½		37	11	11½		367	17	6¼
	100	0	0		600	10	0		0	12	4½
					220	0	6¼		20	10	6
	783	0	4½						1000	0	0
					1462	16	6¾				
									2070	4	8

WEIGHTS, MEASURES, &c.

Here I begin with the firſt Example, going up the Unit's Place in Grains, and find it comes up to 23, which, on a Slate or waſte Paper, I put down the 3, and carry 2 to the Place of Tens, and find it comes to 11, which I ſet down to the 3, and the Whole of the Row of Grains make 113, which I divide by as many Grains as make 1 Penny-weight, viz. 24, and find it will go 4 Times and 17 over, thus 24)113(4 Times, and 17 remains; put down 17 Grains under the Row of Grains, and I carry 4 Penny-weights to the Row of Penny-weights, which I proceed with as in Shillings, as I ſtop at the ſame Number, viz. 20: Likewiſe, wherever I ſtop at 4, 12, or any Number under, I proceed as in Farthings and Pence, according to what Number I ſtop at; but when I ſtop at any Number exceeding 12, as 16, 28, 54, &c. then I proceed as with the Grains above, ſo that it is impoſſible (without greatly perplexing the Learner) to perform theſe Examples before he has learned Diviſion: And I am very ſorry to ſee this Method not practiſed more than what it is, that ſo much of that precious Time of Youth ſhould be laviſhed with perplexing Methods of the Teachers, as it is as improper to learn a Scholar to add theſe Examples before he knows the primary Rules in Integers, as it is to learn a Child to read before he knows his Letters.

(1) oz. 170 dwt. 5 grs. 17 (2) lb. 114 4 0 9

(3)

(3) ℥ 62 5 1 1 (4) 86 lb. 9 0 0 8

(5) Tons 731 14 0 23 (6) lb. 121 11 11

(7) Yards 581 0 1 (8) Eng. E. 599 4 0

(9) E. Fl. 575 2 1 (10) Lea. 528 2 3 10

(11) Yards 976 0 4 0 (12) A. 668 0 6

(13) Tuns 57 0 1 25 0 (14) Pun. 155 62 2 1

(15) Tier. 127 28 7 (16) An. 88 9 3

(17) A. hhds. 118 3 0 (18) B. hhds. 125 53 3

(19) A.B. 124 1 0 06 (20) B. Fir. 125 4 0 0

(21) Qrs. 378 4 0 0 (22) Cha. 138 35 1

(23) La. 78 0 2 1 0 (24) Mo. 110 1 6 10

(25) D. 513 18 0 0 0

QUESTIONS.

(1)

	D.
June	29
July	31
Aug.	31
Sept.	30
Oct.	31
Nov.	30
Dec.	31
Jan.	27

Ans. 240 Days.

(2)

1771
60
Ans. A.D. 1831

(3) £.

11000
1100
11
12111 Youngest's [Fortune.
+1000
13111 Eldest's [Fortune.

Then 12111+13111=25222 £. what the Father left them.

(4)	D.
Jan.	31
Feb.	29
Mar.	31
Apr.	30
May	31
June	30
July	31
Aug.	31
Sept.	30
Oct.	31
Nov.	30
Dec.	31
Ans.	366 Days.

(5)	£.	s.	d.
Pays	17	17	6
Rem. unpaid	82	2	6
Answ.	100	0	0

(6)	£.	s.	d.
	474	17	$11\frac{1}{2}$
	467	0	0
A. £.	941	17	$11\frac{1}{2}$

(7)	£.	s.	d.
Corn Chandler,	123	19	0
Brewer, -	41	10	0
Butcher, -	212	0	6
Baker, -	24	0	0
Tallow Chandler,	13	8	0
Taylor, -	137	9	9
Draper, -	74	13	6
Coach-maker,	214	16	6
Wine-merchant,	68	12	0
Confectioner,	16	2	0
Rent 82l.+82s.=	86	2	0
Servants Wages,	46	5	0
Ex. on the Road,	50	0	0
Answ. £.	1108	18	3

(8)	£.	s.	d.
Oats, -	46	7	6
Beans, -	100	0	0
Pease, -	16	16	0
Barley, -	73	0	8
Wheat, -	56	9	10
Rye, -	4	4	6
Water carriage,	13	2	7
Charges, -	1	13	0
Gains, -	18	18	0
Ans. £.	330	12	1

(9)	£.	s.	d.
A paid -	13	2	6
B - -	2	13	0
C - -	0	14	4
D - -	1	9	8½
E - -	11	0	6¼
F - -	0	17	6
G - -	0	12	2
H - -	1	10	6
I 27s.+13s.=	2	0	0
K 23+23+25s. +1s.=	3	12	0
L - -	9	13	4
M - -	12	12	0
N - -	15	0	0
O. $\overline{5\times3}$+10s.=	1	5	0
Anſw. £.	76	2	6¾

(10)	£.	s.	d.
To Merc. Wares,	418	2	6
Cheſhire Cheeſe,	52	18	0
Broad Cloth,	317	12	10
Lead, -	320	0	0
Bar Iron, -	173	0	3
Copper, -	1110	10	1
Accepted a Bill,	88	14	0
Do. on Honour,	50	0	0
Morocco Skins,	28	15	4
Conv. Inſur. &c.	43	0	0
Warehouſe Room, &c.	5	5	0
Factorage,	112	6	0
Anſw. £.	2720	4	0

(11)	oz.	dwt.
Fourteen Diſhes, wt.	193	6
Thirty-ſix Plates,	421	11
Four Doz. Spoons,	104	6
Six Salts, -	32	0
Knives and Forks,	83	9
Four Preſenters,	113	4
Mugs, Tumblers, Beakers, &c.	264	18
Tea-kettle & Lamp,	126	9
And the reſt of the Equipage,	93	2
Anſw.	1432	5

(12)	Cwt.	qr.	lb.
Bags No. 1.	2	2	10
2	2	1	16
3	2	0	24
4	0	1	16
Pocket, 58½lb.=	0	2	2½
Ditto, -	0	2	2½
Anſw. wt.	8	2	15

(13)	M.	f.	p.
From hence to B,	39	6	0
thence to C,	46	0	24
- - D,	60	4	39
- - E,	37	6	0
A. from A to E is	184	1	23

(14)	Years.	m.	d.
	28	0	0
	2	10	16
	1	11	0
	3	7	25
	16	9	27
Anſw. Years	53	[illegible]	12

(15)

A is now	16 Years old,
B born	14 Years hence,
Answ.	30 Years.

(16)

17+29+23=69 Years the Answer.

(17)

	Years.
When the Eldest was born the Father was	24
Then 19 Years and 19 Half-years = 19+9½ =	28½
And the Youngest is now	21
Answ. Father's Age,	73½

(18)

20 left.
21, 42, 84 } stolen the { 3, 2, 1 } Night.

A. 167 Sheep at first.

(19)

When Seth was born	Adam was	—	130 Years old,
Enos	Seth	—	105
Cainan	Enos	—	90
Mahaleel	Cainan	—	70
Jared	Mahaleel	—	65
Enoch	Jared	—	162
Methuselah	Enoch	—	65
Lamech	Methuselah	—	187
Noah	Lamech	—	182
And when the Flood happened Noah was			600
		Answ.	1656 Years.

SUBTRACTION.

INTEGERS.

From	476004	276000	40106
Take	120706	106019	17109
Remains	355298	169981	22997

MONEY.

M O N E Y.

As the upper Figures in the first Example are all larger than those which are underneath, so it is only taking the lower Line from the upper without borrowing at all; but in the second Example I begin and say, 2 Farthings from 1 I cannot, but 2 Farthings from a Penny, or 4 Farthings, there remains 2, and 1 the top Farthing makes 3 Farthings, which I set down thus, ¾; then I carry 1 to 11 is 12 Pence, which from 10 Pence I cannot, but 12 from 12 there remains 0, but 10 the upper Figure is to put down, and carry 1 to 11 is 12, from 10 I cannot, but 12 from 20 there remains 8, and the top Figure 10 is 18 Shillings to put down, and carry 1 to the Unit's Place of Pounds, which proceed with as in Integers.

Note, When the Learner is pretty ready at subtracting, he need not make use of the Words *I cannot*, as it is always seen at Sight whether he can or cannot; so that when he sees the lower Figure is greater than that which stands over it, always take the lower Figure from what you borrow, and to the Remainder add the top Figure, and their Sum set down; thus, in the 2d Example, I say 2 from 4, 2 and 1 is ¾ to set down, and carry 1 to 11 is 12, from 12 0, but 10 to set down, and carry 1 to 11 is 12, from 20, 8, and 10 is 18 to set down, and carry 1, &c.

	£.	s.	d.		£.	s.	d.		£.	s.	d.
(1)	21	13	5¼	(2)	33	18	10¼	(3)	87	7	5½
(4)	77	14	5¼	(5)	10	9	11¼	(6)	103	12	11¼

(7)	£.	s.	d.	(8)	£.	s.	d.
Paid	370	17	1	Received	1450	4	9
Rem. unpaid	105	2	11	Rem. unpaid	26	9	9

(9)	£.	s.	d.
Borrowed	2648	19	10½
Paid	1843	14	3
Bal.	805	5	6½

WEIGHTS

WEIGHTS, MEASURES, &c.

(1) lb. 2 10 17 20 (2) oz. 3 13 7 (3) lb. 3 0 5 0 14

(4) T. 1 19 3 21 (5) lb. 2 11 15 (6) Yds. 66 3 1

(7) E. Eng. 2 4 3 (8) E Fl. 58 0 1 (9) Lea. 39 1 4 27

(10) Yds. 50 2 11 (11) A. 67 1 16 (12) T. 1 0 0 43 1

(13) Pun. 3 75 3 0 (14) Tier. 3 40 5 (15) An. 6 6 5

(16) A. hds. 6 41 3 (17) B. hd. 1 48 5 (18) B. fir. 3 4 6

(19) A. Bar. 27 2 5 0 0 (20) Qrs. 6 6 3 (21) Cha. 6 25 2

(22) La. 4 0 4 4 3 (23) Mo. 2 3 4 23

(24) D. 156 16 42 45

QUESTIONS.

(1)	1772	(2) 102	102
	1735	72	72
	Answ. ·· 37 Years.	Diff. 30	Sum 174

(3)	£.	s.	d.	(4)	£.	s.	d.
A's Sum	74	17	0	Borrowed	100	0	0
Diff.	49	15	6	Paid	41	17	6
Answ. B's Sum £.	25	1	6	Answ. Rem. £	58	2	6

(5)	£.	s.	d.		£.	s.	d.
Taxes	140	0	0	Income	600	0	0
Repairs	94	17	6	Pays	234	17	6
Pays in all	234	17	6	Answ. neat £.	365	2	6

(6)

	£.
	11000
	1100
	11
Daughter had	12111

	£.	s.	d.
Left	13111	10	6
Daughter had	12111	0	0
Answ. Son had £.	1000	10	6

(7)

	£.	s.
Both together are worth	35	10
Horse alone	12	12
Furniture	22	18

	£.	s.
then	22	18
—	12	12
Answ. £.	10	6

(8)

Dr.	£.	s.	d.
To A, -	71	12	6
B, -	34	9	9
C, -	16	8	8
D, -	44	0	0
E, -	66	7	6
F, -	11	2	3
G, -	19	19	0
H, 13s.4d.×30=	20	0	0
Dr.	283	19	8
Cr	192	19	3
Answ. Bal. lost £.	91	0	5

Cr.	£.	s.	d.
By Cash, -	3	13	6
Commodities,	23	10	0
Housh. Furnit.	13	8	6
Plate, -	7	18	5
Tenement,	56	15	0
Book Debts,	87	13	10
Cr. £.	192	19	3

(9)

	£.	s.	d.
First Bond,	114	10	0
Interest	19	0	0
Amount, £.	133	10	0
Paid off,	42	0	0
Second Bond,	91	10	0
Interest, -	13	4	8
Amt. £.	104	14	8
Paid off,	37	14	2
Third Bond,	67	0	6
Interest, -	9	11	3
Answ. Amt. £.	76	11	9

(10)

	£.	s.	d.
Altog. are worth	50	0	0
Horse & Harness	38	16	6
Chaise, £.	11	3	6
Chaise & Harness,	13	13	0
Chaise, -	11	3	6
Harness, - £.	2	9	6
Horse & Harness,	38	16	6
Harness, -	2	9	6
Horse, £.	36	7	0

(11)

(11) Stock, Dr. Per Contra, Cr.

	£.	s.	d.		£.	s.	d.
To D. E. -	713	13	0	By Brandy,	874	10	6
M. F. -	352	10	8	Claret,	754	4	0
L. P. -	180	12	0	Corn,	675	17	3
I. B. -	57	12	10	Canary Seed,	113	0	0
Insurance,	190	0	0	Indigo,	632	12	0
				Saffron,	253	5	0
£.	1494	8	6	W. P.	384	10	0
				Wines p. F. G.	1011	10	0
				Pepper p. S. Q.	1552	16	8
				Bond on R. O.	300	0	0
				Note on T. M.	260	14	0
				India Bonds,	459	0	0
				Interest,	25	14	6
				Bank Stock,	2134	4	6
				Banker,	1892	17	6
				Cr.	11324	15	11
				Dr.	1494	8	6
				An. Pres. Worth	9830	7	5

(12) Stock, Dr. Per Contra, Cr.

	£.	s.	d.		£.	s.	d.
To Debts,	280	0	0	By Cash, &c.	11505	10	0
				By Trade he cleared	393	13	1
				Cr.	11899	3	1
				Dr.	280	0	0
				Answ. Bal. £.	11619	3	1

(13) Factor, Dr. Per Contra, Cr.

	£.	s.	d.		£.	s.	d.
To Tin, -	197	12	0	By Wines,	226	16	6
Bees-wax,	71	7	6	Figs, -	157	11	3
Stockings,	47	3	6	Fruit, -	104	6	0
Tobacco,	943	15	10	Olives, -	136	10	0
Cotton,	123	3	7	Oil, - -	193	17	0
Wheat,	116	5	6	Raisins, -	143	0	4
				Wool, -	75	13	8
Dr. £.	1499	7	11	Commission,	71	18	11
Cr.	1109	13	8				
				£.	1109	13	8
Employer draws for £.	389	14	3				

(14)

	Y.	m.	w.	d.	h.	m.
He was to serve - - -	14	0	0	0	0	0
Of which he accomplished Y. m. w. d. h. m. 11 11 11 11 11 11 =	12	1	0	4	11	11
Answer, to serve, Year	1	11	6	2	12	49

(15) A. Dr. Per Contra, Cr.

1779.	£.	s.	d.	1779.	£.	s.	d.
Jan. 21, To Cash,	9	10	0	Jan 11, By Cash,	17	17	0
Feb. 19, Ditto,	5	6	8	Feb. 24, Ditto,	42	0	0
Mar. 17, Ditto,	12	2	6	Mar. 24, Ditto,	11	18	4
19, Ditto,	5	5	0				
				Cr.	71	15	4
£.	32	4	2	Dr.	32	4	2
				Bal. to A. £.	39	11	2

B.	Dr.			Per Contra,	Cr.		
	£.	s.	d.		£.	s.	d.
Jan. 24, To Cash,	6	10	0	Jan. 11, By Cash,	34	11	6
Mar. 2, Ditto,	6	6	0	Feb. 12, Ditto,	16	14	0
19, Ditto,	5	5	0	Mar. 14, Ditto,	17	8	8
				24, Ditto,	11	18	4
£.	18	1	0				
				Cr.	80	12	6
				Dr.	18	1	0
				Bal. to B. £.	62	11	6

C.	Dr.			Per Contra,	Cr.		
1779.	£.	s.	d.		£.	s.	d.
Jan. 30, To Cash,	19	8	4	Jan. 11, By Cash,	28	18	10
Mar. 19, Ditto,	5	5	0	21, Ditto,	12	5	0
				Mar. 2, Ditto,	21	0	0
£.	24	13	4	24, Ditto,	11	18	4
To A.	39	11	2	Cr.	74	2	2
B.	62	11	6	Dr.	24	13	4
C.	49	8	10				
				Bal. to C. £.	49	8	10
Jointly, £.	151	11	6				

(16)

	C.	qrs.	lb.			C.	qrs.	lb.
Gross	12	0	19		Gross	8	2	2
Tare	1	3	26		Tare	1	1	15
Net	10	0	21	first Venture.	Net.	7	0	15
	7	0	15	second Venture.				
Answ. C.	3	0	6	Difference.				

(17)

(17) W.	Dr.			Per Contra,	Cr.		
1779.	£.	s.	d.	1779.	£.	s.	d.
June 30, To Cash,	66	3	0	June 4, By Cash,	47	18	2
July 5, Ditto,	15	10	9	Note,	200	0	0
12, Ditto,	10	10	0	24, Ditto,	14	12	10
				July 15, Ditto,	52	0	0
£.	92	3	9				
				Cr.	314	11	0
				Dr.	92	3	9
				B. due to W. £.	222	7	3

X.	Dr.			Per Contra,	Cr.		
June 24, To Cash,	47	10	8	June 4, By Bill,	33	14	9
July 7, Ditto,	7	3	1	Cash,	66	5	3
19, Ditto,	38	18	10	July 24, Ditto,	19	19	0
£.	93	12	7	Cr.	119	19	0
				Dr.	93	12	7
				Bal. due to X. £.	26	6	5

Y.	Dr.			Per Contra.	Cr.		
June 16, To Cash,	43	12	6	June 4, By Bal.	116	14	10
30, Ditto,	88	13	4	11, Ditto,	120	0	0
July 12, Ditto,	81	19	8	28, Ditto,	18	5	0
				July 15, Ditto,	42	0	0
£.	214	5	6				
					296	19	10
					214	5	6
				Bal. due to Y. £.	82	14	4

Z.	Dr.			Per Contra,	Cr.		
June 20, To Cash,	11	11	0	June 4, By Cash,	70	8	0
July 7, Ditto,	12	8	3	July 15, Ditto,	31	12	4
				Assignment, -	63	4	8
£.	23	19	3				
				Cr.	165	5	0
				Dr.	23	19	3
				Bal. due to Z. £.	141	5	9

	£.	s.	d.
To W. -	222	7	3
X. -	26	6	5
Y. -	82	14	4
Z. -	141	5	9
Jointly, £.	472	13	9

Feet. *Feet.* *Years.*

(18) 57 127 had at first, (19) 42
42 99 let out, 14

99 Answ. 28 Feet. Answ. 28 Years.

(20) Grandsire's Age 134 Years. then 112
Older than the Son by 93 —41

Son's Age, 41 Father's Age, 71Y.

(21)

When C. was 25 Years old; then when B. was 25 Years old,
B. 14 C. will be + 11

Diff. 11 Years. Answ. Sum 36 C's Age.

(22) 1772 B. born A. D. 1785
+13 A. born A. D. 1693

B. born A. D. 1785 Answ. Diff. . . 92 Years A. older [than B.

lb.

(23) In fair Weather, 33905
foul, 30624

Answ. . 3281 lb. Avoirdupoise.

(24)

(24)	*Years.*	*Years.*	*Years.*
Before Christ	200	50	1769
Since -	1769	+1769	140
Hip. and Arch. flourished	1969	Passidius 1819	and Ptolemy 1629 Y.

(25)	*lb.*		*lb.*
Pekin Bell weighs	120000	Nankin Bell, -	50000
From which take	94600	Erford Bell, -	25400
Rem. Erford Bell,	25400	Answ. Diff.	24600

(26)	*Years.*		*Years.*
Grandfather's Age,	119	Father's Age is -	65
From which take -	83	From which take -	36
Leaves your Age -	36	Answ. Diff.	29

(27)	*Feet.*	
Goes up the first Day,	8	
Comes down at Night,	4	
	4	Feet made good the first Day.
Goes up the second Day,	8	
	12	
Comes down at Night,	4	
	8	made good the second Day.
Goes up the third Day,	8	
	16	
Comes down at Night,	4	
	12	made good the third Day.
Goes up the fourth Day,	8	
	20	the fourth Day at Night, Answ.

(28) From 1772 then $25+24=49$ C.'s
Take 1747 and $49+17=66$ A.'s Age.
also $66+13=79$ B.'s

Rem. . . 25

(29) Conqueſt, - - Anno 1066
To which add $134+10=$ 144

Edifice finiſhed, - Anno 1210

The Peace of Utretcht, Anno 1713
From which deduct - - 70

Demolition, - - Anno 1643
From which deduct - 1210

Anſwer, Duration, - 433 Years.

(30)

	Years.		Years.
Revolution, Anno	1688	Forfeited in Anno	1551
Forfeited	137	Granted -	1239
Forfeited in Anno	1551	Anſw. ſubſiſted	312

(31)

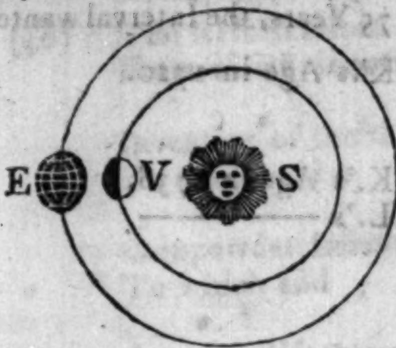
Moſes born Anno	2433	Chriſt born Anno	4000
To which add -	832	From which take	40
Hamer born Anno	3265	Cæſar born Anno	3960
		Erom which take	312
		Alexander born Anno	3648

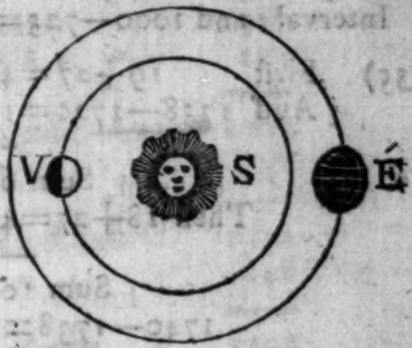
Then $\left.\begin{matrix}3960\\4000\\3648\end{matrix}\right\}-3265=\left\{\begin{matrix}695\\735\\383\end{matrix}\right.$ from Homer to Cæſar. ——— Chriſt. ——— Alexander.

Anſwer 1813 Years, Sum of the Intervals.

(32)

(32) Fig. 1. Fig. 2.





Suppose S. the Sun; V. Venus; and E. the Earth; then Fig. 1. will represent them when in Perigæo; and Fig. 2, when they are in Apogæo.

∵ 18000000—59000000=22000000=S. E.—S.V.=E.V. Fig. 1, or the Distance the Earth is from Venus, when in Perigæo.

And 81000000+59000000=140000000=V. S.+S. E.= V. E. Fig. 2, or the Distance the Earth is from Venus, when in Apogæo.

Therefore 140000000—22000000=118000000 Miles the Answer.

(33) First 23+ 8=31 A.'s }
And 31+19=50 B.'s } Age.
Also 50—14=36 C.'s }

Sum - - - - - 117
From which take - 22

Answ. 95 D.'s Age.

(34)

When Selah was born	Arphaxad was	—	35	Years old.
Eber	Selah —	—	30	
Peleg	Eber —	—	34	
Eber lived after the Birth of Peleg		—	430	Years.
From the Birth of Arphaxad to the Death of Eber was - - - - -			529	
Shem died after the Birth of Arphaxad,			500	
Eber was the Survivor by	-	-	29	Years.

Then

Then per Queft. 2+500+29+194=725 Years, the whole Interval; and 1000—725=275 Years, the Interval wanted.

(35) Firft 19+27=46 K'.s Age in 1720.
And 1738—1720=18

Sum 64 K.'s Age in 1738.
Then 18+27=45 L.'s ——— —.

Sum 109
1740—1738=2

Sum 111—24=87 Years the Age of M. [in 1740.

(36) Firft 318+207=525 B.
Then 525—104=421 C.
And 421— 84=337 D.
Then 337+112=449 E.
Alfo 449+ 47=496 F.
} Flourifhed.

(37) Sam was born before Toby 28 Years.
Toby died at - - 12 —— old.
After which Sam lived - 19

Sam's Age, - - 59 Years.
From which deduct 16+11=27

Leaves Rachel's Age, - 32 Years.
To which add - 7+4=11

Gives Jofhua's Age - 43 Years.

Then 12+59+32+43=146, the Sum of their Ages.

(38)

					Years.
B. born A. D.	-	1108	Z. born A. D.	-	1527
Lived before C.	-	48	After which Y. was born		74
C. born	- -	1156	Y. born Anno	-	1601
Lived before D.	-	113	Before wh. X. was born		114
D. born Anno	-	1269	X. born Anno	-	1487

(39)

(39) I ſhall be 34 + 17 = 51 Years of Age.
And you will be 70 — 34 = 36 ———

(40)

The Reformation, Anno	1517
From which deduct - -	215
Invention of the Compaſs, Anno	1302
To which add - - -	42
Gunpowder Invented, Anno	1344
To which add - - -	148
America diſcovered, Anno -	1492
To which add - - -	77
Painting invented, Anno -	1569

(41)

Reſtoration, Anno - -	1660
Grant made before, - -	—33
Firſt Grant made, Anno -	1627
Duration, - - -	—210
Firſt Grant ended, Anno -	1837
Reverſionary Grant's Continuance,	+99
Its Expiration, Anno -	1936

(42) When B. is 41, A. will be 41 — 18 = 23 Years old.
But when A. is 72, B. will be 72 + 18 = 90 Years old.

(43)

Temple built, Anno	3000	Chriſt born A. D.	4000
Troy before, -	443	Rome built before,	—744
Troy built, Anno	2557	———— A.D.	3256
London after, -	260	Carthage built before,	—113
London built, Anno	2817	———— A. D.	3143
		London built, -	—2817
		London older than Carthage by	326

(44)

(44)

Suppose E. the Earth, M. the Moon, and S. the Sun; then an Eclipse of the Sun will be represented by Fig. 3, and that of the Moon by Fig. 4.

Therefore 81000000—240000=80760000=S M. Fig. 3, or the Distance these two luminaries are asunder, in an Eclipse of the Sun.

Lixewise 81000000+240000=81240000=S M. Fig. 4, or the Distance these two Luminaries are asunder, in an Eclipse of the Moon.

Fig. 3.

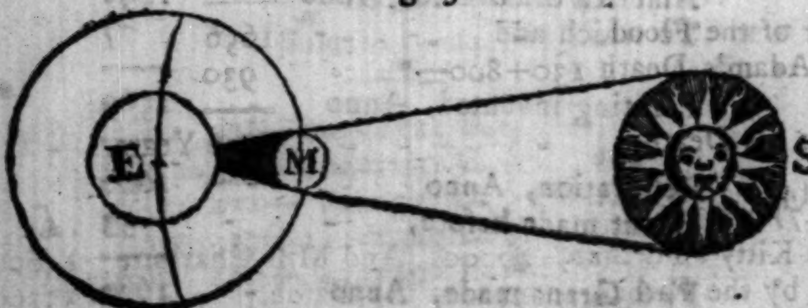


Fig. 4.

(45)	From the Creation to the Flood, - -	1656
	thence to the Building of Solomon's Tem.	1336
	to Mahomet, - - - -	1630
	Together, - - -	4622
	Mahomet after Christ, - - -	622
	Answ. Christ born, A. D.	4000

(46)

(46)

When	Seth was born	Adam was —	130	Years old.
	Enos	Seth —	105	
	Canaan	Enos —	90	
	Mahaleel	Canaan —	70	
	Jared	Mahaleel —	65	
	Enoch	Jared —	162	
	Methuſelah	Enoch —	65	
	Lamech	Methuſelah —	187	
	Noah	Lamech —	182	
When	the Flood happened	Noah was	600	Years.

Year of the Flood, - - - - 1656
To Adam's Death 130+800= - 930

After his Death, - - - 726 Years.

(47)

	£.		£.
Miſs Kitty's Fortune,	20000	And Miſs Charlotte	15000
Left by the Father,	13200	By Father, -	13200
Left by the Grandmo.	6800	By Grandmother, £.	1800
	+1800		
Anſw. Left them,	£. 8600		

(48) Reformation was in Anno 1517
To which add - - - 88

Powder Plot was diſcovered, 1605
To which add - - - 43

King Charles murdered, Anno 1648

Then 1714—54=1660 King Charles II. returned.
And 1660—1648=12 Years, the Anſwer.

(49) Firſt 180— 47=133 Years ſince B. died.
Then 161—133= 28 Years, B.'s Age.
And 47+ 43= 20 ——— C.'s Age.

Sum, 181 ——— the Anſwer.

(50)

(50) A. D.

First, 1772—50=1722.
And 1722+28=1750 Jacob.
Also, 1750—26=1724 Timothy.
Likewise, 1724+17=1741 Sampson.

(51) First, 1733— 445=1288 A. born.
Then, 1362—1288= 74 A.'s Age.
And, 37+ 18= 55 B.'s Age.
Also, 256— 197= 59 C.'s Age.
Again, 1733— 75=1658 D. born.
Then, 1658—1578= 80 D.'s Age.

(52) Reformation, Anno - - 1517
From which deduct - - 23

A. died, Anno - - 1494
born - - - - 1441

53 A.'s Age

Then 1517+49=Anno - 1566 B. died.
And 1494— 7=— - 1487 B. born.

79 B.'s Age.
Also, $36-\overline{7+9}=$ - - 20 C.'s Age.
Then, 53+79+20=152 the Answer.

(53) Anno A. D.
A. born 1438, lived 48 Years; then 1438+48=1486 died.
B. died 1502, bo. bef. 77 Yrs. —— 1502—77=1425 born.
C. in 1577, was 22 Yrs. old, —— 1577—22=1555 —— .
And in 1577+54=1631 died.
D. died 1648, and in 1616 had lived $\frac{1}{2}$ his Time; then 1648—1616=32 Half his Age double of which is 64.
Then 1684, born bef. 64 Yrs. then 1648—64=1584 died.
E. in 1648 was 13 Years old, —— 1648—13=1635 born.
And in 1635+13+14+31+7=1700 died.
F. in 1635 born after 27 Yrs. then 1635+27=1662 born.
G. in 1662 —— 31 —— 1662+31=1693 —— .

(54)

(54) Anno Anno

1727—42=1685	A. died.	Aged 47 Years.	
1685—47=1638	— born.		
1638+13=1651	B. born.	Aged 53 —.	
1712— 8=1704	— died.		
1638—17=1621	C. born.	Aged 91 —.	
——1712	— died.		
1621—23=1598	D. born.	Aged 64 —.	
1598+64=1662	— died.		
1704+11=1715	E. born.	Aged 30.—.	
1733+12=1745	— died.		

1638—1598=40, Half of which is 20.

1638—20=1618	F. born.	Aged 113 Years.
1745—14=1731	— died.	

The Sum of all their Ages is 398 Years.

Then 113—91=22 Years F. Survivor.

MULTIPLICATION.

INTEGERS.

(1) 14276084 × 4	(2) 20749509 × 9	(3) 1204674 × 12
Prod. 57104336	186745581	14456088
(4) 4074746 × 16	(5) 147624 × 69	(6) 42768 × 748
Prod. 65195936	10186056	31990464
(7) 10646 × 5278	(8) 14276 × 39674	(9) 3142708 × 467852
Prod. 56189588	566386024	1470322223216

(10)	27680709	(11)	2142760	(12)	21700
	40700609		4100		954000
Prod.	1126621713851781		8785316000		20701800000

The above Examples are performed as in Sect. 4.

CONTRACTIONS.

1. When the Multiplier consists of the same Figure in all the Places, that is, all Nines or all Sevens, &c. then for each Figure in the Multiplier annex a Cypher to the Multiplicand, and from that Number subtract the Multiplicand, and if the repeating Figure is 9, the Remainder is the Product; but if any other Figure, multiply it into the 9th Part of the Remainder; or, for the Figure 3, take the 3d Part of the Remainder, and for 6 multiply the 3d Part by 2.

EXAMPLES.

(1) Mul. 47627 by 9999.
Thus, 476270000
47627
Prod. 476222373

(2) Mul. 27464 by 11111.
Thus, 2746400000
27464
9)2746372536
Prod. 305152504

(3) Mul. 4674 by 2222.
46740000
4674
9)46735326
5192814
2
Prod. 10385628

(4) Mul. 47694 by 7777.
476940000
47694
9)476892306
52988034
7
Prod. 370916238

5. Mul.

(5) Mul. 74769 by 3333.
Thus, 747690000
74769

3)747615231

Prod. 249205077

(6) Mul. 42763 by 6666.
Thus, 427630000
42763

3)427587237

142529079
2

Prod. 285058158

2. In many Cases the Work may be performed as in the following Examples.

(1) Mul. 4276
By 63

12828 × 2 *
25656

Prod. 262388

(2) Mul. 6946
486

41676 × 8 *
333408

Prod. 3375756

COMPOUND MULTIPLICATION.

MONEY.

CASE I.

	£.	s.	d.
(1)	29	15	10
(2)	843	0	3
(3)	155	17	0

	£.	s.	d.
(2)	3	0	6
			5
£.	15	2	6

	s.	d.
(3)	7	10
		7
£. 2	14	10

	s.	d.
(4)	18	9½
		8
£. 7	10	4

* Because 3×2=6, and 6×8=48, Figures of the Multipliers.

	£.	s.	d.			£.	s.	d.			£.	s.	d.
(5)		12	8		(6)	2	6	4		(7)		12	7
			9					10					11
£.	5	14	0		£.	23	3	4		£.	6	18	5

CASE II.

	£.	s.	d.				£.	s.	d.	
(8)	1	17	6¾			(10)			10¼	
			12							$6 \times 3 = 18$
£.	17	10	9			£.	0	15	4½	

	£.	s.	d.				s.	d.	
(11)	2	9	6			(12)	4	11½	
			$9 \times 3 = 27$						$6 \times 5 = 30$
£.	66	16	6				7	8	9

	s.	d.				d.	
(13)	10	8			(14)	2½	
		$6 \times 6 = 36$					$9 \times 5 = 45$
£.	19	4	0		£.	0	9 4½

	£.	s.	d.				s.	d.	
(15)	1	7	0			(16)	2	7½	
			$10 \times 5 = 50$						$8 \times 7 = 56$
£.	67	10	0				7	7	0

	£.	s.	d.				s.	d.	
(17)	1	11	0			(18)	15	9	
			$8 \times 8 = 64$						$9 \times 8 = 72$
£.	99	4	0			£.	56	14	0

(19) *d.* $11\frac{3}{4}$

$10 \times 8 = 80$

£. 3 18 4

(20) *s.* 9

$12 \times 7 = 84$

£. 37 16

(21) *s.* *d.* 1 $10\frac{3}{4}$

$12 \times 8 = 96$

£. 9 2

(22) *s.* *d.* 18 $11\frac{1}{2}$

$11 \times 9 = 99$

£. 93 16 $10\frac{1}{2}$

(23) *s.* *d.* 11 10

$10 \times 10 = 100$

£. 59 3 4

(24) *s.* *d.* 2 6

$12 \times 10 = 120$

£. 15 0 0

(25) *s.* *d.* 1 10

$12 \times 11 = 132$

£. 12 2 0

(26) *s.* *d.* 1 $7\frac{3}{4}$

$12 \times 12 = 144$

£. 11 17 0

CASE III.

(28) *s.* *d.* 19 $11\frac{1}{4}$

$6 \times 3 + 1 = 19$

£. 18 19 $7\frac{1}{4} = 1 + 18 = 19$

(29) *s.* *d.* 1 $7\frac{1}{2}$

$7 \times 4 + 1 = 29$

£. 2 7 $1\frac{1}{2} = 29$

(30) £. *s.* *d.* 2 17 6

$6 \times 6 + 2 = 38$

£. 109 5 0

(31) *s.* *d.* 5 10

$9 \times 5 + 2 = 47$

£. 13 14 2

(32) s. 10 d. $4\frac{1}{2}$

$8\times7+2=58$

£. 30 1 9

(33) £. 5 s. 17

$8\times8+3=67$

£. 391 19

(34) s. 6 d. $4\frac{1}{2}$

$9\times8+3=75$

£. 23 18 $1\frac{1}{2}$

(35) s. 19 d. $11\frac{3}{4}$

$9\times9+5=86$

£. 85 18 $2\frac{1}{2}$

(36) s. 3

$10\times10+6=106$

£. 15 18

or thus, s. 106

3

£. 15 18

CASE IV.

(38) £. 3 s. 0 d. 6

$7\times5+\frac{1}{2}=35\frac{1}{2}$

£. 107 7 9

(39) £. 1 s. 12 d. 10

$9\times8+4\frac{3}{4}+\frac{1}{4}=76\frac{3}{4}$

£. 125 19 $11\frac{1}{2}$

(40) £. 1 s. 16 d. $6\frac{1}{2}$

$4\times4+1+\frac{1}{4}=17\frac{1}{4}$

£. 31 10 4

(41) £. 4 s. 6 d. 7

$8+\frac{1}{2}=8\frac{1}{2}$

£. 36 15 $11\frac{1}{2}$

(42)

8
4
32
4
128

(42)

£.	s.	d.
26	17	6

$10 \times 10 + \frac{1}{2} = 100\frac{1}{2}$

£. 2700 18 9

(44)

s.	d.
1	$10\frac{1}{2}$

$8 \times 4 \times 4 = 112\frac{1}{2}$

£. 720 0 0

(45)

s.	d.
2	5

$8 \times 7 \times 6 = 336$

£. 40 12 0

(46)

d.
$11\frac{3}{4}$

$10 \times 7 \times 5 = 350$

£. 17 2 $8\frac{1}{2}$

WEIGHTS, MEASURES, &c.

Proceed here in the like Manner with the Product of each Denomination, as you did with the Sum of each Denomination in Addition.

(1)

lb.	oz.	dwt.	grs.
14	10	0	21
			4
59	4	3	12

(2)

T.	C.	qrs.	lb.
17	17	0	24
			2
35	14	1	20

(3)

C.	qrs.	lb.	oz.	drs.
14	0	21	0	14
				7
99	1	7	6	2

(4)

lb.	℥	ʒ	℈	grs.
10	6	4	1	17
				9
94	11	1	1	13

(5)

Yds.	qr.	na.
127	0	3
		12
1526	1	0

(6)

E.E.	qr.	na.
40	4	2
		11
449	4	2

(7)

Lea.	m.	fu.	p.
120	0	7	24
			5
601	1	6	0

(8)

	Yds.	*f.*	*in.*	*b.c.*
(8)	147	2	11	2
				6
	887	2	10	0

	W.hds.	*ga.*	*pts.*
(9)	46	47	7
			3
	140	17	5

	Tu.	*p.*	*hds.*	*g.*	*qt.*
(10)	6	1	1	46	3
					8
	55	0	1	59	0

	Tier.	*g.*	*qts.*
(11)	27	41	2
			6
	167	39	0

	B.hds.	*g.*	*pts.*
(12)	4	47	6
			9
	43	51	6

	A.hds.	*gal.*	*q.*	*p.*
(13)	10	17	3	1
				4
	41	23	2	0

	B.b.	*fi.*	*g.*	*p.*
(14)	12	2	7	7
				6
	76	1	2	2

	A.	*r.*	*p.*
(15)	140	2	29
			5
	703	1	25

	La.	*qr.*	*bu.*	*p.*
(16)	74	7	4	1
				7
	523	2	5	3

	D.	*h.*	*m.*	*sec.*
(17)	365	5	48	57
				12
	4382	21	47	24

QUESTIONS.

(1) First $54\times54=2916$
And $46\times19=\ 874$

Answ. 2042

(2) £. 142
50

An. £. 7100

(3) $12\times12\times6=864=$ Six Dozen Dozen.
$12\times6=\ 72=$ Half a Dozen Dozen.

Answer, $\begin{cases} 792 \text{ Difference,} \\ 936 \text{ Sum.} \end{cases}$

(4)

(4)

$$\begin{array}{r l} 52 & \\ 7\times6=42 & \\ \hline 364 & \\ 6 & \\ \hline 2184 & \text{Parishes.} \\ 246 & \\ \hline 13104 & \\ 52416 & \\ \hline 537264 & \text{Houses.} \\ 10 & \\ \hline 5372640 & \text{Persons.} \end{array}$$

(5)

$$\begin{array}{l r} \text{First } 28\times2 & =56 \\ \text{And } 8\times2+20 & =36 \\ \hline \text{Answ. Diff.} & 20 \end{array}$$

$$\begin{array}{l r} \text{Again } 55\times2= & 110 \\ \text{And } 5\times2+50 & =60 \\ \hline \text{Answ.} & 50 \text{ Diff.} \end{array}$$

(6)

	Years.		£.		£.
The first	4	he cleared	364	per Ann. $=364\times4=$	1456
The next	3	———	586	$=586\times3=$	1758
And the last	3	———	873	$=873\times3=$	2619
				His whole Gain, £.	5833

Then $13000-5833=7167$ £. his original Stock.

To find the State of his Fortune at each Year's End, thus: He begun with 7167 £. cleared the first Year 364 £. which, added to 7167 £. $=7531$ £. his Worth at the first Year's End; then $7531+364=7895$ £. at the second Year's End; and in this Manner proceed by a continual adding the preceding Year's Gain.

(7)

$$\begin{array}{r l} 20 & \text{Remainder.} \\ 423 & \text{Quotient.} \\ 19+ & \\ \hline 462 & \text{Divisor.} \end{array}$$

$$\begin{array}{r r l} & 423 & \text{Quotient.} \\ & \times462 & \text{Divisor.} \\ \hline & 195426 & \text{Prod.} \\ & +20 & \text{Rem.} \\ \hline \text{Ans.} & 195446 & \text{Dividend.} \end{array}$$

(8)

(8) £. s. d. £. s. d.
Each Parish paid 37 16 4 then 373 14 8 assessed.
Hamlets - 28 3 10 343 16 8 paid.
Townships - 19 19 0
Ans. £. 29 18 0 the Desic.
Sum 85 19 2
×4
£. 343 16 8 paid in all.

(9) £. s. d.
Rent 7 7 6 per Quar.
Repair 0 18 6 ———
Tax 0 8 9 ———
Sum 8 14 9 ———
4
An. £. 34 19 0 per Ann.

(10) C. qrs. lb.
C. qr. 137 2 10 Gr. Wt.
13 1×3= 39 3 0 Tare.
Answ. C. 97 3 10 Net W.]

(11) £. s.
1780 0 to the Widow.
1250× 4=5000 0 ——— Children.
31*l.* 10*s.*×5×3(15)= 472 10 ——— Relations.
150 0 ——— Charity.
£. Died worth 7402 10
126×5×5+½(25½)=3213 3 gained by Trade.
Answer, £. 4189 10 begun with.

£. s. d.
(12) 5000 0 0 Widow.
£. 846 10 0 Charity.
1230×3=3690 0 0 Nephews.
1050×4=4200 0 0 Nieces.
5*l.* 5*s.*×5×4(20)= 105 0 0 House-keepers.
200 Guin.= 210 0 0 Executor.
Answ. £. 14051 12 0 died worth.

(13)

(13)

	£.	s.	d.
	100	0	0
	1	7	0
	0	10	6
In each Diviſion was	101	17	6
			×6
In each Drawer was	611	5	0
		×12	0
Anſw. £.	7335	0	0

(14)

		s.	d.	
		19	11	ſpends daily.
				$9\times\overline{8\times5}+5=365$
£.	363	9	7	ſpends per Annum.
	500	0	0	Income.
Anſw. lays up, £.	136	10	5	

(15)

	£.	s.	d.	
	1	12	6	ſpends daily.
				$9\times8\times5+5=365$
	593	2	6	ſpends.
	294	12	6	lays up.
Anſw. £.	887	15	0	per Annum.

(16)

(16) *Yards.*

D.	73726 at one Circumvolution.
365 in the Y.	3
63 played.	221178 in a Minute.
Days 302 worked.	60
	13270680 in an Hour.
	10
	132706800 in Day.
	302 Days worked.
	2654136
	3981204
	Answ. 40077453600 Yards.

(17) First 20 × 60 = 1200 Feet, Piers stand on.
And 21 × 170 = 3570 —— Arches Span.

Width of the Danube, 4770
—————— Thames, 1200

Answ. 3570 Feet the Difference.

(18) 2 × 3 × 4 × 5 × 6 = 720, then 720 + 1 = 721 the Answer, or any Term of the Arithmetical Progression, 301, 721, 1141, &c. See page 358 and 359.

(19) 187 the lesser Number.
187 + 34 = 221 the great Number.
221 × 187 = 41327 their Product.
41327 × 41327 = 1707920929 Square of their Products.
187 + 221 = 408 their Sum, and 408 × 408 = 166464 Square of their Sum.
221 — 187 = 34 their Difference, and 34 × 34 = 1156 Square of their Difference.
And, lastly, 1707920929 + 166464 + 1156 = 1708088549 the Sum of those Squares.

(20)

(20) $109 \times 73 = 7957$ the greater Number.
$28 \times 17 = 476$ Difference.

7481 leſſer Number.

Then $7957 + 7481 = 15438$ their Sum,
And $7957 + 7481 = 59526317$ their Product.

(21)

	Acres.
A. had	757
B.	2104
C.	16410
D.	12881
E.	11008
F.	9813
H.	13800
I.	8818
Wanted,	416
Sum,	$76007 = \frac{1}{5}$ of the Whole.
	5
Anſwer,	380035 Acres.

(22)

One comes up	6 Ways.
	6
Two comes up	36
	6
Three comes up	216
	6
Four comes up	1296 the Anſ.

(23)

	£.	s.	d.		£.	s.	d.
100 *G.* =	105	0	0				
Ten Marks =	6	13	4				
W. had	98	6	8	-	98	6	8
	—0	16	8				
R.	97	10	0	-	97	10	0
	—6	6	0				
S.	91	4	0	-	91	4	0
	+3	17	2				
T.	95	1	2	-	95	1	2
				Anſw. £.	382	1	10

DIVISION.

INTEGERS.

(1) 4)14076893

3519223—1

(2) 12)30742165

2561847—1

(3)

84 { 7)2410296

12) 344328

28694

(4) Quot. Rem.

576)984220649(170869—105

(5)

3029)308763705(101935—2590

Quot. Rem.

(6) 46058)16222121249 9(352190—45479

(7) 127345)51799555(406—97485

(8) 3090807)78855994985(25513—235994

(9) 37,00)12764214,27(344978—2827.

(10) 827,000)4074954,478(4927—3[illegible]5478

(11) 94,00)247698514,00(2635090 $\frac{54}{94}$.

CONTRACTIONS.

When the Divisor consists of the same Figure in all the Places, that is, all Nines, or all Sevens, &c. annex as many Cyphers to Unity or 1, as there are 9's or 7's, &c. in the given Divisor, for a new Divisor, and if the repeating Figure is 9, divide the Dividend by that Divisor, and multiply the integral Quotient by the Difference; do the same with the Product, and so proceed till you get o for an integral Quotient; then add all the Overplusses together, and divide that Sum by the given Divisor, the Overplus thence arising is that required, and the Sum of all the integral Quotients is the Quotient required; for any other Figure divide 9 Times the Dividend so, and the integral Quotient by the repeating Figure, this gives the true integral Quotient; and if the 9th Part of the first Overplus be added to the second, repeated as the given Figure, the Sum will be the true Overplus.

EXAMPLES.

EXAMPLES.

(1) Divide 4677823 by 999.

1,000)4677	823
4	677
	4
4681	1504

Then 1504 the Sum of the Overpluſſes, divided by 999 the Diviſor gives 1, and 505 remains.

So $4681 + 1\frac{505}{999} = 4682\frac{505}{999}$, the Quotient required.

(2) Divide 2692464 by 1111.

First, 2692464
×9

2423	2176
	2423
2423	4599

Then $4599 \div 9 = 511$, the Remainder
So $2423\frac{511}{1111}$ is the Quotient required.

(3) Divide 4769042 by 7777.

First, 4769042
×9

1000)4292	1378
	4292
7)4292	5670
631—1	

Then $5670 \div 9 = 630$, and $630 + 1111 = 1741$ the Remainder; ſo the Quotient is $613\frac{1741}{7777}$.

COMPOUND DIVISION.

MONEY.

CASE I.

Here, in the 3d Example, I divide by 8, ſaying the 8's in 17 is 2 Times and 1 over, the 8's in 11 is 1 and 3 over, which is 3£. this 3£. I carry to the 11*s*. and it is 3£. 11*s*. or

or 71*s.*; then I say the 8's in 71 is 8 Times 8 is 64, and 7*s.* over, which I carry to the 4 Pence, and it is 7*s.* 4*d.* or 88 Pence; then I say the 8's in 88 is 11 Times, and Nothing over, and so the Quotient is £. 21 8*s.* 11*d.*; in the same Manner proceed with all the Rest.

	£.	s.	d.		£.	s.	d.		£.	s.	d.
(1) Qu.	7	8	5¼	(2)	38	2	10½	(3)	21	8	11

	£.	s.	d.		£.	s.	d.		£.	s.	d.
(4) Quot.	35	4	6	(5)	0	11	6½	(7)	3)7	11	6
									2	10	6

	£.	s.	d.		£.	s.	d.
(8)	10)3	17	1	(9)	12)14	14	0
Quot. £.	0	7	8½	£.	1	4	6

CASE II.

(2) 24 { 4)3 13 — 6)0 18 3 } — £. 0 3 0½

(3) 36 { 6)64 19 — 6)10 16 6 } — £. 1 16 1

(4) 42 { 6)190 4 6 — 7) 31 14 1 } — £. 4 10 7

(5) 48 { 6)37 14 8 — 8) 6 5 9¼ } — £. 0 15 8½

(6) 32 { 4)78 16 8 — 8)19 14 2 } — £. 2 9 3½

(7) 55 { 5)4567 0 10 — 11) 913 8 2 } — £. 83 0 8½

(8) 63 { 7)264 12 — 9) 37 16 } — £. 4 4

72 { 8)18 18 — 9) 2 7 3 } — £. 0 5 3

(10)

(10) £. s. d.
81 { 9) 121 12 6
9) 13 10 $3\frac{1}{4}$
1 10 0

(11) £. s. d.
120 { 10) 174 1 8
12) 17 8 2
1 9 $0\frac{2}{12}$

CASE III.

(13) £. s. d.
26)83 17)3 4 6
78
5
20
26)117(4*s.*
104
13
12
26)156(6*d.*
156
...
Answ. £. 3 4 6

(14) £. £. s. d.
145)467(3 4 $4\frac{3}{4}$ Answer.
435
Rem. 32£.
20
145)640(4*s.*
580
Rem. 60*d.*
12
145)720(4*d.*
580
Rem. 140 *d.*
4
145)560($\frac{3}{4}$
435
Rem. 125 *qrs.*

(15) £. s.
112 { 4) 30 2
4) 7 10 6
7) 1 17 $7\frac{1}{2}$
£. 0 5 $4\frac{1}{2}$

(16) £.
350 { 5) 1009
7) 201 16
10) 28 16 $6\frac{3}{4}$
£. 2 17 $7\frac{1}{4}$

(17) £. s.

224 { 4)610 8
7)152 12
8) 21 16

£. 2 14 6

(18) £.

640 { 8)1426
8) 178 5
10) 22 7 7

£. 2 4 $6\frac{3}{4}$

(20) £. s. d.

$6\frac{1}{4}$
4
25

{ 5)12 12 11
5) 2 10 7

0 10 $1\frac{3}{4}$ per Qr.
4

£. 2 0 5

(21) £. s. d.

$10\frac{1}{2}$
2
21

{ 3)1061 8 $9\frac{3}{4}$
7) 353 16 $3\frac{1}{4}$

50 10 $10\frac{1}{2}$
2

£. 101 1 $9\frac{1}{2}$

(22) £. s. d.

$8\frac{3}{4}$
4
35

{ 5)450 13 $11\frac{1}{2}$
7) 90 3 9

12 17 $6\frac{1}{2}$
4

£. 51 10 2

WEIGHTS, MEASURES, &c.

	lb.	*oz.*	*dwt.*	*grs.*
(1)	2)8	1	5	8
Quot.	4	0	12	16

	T.	*C.*	*qrs.*	*lb.*
(2)	3)24	14	0	14
	8	4	2	$23\frac{1}{2}$

	C.	*qr.*	*lb.*	*oz.*	*drs.*
3)	4)17	2	27	14	15
Quot.	4	1	20	15	$11\frac{3}{4}$

	lb.	℥	ʒ	℈	*grs.*
(4)	5)4	11	4	2	12
	0	11	7	1	$2\frac{2}{5}$

	Yds.	*qr.*	*na.*
(5)	9)214	3	2
Quot.	23	3	2

	E.E.	*qr.*
(6)	8)120	4
	15.0	0

	Lea.	*m.*	*fu.*	*p.*
(7)	9)12	2	0	26
	1	1	1	3

	Yds. f. in. b. c.		*W. hds. ga.*		*Tu. p. hd. g. qt.*
(8)	10)147 2 11 2	(9)	11)24 57	(10)	8)10 1 1 60 3
	14 2 4 $2\frac{3}{10}$		2 16 $5\frac{1}{11}$		1 0 1 13 $1\frac{3}{8}$

	Tier. g. pts.		*A. hds. gal.*		*B. hds. g. qts.*
(11)	6)16 20 7	(12)	5)76 27	(13)	4)12 49 2
	Quot. 2 31 $5\frac{5}{6}$		15 15		3 12 $1\frac{1}{2}$

	B. b. fi. g.		*A. r. p.*		*La. qr. bu. p.*
(14)	3)61 2 6	(15)	12)140 2 26	(16)	7)60 6 7 2
	Quot. 20 2 2		11 2 $35\frac{1}{2}$		8 6 5 $2\frac{4}{7}$

D. h. m. sec.

(17) 6)146 23 24 56

Quot. 24 11 54 $9\frac{1}{3}$

QUESTIONS.

	£.		*M.*
(1)	1,0000)22,0000	(2)	12)336
	Answ. 22 £. each.		Answ. 28 Miles per Day.

(3) First 4429÷43=103, then 240−103=137 the Answ.

(4) First 2262÷26=87, and 2262÷87=26

Then 87−26=61 the Number required.

(5) 5190048÷72084=72 the Number required.

(6) First 419844÷3=139948 the Remainder.

And 419844×9494=3985998936 Pro. of Div. and Quot.

Then 3985998936+139948=3986138884 the Answer.

(7) First 360−144=216 the greater Number.

And 216−144=72 their Difference.

Also 216×144=31104 Product.

Likewise 216÷144=$1\frac{72}{144}$ or $1\frac{1}{2}$ the larger Quote.

(8)

(8) First 3 Tons $\times 20 = 60$ Cwt. then $60 \div 15 = 4$ Cwt. per Man, the Answer.

(9) First 25 $\left\{ \begin{array}{l} 5)22525 \\ \hline 5)\ 4505 \\ \hline \end{array} \right.$ and 16 $\left\{ \begin{array}{l} 4)9696 \\ \hline 4)2424 \\ \hline \end{array} \right.$

25th Part $= 901$ 16th Part $= 606$

Then $1440 + 901 = 2341$, and $2341 - 606 = 1735$ the Answ.

(10) First $134 \times 71 = 9514$, consequently $9514 \div 57 = 166\frac{52}{57}$ the Answer.

(11) Fourscore and thirteen Millions $= 93000000$. Then $93000000 \div 30074 = 3091$, and the Remainder is 25811, from which deduct 21180 leaves 4631 excess the Answer.

(12)

2,0)1640 Guineas.			£.	s.
+82				
£. 1722	-	-	1722	0 College.
+4				
£. 6888	-	-	6888	0 Wife.
—1383				
2)5505	-	-	5505	0 Daughter.
+2752 10				
£. 8257 10		-	8257	10 Eldest Son.
2)12393 0 Sum of the Mo. & Sist.				
£. 6196 10	-	-	6196	10 Youngest Son.
			988	10 Expences on his Burial.
		£.	29557	10 died worth.

Then from £. 30000 take £. 29557 10 leaves £. 442 10s. the Answer.

(13)

(13)

	£.	s.	
	8)12	8	Worth of both.
	1	7	Value of the Purse.
Answ.	£. 11	1	Cash in the Purse.

(14)	£.	s.	d.	£.	s.	d.	
Brother owed at first,	74	18	2				
Paid in Part, - -	—41	14	8				
Remains on Balance,	2)33	3	6	33	3	6	Bro.
Half of which is	+16	11	9				
Sister owed at first,	49	15	3				
Paid in Part, -	—13	12	10				
Remains on Balance,	36	2	5	36	2	5	Sist.
To which add the Brother's,	33	3	6				
Uncle William owed,	69	5	11				
Paid in Part, -	—24	7	3				
Rem. on B. unpaid by him,	44	18	8	44	18	8	Uncle.
All together owed, - -			£.	114	4	7	

Then from 150 £. take £. 114 4 7 leaves £. 35 15 5

(15	C.	qrs.	lb.			£.	s.	d.
Weight together,	9	3	16		Cost,	97	17	6
Difference,	—1	2	16		Diff.	8	13	3
	2)8	1	0	Rem.		2)89	4	3
Lesser Weight,	4	0	14		Cost, £.	44	12	1½
Difference,	+1	2	16		Diff.	8	13	3
Greoter Weight,	5	3	2		Cost, £.	53	5	4½

(16)

(16) £. £.

In Cash,	-	10000		Burial,	30
Bills, £. 54 10 6×8=		436	4	Debts,	260
Left in all,	£.	10436	4	£.	290
From which take		290	0 Rem.		
	9)	10146	4 0		
Daughter had,	£.	1127	7 $1\frac{1}{3}$		
	7)	9018	16 $10\frac{1}{3}$ Rem.		
Sons had each,	£.	1288	8 $1\frac{13}{27}$		

(17) 14 { 2)2072(; 7)1036 }

148 Elms in each Row, then 148—1=147 Vacancies, and 147×25=3675 Feet, which ÷3=1225 Yards, the Length of the Grove required.

(18) 32 { 4)384 ; 8) 96 }

Ans. in Rank 12 Men.

(19) First 129+178=307
Then 1000—307=693
And 693÷ 3=231 B's
Also 231+129=360 A's
Likewise 231+178=409 C's } Crowns

(20) First 37—28=9, then 250—9=241.
∴3)241

gives $80\frac{1}{3}$ B.'s
And 80+37=$117\frac{1}{3}$ A.'s } £.
Also 80—28= $52\frac{1}{3}$ C.'s

(21) 207+213+189+234+222+250=1315 the Sum, which being divided by 5, the Times each being mentioned, viz. - - - 5)1315

Quotient, - - -	263
To which add Four-score and 13=	93
Sum, - - - - - -	356
From which deduct - -	1
Leaves, Answer, - - -	355

(22)

(22) £.

First 10000—850 =1500 O.'s - - - -
And 8500—6050=2450 L.'s - - -
Then 6050—420=5630, and 5630÷2=2815M.'s
Consequently 2815+420=3235 N.'s - -
} Subscription.

(23) 46)1610(35 the lesser N°. then 46+35=81 their Sum, and 46—35=11 their Difference; also 46÷35 $=1\frac{11}{35}$ their Quotient.

Then 46×46=2116 the Square of greater Number.
And 35×35=1225 ———— lesser.

3341 Sum of those Squares.

Again, 11×11×11=1331 the Cube of their Difference.

(24)

First 7050÷ 94=75 the lesser Number.
And 7050×7050=49702500 the Square of the greater.
Also 75 ×75 =5625 Ditto of the lesser.

49696875 Difference of those Squares.

Again 7050+75=7125 their Sum, and 7050—75=6975 the Difference; then 7125×6975=49696875 the Product of their Sum, and Difference.
Consequently 49696875×49696875 = 2469779384765625 the Square of their Product, of their Sum and Difference.

(25) Suppose the Expence or Profit to be 2.
Then 2×2=4 Double the Expence.
And 2—2=1 Half the Profit.
Answer, Difference 3, or as 4 to 1.

(26) First B. is to have 72 } more than A.
And C. 72+112=184 }

Sum, 256

Then

Then $1500-256=1244$.

$\because \quad 3)1244$

gives	$414\frac{2}{3}$ A.'s	Share.
And $414\frac{2}{3}+72$	$=486\frac{2}{3}$ B.'s	
Also $486\frac{2}{3}+112$	$=598\frac{2}{3}$ C.'s	

(27) $25000+33000+30000+28000+32000=148000$ their Sum; then each being repeated four Times, that Sum must be divided by 4, viz.

$4)148000$

Which gives 37000£. Sum of their Fortune, then

From	Take	Rem.	
£. 37000	25000=	12000£.	the Youngest's Fortune.
	33000=	4000	—— Eldest's.
	30000=	7000	—— Second's.
	28000=	9000	—— Fourth's.
	32000=	5000	Miss Charlotte's Fortune.
	Proof	37000£.	

(28)

	d.	s.	d.	
First 120 at 2 for 1*d*. $=120\div2=$	60 or	5	0	
And 120 at 3 —— $=120\div3=$	40 —	3	4	
		8	4	Cost.
Then 240 at 5 for 2*d*. $=240\div5=48$ Two-pences, or -		8	0	Sold for.
	Difference	0	4	Lost.

(29)

	£.	s.	d.
A. and B. had	13	10	0
B. and C.	12	12	0
A. and C.	11	16	6
Sum, £.	37	18	6

Which being divided by the Number of Players at each Time will give the Sum won, viz.

£.

viz. £. 37 18 6÷6=18 19 3 what was won.

		£. s. d.		£. s. d.		
		13 10 0		5 9 3	C.'s	
Then from £. s. d. 18 19 3.	Take	12 12 0	Rem.	6 7 3	A.'s	Gain.
		11 16 6		7 3 9	B.'s	

		£.	s.	d.
(30)	W. X. and Y. advanced	350	10	
	W. X. and Z. -	344	10	
	X. Y. and Z. -	400	0	
	W. Y. and Z. -	378	4	
	They being mentioned each 3 Times, - -	3)1473	4 Sum.	
		£. 491	1	4 joint Property.
	Sold for 450 Guineas, or	472	10	0
	Anſwer, Loſt £.	18	11	4

		£.	s.	d.
(31)	Worth at the End of $3\frac{1}{2}$ Years,	3179	11	8
	4)100£.=	25	0	0
	$3\frac{1}{4}$ Year =13 Quarters,	13)3204	11	8
		— 246	10	$1\frac{1}{2}$
	Worth at the End of 3 Years £.	2958	—	$6\frac{1}{2}$
		+ 100	0	0
		4)3058	—	$6\frac{1}{2}$
		+ 764	10	$4\frac{1}{2}$
	Worth at the end of 2 Years, £.	2293	11	2
		+ 100	0	0
		4)2393	11	2
		— 598	7	$9\frac{1}{2}$
	Worth at 1 Year's End, £.	1795	3	$4\frac{1}{2}$
		+ 100	0	0

Brought over	4)1895	3	$4\frac{1}{2}$
	— 473	15	10
Answer, He began with,	£. 1421	7	$6\frac{1}{2}$

(32) £. £.
First, $50 \div 5 = 2$ each Man's Share, supposing they had attended equally.

And 2£. $\div 7 =$ 5*s*. $8\frac{4}{7}$*d*. each Man's daily Pay.

Then 5*s*. $8\frac{4}{7}$*d*. $\times 2 =$ 11*s*. $5\frac{1}{7}$*d*. what C. and D. must each forfeit.

$\therefore$ 11*s*. $5\frac{1}{7}$*d*. $\times 2 =$ £. 1 2*s*. $10\frac{2}{7}$*d*. C and D.'s whole Defaults.

And £. 1 2*s*. $10\frac{2}{7}$*d*. $\div 3 =$ 7*s*. $7\frac{3}{7}$*d*. what A. B. and E each received of C. and D.'s Default.

Also 5*s*. $8\frac{4}{7}$*d*. $\div 4 =$ 1*s*. $5\frac{1}{7}$*d*. what A. B. C. and D. each received of E.'s Default.

		£.	s.	d.		
Therefore,	2£. + 7*s*. $7\frac{3}{7}$*d*. + 1*s*. $5\frac{1}{7}$*d*. =	2	9	$0\frac{4}{7}$	A.	Received.
	Ditto, - - -	2	9	$0\frac{4}{7}$	B.	
	2£. + 1*s*. $5\frac{1}{7}$*d*. less 11*s*. $5\frac{1}{7}$*d* =	1	10	0	C.	
	Ditto, - - -	1	10	0	D	
	Also, 2£. + 7*s*. $7\frac{3}{7}$*d*. less 5*s*. $8\frac{6}{7}$*d*. =	2	1	$10\frac{4}{7}$	E.	

(33) First $12 - 1 = 11$, and $11 \times 2 = 22$ she had before she met the last Boy; then $22 - 2 + 10 = 30$ she had before she met the second Boy; consequently $30 - 10 = 20$ what she had before the first returned her back 10; so that 20 being multiplied by $2 = 40$, the Number of Apples she had at first.

SECT. II. REDUCTION.

MONEY.

(3)

£.	s.	d.
80	15	$11\frac{1}{2}$
20		
1615		
12		
19391		
4		
77567 Farthings.		

(4)

qrs.

4)16921

12)$4230\frac{1}{4}$

2,0)35,2 6

£. 17 12 $6\frac{1}{4}$

£.

(5)
£. s. d.
110 0 6½
20
——
2200
12
——
26406
2
——
52813 Half-pence.

(6) 2)20553 Half-pence.
12)10276½
2,0)85,6—4
£. 42 16 4½

(7)
£. s. d.
107 10 8
20
——
2150
6= Two pences =1*s*.
——
12904 Two-pences.

(8) 6)5348 Two-pences.
2,0)89,1 2 over =4*d*.
£. 44 11 4

(9)
£. s.
6 17
20
——
137
4 Three pences =1*s*.
——
548 Three-pences.

(10) 4)2782 Three-pences.
2,0)69,5—2 or 6*d*.
£. 34 15 6

(11)
£. s. d.
10 10 8
20
——
210
3
——
632 Four-pences.

(12) 3)3859 Four-pences.
2,0)128,6 4
£. 64 6 4

(13) £. s. 200 17
20

4017
2

8034 Six-pences.

(14) 2)705 Six-pences.
2,0)35,2—6
£. 17 12 6

Any Number of Pounds, &c may be reduced to Six-pences, Four-pences, Three-pences, or Two-pences, by multiplying the Pounds by as many of each Denomination that make one Pound; that is, to reduce Pounds, &c. to Six-pences, multiply by 40, if to Four-pences by 60, if to Three-pences by 80, and if to Two-pences by 120, observing to add in the odd Shillings and Pence, if any; accordingly, on the contrary, any Number of Six-pences, Four-pences, &c. may be reduced to Pounds, by dividing by as many as make one Pound, observing to value the Remainder (if any) right; thus, the two last Examples.

(13) £. s.
200 17
40+17 s.=34 Six-pences.

8034 Six pences.

(14) *Six-pences.* *s. d.*
4,0)705 and 25 over = 12 6
£. 17 12

(15) 21
$7 \times 3 = 21$

147
3
---
441 Shillings.
12

5292 Pence.
4

21168 Farthings.

(16) 4)24192
12)6048
21 { 3)504
7)168
24 Guineas.

(17) 12 Moidores.
27
324
12
3888
4
15552 Farthings.

(18) 12)3240
27)270
10 Moidores.

(19) 30 £.
4
120 Crowns.
2
240 Half-Crowns.
30
7200 Pence.

(20) 3,0)2016,0 Pence.
2)672 Half-Crowns.
4)336 Crowns.
84 £.

(21) 25 Crowns.
5
125 Shillings.
3
375 Groats.
4
1500 Pence.

(22) 4)25200 Pence.
3)6300 Groats.
5)2100 Shillings.
420 Crowns.

(23) 25 £.
20
5)500 Shillings.
100 Crowns.
60
6000 Pence.

(24) 80 Crowns.
5
2,0)40,0 Shillings.
20 £.

(25)

	d.
A Crown	=60
Half dit.	=30
Shilling	=12
One of each	102

£.	s.	d.
213	15	6
20		
4275		
12		

102)51306(503 of each.
510
306
306
...

(26)

£.
120
8
2)960 Half Crowns.
480 Crowns.
15
3)7200 Groats.
2400 Shillings.

(27)

	s.	d.
A Crown	=5	0
Half dit.	=2	6
Shilling	=1	0
Groat	=0	4
Sum, 1 of each,	=8	10

6×6=36

	2	13	—
		6	
£.	15	18	—

(29)

Guin.
240
7×3=21
1680
3
5)5040 Shillings.
4)1008 Crowns.
252 £.

(30)

	£.	s.
21 Guineas	=22	1
A Crown	= 0	5
A Moidore	= 1	7
In each Purse	23	13
		7
7×3=21	165	11
		3
Answer, £.	496	13

(33)

First 17*s*. 6*d*.=35 Six-pences.
And 1£.=40 ditto.

Pist.
Then 470
7×5=35

3290
5

4,0)1645,0

Answ. £. 411 5*s*.

(34)

First 10*s*. 6*d*.=21 Six-pences
Then 270 Half-Guin.
7×3=21

1890
3

4,0)567,0 Six-pences.

£. 141 15*s*.

Or thus: 10*s*. 6*d*. ×10×9×3(270) will give the Answer.

(35)

First 6*s*. 4*d*.=19 Four-pences.

Ducatoons.
Then 427
19

3)8113 Four-pences.

2,0)27,0 4*s*. 4*d*.

Answer, £. 135 4 4

(36)

First 5*s*. 3*d*.=21 Three-pences.

Qr. Guin.
Then 2740
7×3=21

19180
3

4)57540 Three-pences.

2,0)1438,5

Answ. £. 719 5*s*.

(38)

First 3*s*. 2*d*.=19 Two-pences.

£. *s*. *d*.
387 18 4
20

7758
6

19)46550

2450 Florins.

(39)

First 13*s*. 4*d*.=40 Three-pences.

£. *s*. *d*.
Then 496 18 4
20

9938
3

4,0)2981,5 Three-pences.

745 Mar. 5*s*.

(40)

(40)

First 4*s*. 3*d*.=17 Three-pences.

```
£.  s. d.
262 8  9
 20
----
5248
   4
-----
17)20995
-----
1235 Dollars, Answer.
```

(41)

First 18*s*. 6*d*.=37 Six-pences.

```
     £.
Then 247
      40
   -----
37)9880
   -----
   267 Pist. and 6d. over.
```

(43) *Moidores.*

```
  1240
     9×3=27
  -----
  11160
      3
  -----
   { 3)33480 Shillings.
21 {   -----
   { 7)11160
       -----
Answ.  1594 Guin. 6s.
```

(44)

First 5*s*. 6½*d*.=133 Half-pence.
And 4*s*. 7*d*. =110 ditto.

```
      Cobs.
Then  476
      110
     -----
133)52360
    -----
    393 Duc. & 3s. 9½d.
```

WEIGHTS, MEASURES, &c.

(1) *lb*.

```
 14
 12
---
168 Ounces.
 20
----
3360
  6×4=24
-----
20160
    4
-----
80640 Grains.
```

(2)

```
   { 4)138240
24 {   ------
   { 6) 34560
        -----
   2,0)576,0
       -----
    12)288
       ---
       24 lb.
```

(3)

(2) *lb. oz. dwt. grs.*
19 10 17 22
12

238
20

4777
$6\times4=24$

28662
4

114670 Grains.

(4) *grs.*
$$24\begin{cases}4)74342\\6)18585-2\end{cases}$$
2,0)309,7—3 } 14 grs.
12)154—17 dwts.

Answ. *lb.* 12 10 17 14

(5) *lb. oz. dwt. grs.*
4 6 0 22
12

54
20

1080
$6\times4=24$

6480
4

25942 grs. = 1 Ingot.
4

A. 103768

(6) *lb. oz. dwts.*
6 11 14
12

83
20

1647
$6\times4=24$

10044
4

40176)251056

6 Ingots.

(7)

	lb.	*oz.*	*dwt.*	*grs.*
One Dozen of Dishes, each Weight	2	1	15	0
Ditto ——— Plates, ————	1	3	15	22
One of each, - - -	3	5	10	22
				12
Answer, *lb.*	41	6	11	0

(8)

(8)

	oz.	*dwt.*	*grs.*
Bowls, each	24	4	0
Tankards, -	11	14	0
Tea Pots, -	10	10	0
Lamps, - -	20	17	21
Pl. per Dozen,	127	11	0
Spoons ditto,	36	17	23
One of each,	231	14	20

20
4634
6×4=24
27804
4
111236)218440(1 of each.

oz.	*dwt.*	*grs.*
455	1	16

20
9101
6×4=24
54606
4
218440 grs.

Rem, 107204 *grs.* = 223 *oz.* 6 *dwts.* 20 *grs.*
Answer 1 of each Sort, and 223 *oz.* 6 *dwts.* 20 *grs.* over.

(9)

lb.	℥	ʒ	℈	*grs.*
4	10	4	1	12

12
58
8
468
3
1405
20
28112 Grains.

(10)

grs.
2,0)5993,4
3)2996—14 grs.
8)998—2℈.
12)124—6ʒ
lb. 10 4 6 2 14

(11)

(11) 6 Tons.
20
120 Cwts.
4
480 Qrs.
$7 \times 4 = 28$
3360
4
13440 *lb.*

(12) 28 { 4) 26880 *lb.*
7) 6720 }
4) 960
2,0) 24,0
12 Tons.

(13) C. *qrs.* *lb.* *oz.* *drs.*
74 2 16 0 7
4
298
$7 \times 4 = 28$
2086
4
8360
16
133760
16
2140167 drs.

(14) *oz.*
16 { 4) 29768
4) 7442 } 8 oz.
28 { 4) 1860—2
7) 465 } 12 lb.
4) 66—3
C. 16 2 12 8

(15) *lb.* *oz.* *drs.*
67 12 15
16
1084
16
17359 drs.

(16) C. *qrs.* *lb.*
6 3 27
4
27
$7 \times 4 = 28$
189
4
783 lbs. in 1 Hhd.
6
Anſwer 4698 lb.

(17)

(17) *C. qrs.*
11 2
4
—
46
7 × 4 = 28
—
322
4
— *lb.* *Hds.*
1288)12880(10 Answ.
—

(18) *C.*
A Fother = $19\frac{1}{2}$
× 2
—
39

507
2
—
1014(26 Fother.
—
234
—
...

(19) *C. qrs. lb.*
12 3 12
4
—
51
7 × 4 = 28
—
357
4
—
12)1440
—
Ans. 120 Canisters.
—

(20) *lb.*
$126\frac{1}{2}$
2
—
253

C. qrs.
8 3
4
—
35
7 × 4 = 28
—
245
4
—
980 lb.
2
—
253)1960(7 Parcels, and
Remains 189 half-lbs. = 3 qrs. $10\frac{1}{2}$ lb. over.

(21) 6 + 8 + 12 + 16 = 42 *lb.*
C. qrs. lb.
4 3 24
4
—
19
7 × 4 = 28
—
133
4
—
42 { 6)556
—
7) 92 — 4
—
31 — 1 } 10 lb.
Answ. 12 of each, and 10 lb. over.

(22) *lb.*
24 great.
3
—
2)72
—
36 lb. common.
—

(23) *lb.*
120
2
—
3)240
—
80 lb. great.

(24) *Yds. qrs.*
27 3
4
111
4
444 Nails.

(25) *Nails.*
4)352
4)88
22 Yards.

(26) *E.Eng.qrs.na.*
30 4 3
5
154
4
619 Nails.

(27)
4)569
3)142—1
E. Fl. 47 1 1

(28) *Yards.*
24
4
96
4
384 Na. = 1 Piece.
14
Anſ. 5376 Nails.

(9) *Nails.*
4)24768
4)6192
12)1548
Anſ. 129 Pieces.

(30) *E. Fl.*
20 each.
3
60 qrs. in 1 Piece.
12
5)720
144 Ells Engliſh.

(31) *Yards.*
24 227
3 4
72 { 8)908
9)113—4 }
12—5 } 44 Qrs.
Anſ. 12 Ps. 11 Yards.

(32) 24 *E. Fl.*
3
72 Qrs. in 1 Ps.
12
864 Qrs. in 12 Pieces.
4
5)3456 Qrs. in 4 Bales.
E. Eng. 691 1 Qr.

(43) 60 *Miles.*
8
480 Furlongs.
40
19200 Poles.

(34) *Poles.*
4,0)1280,0
8)320
Answ. 40 Miles.

(35) *Miles.*
16
1760 Yards in 1 Mile.
28160 Yards.
3
84480 Feet.
12
1013760 Inches.
3
3041280 Barley Corns.

(36) *B. C.*
3)2280960
12)760320
3)63360
176,0)2112,0(12 Miles.
352
. . .

(37) *Miles.*
276
1760
1656
4992
485760 Yards.
6 x 6 = 36.
2914560
6
17487360 Inches.
3
52462080 B. C.

(38) *Feet.*

First $18\frac{1}{2}=37$ Half-Ft.

Miles.

$$\begin{array}{r} 197 \\ 1760 \\ \hline 1182 \\ 3349 \\ \hline 346720 \text{ Yards.} \\ 6 \\ \hline 2080320 \text{ Half Ft.} \end{array}$$

which divided by $37=56224\frac{32}{37}$ Times, the Answer.

(39) *M. fur.*

$$\begin{array}{r} 69\ 4 \\ 8 \\ \hline 556 \\ \hline 6116 \\ 20 \\ \hline 122320 \text{ Yards in } 1^\circ \\ 6\times6=36 \\ \hline 733920 \\ 6 \\ \hline 4403520 \text{ Inches in } 1^\circ \\ 3 \\ \hline 13210560 \text{ Bar. C. in } 1^\circ \\ 6\times60=360^\circ \\ \hline 79263360 \\ 60 \\ \hline \end{array}$$

[$=1$ Fur. $11\times20=220$ Yards

Answ. 4755801600 Barley-Corns.

(40) *A.*

$$\begin{array}{r} 64 \\ 4 \\ \hline 256 \text{ Roods.} \\ 40 \\ \hline 10240 \text{ Poles.} \end{array}$$

(41) *P.*

$$\begin{array}{r} 4,0)2176,0 \\ \hline 4)544 \\ \hline \end{array}$$

Anf. 136 Acres.

(42) *A.*

```
        774
          4
       ----
       3096
        4,0
      -----
    { 3,0)12384,0(
270 {   ---------
    {   9)4128
         ----
```

Anſwer, 458 and 180 Poles over.

(43) *A.* *r.*

```
Rents  200  0
Tills   96  2
     --------
Rem.   103  2
         4
      ----
       414
        40
      ----
```

Anſ. 16560 Perches.

(44) *Tierce.*

```
   12
   42
  ---
  504 Gal.
    8
 ----
 4032 Pts.
 ----
```

(45) *Pts.*

```
     8)6048
      -----
   { 6) 756
42 {   -----
   { 7) 126
       -----
         18 Tierce.
```

(46) *Hhd. gal. qts.*

```
     4  42  2
    63
   ---
   294
     4
  ----
```

Anſ. 1178 Qts.

(47) *Pts.*

```
     8)5746
      -----
   { 7)718—2 Pts.
63 {   ---
   { 9)102—4 }
        ---    } 25 Gal.
         11—3 }
```

Anſw. 11 hhds. 25 gal. 1. qt.

(48) *Gal.*

```
A Pipe=126
         4
      ----
   12)504
      ----
```

Anſw. 42 Doz.

(49)

```
A Quart=4 Half-Pts.    Tun=252 Gal.
A Pint =2 ————             4
Half P. =1 ————        -------
        ---            1008 Qts.
         7                 4
                       -------
                       7)4032
                       -------
```

Anſ. 576 of each.

(50)

(50) *Gal.*

A Pipe = 126
A Punch. = 84
A Hogsh. = 63
A Tierce = 42

Sum, 315)1890(6 of each.
....

(51) *A. Bar.*

12
32

384 Gal.
8

3072 Pints.

(52)

8)1704

32)213(6 Bar. 21 Gal.

Rem. 21 Gal.

(53) *B.Bar.fir.gal.*

6 2 7
4

26
9

Answ. 241 Gal.

(54) *A.hds.gal.pts.*

10 42 4
48

522
8

4180 Pints.

(55) *Qts.*

4)2017

48 { 6) 504—1 Qt.
8) 84

Hhds. 10 24 1

(56) *Hhds.*

12
3

2)36 Half Barrels.

Answ. 18 Barrels.

(57) *Bar.*

18
2

3)36 Half Barrels.

Answ. 12 Hhds.

(58) *T. b. bds. gal.*
Gal. 4 1 1 49
A Hhd = 54 2
A Bar. = 36
A Fir. = 9 9
2
Sum, 99
19
54

99)1075(10 of each, and 85 Gal. over.
Rem. 85 Gal.

(59) *qrs.*
24
8
192 Bush.
4
768 Pecks.
2
1536

(60) *Gal.*
2)3360
4)1680
8)420
Answ. 52 qrs. 4 bu.

(61) *La. qrs. bu.*
42 4 7
10
424 Qrs.
8
3399 Bushels.

(62) *Cha.*
40
36
1440 Bush.
4
Answ. 5760 Pecks

(63) *Cha.*
36 { 6)4762
6) 793—4 }
132—1 } 10 Pks.
Cha. 132 10

(64) *Cha. bu.*
47 30
6×6=36
282
6
3)1722
Answ. 574 Sacks.

(65) *Sacks.*
12)6450
Answ. 537 Cha. 6 Sacks.

(66)

(66) *D.* *H.*
365 6
6×4=24

2190
4

8766
60

Anſw. 525960 Minutes.

(67) *Sec.*
6,0)207360,0

6,0)3456,0

$24\left\{\begin{array}{l}4)576\\6)144\end{array}\right.$

Anſw. 24 Days.

(68) *D.* *h.* *m.* *ſec.*
27 7 43 5
6×4=24

162
4

655
60

39343
60

Anſ. 2360585 Seconds.

(69) *D.* *h.* *m.* *ſec.* '''
365 5 48 57 39
6×4=24

2190
4

8765
60

525948
60

31556937
60

Anſwer, 1893416259 Thirds.

(70) 6,0)3155760,0

6,0) 52596,0

$24\left\{\begin{array}{l}4)8766\\6)2191-2\end{array}\right.$

$\left.\begin{array}{l}2191-2\\365-1\end{array}\right\}$ 6 Hours.

Anſw. 365 D. 6 H.

(71) *D. h.*

365 6
$6 \times 4 = 24$

```
      365   6
            6×4=24
      ----
      2190
         4
      ----
      8766 Hours in 1 Year.
      1772 Years since.
      ----
     17532
    61362
  149022
  --------
24 { 4)15533352 Hours in 1772 Years.
   {   --------
   { 6) 3883338
        -------
Answ.  647223 Days since.
       ------
```

(27) *Years.*

```
London built 1108 before the birth of our Saviour.
             1772 since ————————————————
             ----                     Hours.
In all,      2880   Then 365 d. 6 h.=8766 by last Ex.
             ----                    2880
                                    -----
                                    70128
                                   70128
                                  17532
                                  --------
                             24 { 4)25246080
                                {   --------
                                { 6) 6311520
                                     -------
                         Answer, 1051920 Days.
                                 -------
```

12. THE RULE OF THREE DIRECT.

(2) lb. d. lb.

St. thus—If 2 : $18\frac{1}{2}$: : 24

```
      4
    ---
     74
      6×4=24
    ---
    444
      4
  -----
2)1776
  -----
 4)888
   ---
12)222
   ---
Answ. 18s. 6d.
```

lb. s. d. lb.

If 24 : 18 6 : : 2

```
          12
         ---
         222
           2
         ---
   ( 4)444
24 {     ---
   ( 6)111
       -----
Proof, 18½ d.
```

Or whenever the first and second Terms consist of any Numbers that are in the Multiplication Table; then the most expeditious Way will be to multiply and divide as in Sect. 9. and 10.—Thus the above Example will be performed as follows.

lb. s. d. lb.

If 2 : 1 $6\frac{1}{2}$: : 24

```
        6×4=24
     ------
      9   3
          4
  ----------
 2)1  17  0
  ----------
Ans. £. 0  18  6
```

lb. s. d. lb.

If 24 : 18 6 : : 2

```
                 2
     ------------
   ( 4)1  17  0
24 {   ---------
   ( 6)0   9  3
     ------------
Proof,   1s. 6½d.
```

But in teaching, I would advise the Tutor to follow the general Rule, in which to make his Pupil quite perfect; after which the Pupil will, with more Ease, both to himself and Master, learn the Abbreviations.

(3)

lb. *d.* *doz.* *lb.*

(3) Stated thus—If 4 : 30 :: 6 or 72

Then 72 × 30 = 2160, this ÷ 4 = 540 Pence, which ÷ by 12 and 20, will give £. 2 5*s.* the Answer.

C. *s.* *C.*

(4) Stated thus—If 1 : 26 :: 40

$$\begin{array}{r} 40 \\ \hline 2{,}0)104{,}0 \\ \hline \text{Answ. } 52\,£. \\ \hline \end{array}$$

oz. *d.* *C.* *lb.*

(5) Sated thus—If 1 : $7\frac{1}{2}$ :: 1 or 112

Then by Reduction $7\frac{1}{2}$*d.* = 15 Half-pence, and 112*lb.* = 1792 *oz.* consequently 1792 × 15 = 26880 Half-pence, which divide by 2, 12, and 20, will give 56 £. the Answer.

oz. *s.* *d.* *oz.*

(6) Stated thus—If 1 : 5 4 :: 36

Then by Reduction 5*s.* 4*d.* = 16 Four-pences; these × by 36 (the Third Term) = 576 Four-pences, which ÷ by 3, and 20, will give £. 9 12*s.* the Answer.

Yd. *s.* *d.* *Ps.* *Yds.*

(7) Stated thus—If 1 : 17 6 :: 12 each 30

Then by Reduction 17*s.* 6*d.* = 35 Six-pences, and 12 × 30 = 360 Yards, consequently 360 × 35 = 12600 Six-pences, these ÷ by 4,0 will give 315 £. the Answer.

lb. *s.* *d.* *C.* *lb.*

(8) Stated thus—If 1 : 7 9 :: 1 or 112

Then by Reduction 7*s.* 9*d.* = 31 Three-pences, these × by 112 (the last Term) = 3472 Three-pences, which ÷ by 4, and 20, will give £. 43 8*s.* the Answer.

(9) First Cost £. 34 6; gained 6 £. which, added together, = £. 40 6—to be sold for—then

C. *qrs.* *lb.* £. *s.* *lb.*

Stated thus—If 2 1 14 : 40 6 :: 1

Then by Reduction 2*C.* 1*qr.* 14*lb.* = 266*lb.* and £. 40 6*s.* = 806*s.* which ÷ by 266 will give 3*s.* $0\frac{1}{4}$*d.* $\frac{118}{266}$ the Answer.

(10)

(10) First Cost 90£. lost £. 7 10; Difference £. 82 10 sold for. Then,

Yds. £. *s.* *Ell.* *qrs.*

Stated thus—If 200 : 82 10 :: 1 or 5

Then by Reduction 200 Yards =800 Qrs. and £. 82 10= 1650*s.* which × 5 (the last Term) =8250*s.* these ÷ by 8,00 (the first Term) will give 10*s.* $3\frac{1}{4}$*d.* the Answer.

Doz. *lb.* *s.* *lb.*

(11) Stated thus—If 9 or 108 : 45 :: 4

Then 45 × 4=180*s.* these ÷ 108 will give 1*s.* 8*d.* the Answer.

lb. *d.* *Hhs.* *C.* *qrs.* *lb.*

(12) Stated thus—If 1 : $6\frac{1}{2}$ :: 4 each 12 2 14

Then by Reduction $6\frac{1}{2}$*d.*=13 Half-pence, and 12 *C.* 2 *qrs.* 24 *lb.*=1424 *lb.* in one Hogshead, which × by 4 the Number of Hhds.=5696 *lb.* these × by 13 (the second Term) =74048 Half-pence, which ÷2, 12, and 20, will give £. 154 5*s.* 4*d.* the Answer.

(13)

	£.	*s.*
Cost, - -	579	12
Freight, - -	46	0
Loading, &c. -	6	0
Custom, - -	10	0
Cellar, - -	4	0
Gain, - -	360	0

Tuns. ——— *Tuns.*

Stated thus —If 46 : 1005 12 :: 26

Then by Reduction £. 1005 12*s.*=20112*s.* these × by 26 (the third Term) =522912*s.* which ÷ by 46 (the first Term) will give 11367*s.* $7\frac{3}{4}$*d* $\frac{14}{46}$, or £. 568 7*s.* $7\frac{3}{4}$*d.* $\frac{14}{46}$. the Answer.

C. *lb.* *s.* *d.* *C.* *qrs.* *lb.*

(14) Stated thus—If 1 or 112 : 32 6 :: 12 2 14

Then by Reduction 32*s.* 6*d.*=390*d.* and 12 *C.* 2 *qrs.* 14 *lb.* =1414 *lb.* which × by 390*d.*=551460*d.* these ÷ by 112 (the first Term) will give $4923\frac{3}{4}$*d.* or £. 20 10*s.* $3\frac{3}{4}$*d.* the Answer.

(15)

(15) First Cost £. 89 16*s*. 4*d*; Loss 12 £.; Difference £. 77 16*s*. 4*d*.—— then

	C.	lb.	£.	s.	d.	lb.
Stated thus—If	1 or 112 :		77	16	4 ::	1

Then by Reduction £. 77 16*s*. 4*d*.=18676*d*. which ÷ by 112 will give 166¾*d*. or 13*s*. 10¾*d*. the Answer.

(16) Here, as he spends as much in 4 Months as he gains in 3, consequently he will spend in 8 Months what he gains in 6—— then

	M.	£.	s.	d.	M.	£.	s.	d.
Stated thus—If	8 :	185	5	6 ::	12	185	5	6
				12				2
	8)	2223	6	0	Gains	370	11	0 per An.
					Spends	277	18	3
Spends	£.	277	18	3				
					Answ.	92	12	9

(17) First 4*s*. 6*d*. × by $\overline{12 \times 6 \times 6 + 8}$ (36 *Doz*. 8 *Prs*.) will give 99 £. the Value of the 36 Dozen 8 Pair of Stockings;—— then

	s.	d.	P.	£.
Stated thus—If	1	4 :	1 ::	99

Then by Reduction 1*s*. 4*d*.=4 Four-pences, and 99 £.= 5940 Four-pences, which ÷ by 4 (the first Term) will give 1485 Pair; these ÷ 12, will give 123 Dozen and 9 Pair, the Answer.

(18) First 18*s*. 9*d*. ÷ by 5=3*s*. 9*d*. sold for per Yard, and 6*s*. 6*d*. ÷ 2=3*s*. 3*d*. cost per Yard; then from 3*s*. 9*d*. take 3*s*. 3*d*. remains 6*d*. gained per Yard; and he gained as much as 180 Yards cost; thus, cost per Yard 3*s*. 3*d*. or 39*d*.; consequently 39 × 180=7020*d*. his whole Gain — then

	d.	Yd.	d.
Stated thus—If	6 :	1 ::	7020
	6)	7020	

Answer, 1170 Yards.

(19)

(12) *s. d. Ell. qrs.* £. *s.*

Stated thus—If 6 6 : 1 or 5 : : 28 8

Then by Reduction 6*s.* 6*d.* =13 Six-pences, and 28£. 8*s.* =936 Six pences, which multiplied by 5 (the second Term) =4680 *qrs.* these ÷ by 13 (the first Term) will give 360*qrs.* bought: this ÷ by 60, (the Quarters contained in 20 Ells or 1 Piece) gives 6 Pieces, the Answ.

(20) *s. d. oz. dwts.* £. *s. d.*

Stated thus—If 5 10 : 1 or 20 : : 102 16 6

Then by Reduction 5 *s.* 10 *d* =70 *d.* and £. 102 16 6= 24678*d.* which × by 20, (the second Term) =493560 *dwts.* these ÷ by 7,0 (the first Term) gives 7050$\frac{6}{7}$ *dwts.* the Quantity of Silver bought, which, ÷ by 994 *dwts.* (4 *lb.* 1 *oz.* 14 *dwts*) the Weight of one Ingot, will give 7 Ingots, and 92*dwts.* or 4*oz.* 12*dwts.*$\frac{6}{7}$ over, the Answer.

(21) *d. oz.* £. *s.*

Stated thus—If 8$\frac{3}{4}$: 1 : : 426 16

Then by Reduction 8$\frac{3}{4}$ *d.*=35*qrs.* and £. 426 16*s.*=409728 *qrs.* which ÷ by 35, gives 11706$\frac{18}{35}$ *oz.* these ÷ by 16, 28, and 4, will give 6 Par. and 2 *qrs.* 3 *lb.* 10$\frac{18}{35}$ *oz.* over, the Answer.

(22) Here, first 100 Guineas =105£. lays by; then from £. 488 5 *s.* his yearly Income, take 105 £. remains £. 383 5 *s.* what he spends per Annum—then

D. £. *s.* *D.*

Stated thus—If 365 : 383 5 : : 1

20

365)7665(21 Shillings per Day, [the Answer.

£. *s.* *s. d.* £. *s.*

(13) Stated thus—If 1 or 20 : 3 9 : : 564 12

Then by Reduction 3*s.* 9*d.*=45*d.* and 564£. 12*s.*=11292*s.* which ×45*d.* (the second Term) =508140*d.* this, ÷ by 20, (the first Term)=25407*d.* these, ÷ by 12 and 20, will give £.105 17*s.* 3*d.* Taxes, which, subtracted from his Income, viz. £.564 12*s.* leaves £ 458 14*s.* 9*d.* his neat Income, the Answer.

(24) *s. d. oz. dwts.* £.

Stated thus—If 5 9 : 1 or 20 :: 200

Then by Reduction 5*s.* 9*d.*=69*d.* and 200£.=48000*d.* which × by 20, (the second Term)=960000*dwts.* this, ÷ by 69 (the first Term) gives 13913 *dwts.* $\frac{3}{69}$, these *dwts.* ÷ by 692, (2 *lb.* 10 *oz.* 12 *dwts.*) the Pennyweights in an Ingot, will give 20 Ingots, and 73 *dwts.* or 3 *oz.* 13 *dwts.* $\frac{3}{69}$ over, the Answer.

(25) First 6×6×10×30=10800 Yards in the six Packs; then

Yds. £. *s. d.* *Yds.*

Stated thus—If 3 : 2 4 3 :: 10800

Then by Reduct. £. 2 4*s.* 4*d.*=531 Pence, these × by 10800 =5734800*d.* which ÷ by 3, (the first Term) =1911600 Pence, or 7965£. cost him, and 3) 2£. 4*s.* 3*d.* (14*s.* 9*d.* cost per Yard.

(26) *W.* £. *s.* *W.*

Stated thus—If 16 : 14 16 :: 52

By Reduction £. 14 16*s.*=296*s.* which ×52=15392*s.* these ÷ by 16, gives 962*s.* or £. 48. 2*s.* the Answer.

(27)

	£.	s.
Oxen, each, - -	10	0
Cows, - - -	7	0
Calves, - - -	1	10
Sheep, - - -	0	19
One of each cost, - £.	19	9

£. *s.* of ea. £. *s.*

Stated thus—If 19 9 : 1 :: 116 14

20   20

389   )2334(6 of each.

(28) £. £. *s. d.* £.

Stated thus—If 560 : 374 10 6½ :: 1

By Reduction £. 374 10*s.* 6½*d.*=359546 Farthings, which ÷ by 560 gives 642 $\frac{26}{560}$ *qrs.* which, reduced to Shillings, &c. will be 13*s.* 4½*d.* $\frac{26}{560}$, the Answer.

(29)

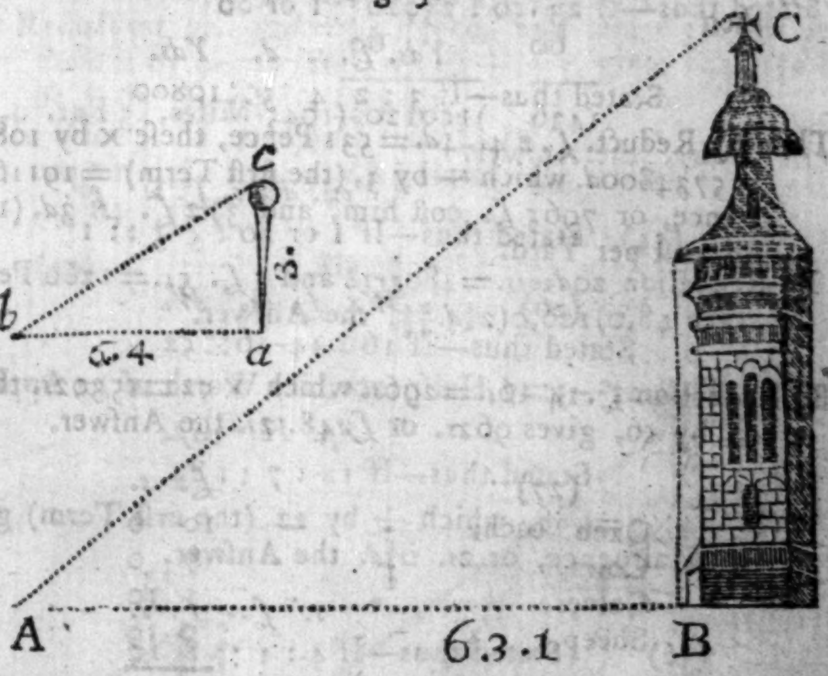
First the Cost, Custom, Fees, Freight, Factorage, and Profit, added into one Sum=£. 14 0s. 2*d.*—then

C. qrs. lb. oz. £. s. d. C. lb.

(29) Stated thus—If 18 3 17 7 : 14 0 2 :: 1 or 112

By Reduction 18 *C.* 3 *qrs.* 17 *lb.* 7 *oz.*=33879 *oz.* and 112 *lb.* =1792 *oz.* Also 14 £. 2 *d.*=3362 Pence; then 3362 *p.* × 1792 *oz* =6024704 Pence, which ÷ by 33879 (the first Term) gives $177\frac{3}{4}d\,\frac{10347}{33879}$, these *d.* ÷ by 12, give $14s.\ 9\frac{3}{4}d.\,\frac{10847}{33879}$, the Answer.

Fig. 5.



(30) Here the above Figures *ac*=3 Feet, the Length of the Staff, *ab*=6 Feet 4 Inches, or 76 Inches, Distance of its Shadow. Also B C = the Height of the Tower, and A B=63 Yards, 1 Foot, or 2280 Inches Distance of its Shadow—then

In. Ft. In.

Stated thus—If 76 : 3 :: 2280

Then 2280 × 3=6840 Feet, which ÷ by 79=90 Feet, the Answer. For as *a c* : *a b* :: B C : A B.

D. H. M. H.

(31) Stated thus—If 6 4 : 285 :: 1

12

76)285(3 Miles 6 Fur. the Answ.

 (32)

Pipe G. £. s.

(32) Stated thus—If 1 or 126 : 44 2 :: 1 Pt.

By Reduction 126 Gal.=1008 Pts. and 44 £. 2s.=10584 Pence, which ÷ by 1008 (the first Term) gives $10\frac{1}{2}d.$ per Pint, the Answer.

(33) First $360 \times 69\frac{1}{2} = 25020$ Miles, the Circumference of the Globe—then

H. M. *Miles.* H. M.

Stated thus—If 23 56 : 25020 :: 1 or 60

```
         60             60
      ----------    ----------
       1436     )1501200(1045 Miles, 3 Fur. 9 316/1436
```

Poles, the Answer.

oz. dwts. £. s. *gr.*

(34) Stated thus—If 1 or 20 : 5 5 :: 1

By Reduction 20*dwts.*=480*grs.* and 5 £. 5s.=1260 Pence, then 48,0)126,0($2\frac{1}{2}d.\frac{24}{48}$, the Answer.

(35) First 21÷3=7 Halfpence, the Worth of 12 Apples.

Ap. H. *Ap.*

Stated thus—If 12 : 7 :: 84

Then 84×7=588, which ÷ by 12 (the first Term) gives 49 Halfpence, or 2s. $0\frac{1}{2}d.$ the Answer.

s. £. £. s.

(36) Stated thus—If 4 : 1 :: 8 10

By Reduction 8 £. 10s.=170s. which ÷ by 4 gives 42 £. 10s. the Answer.

(37) First $184 \times \overline{7 \times 4 + \frac{1}{2}}(28\frac{1}{2}) = 5244$ Yards of the former.

Yds. Yds. Yds.

Stated thus—If 19 : 14 :: 5244

```
                       14
                  ----------
                19)73416(3864 Yds. the Answer.
```

(38) First 406×117=47502 Pieces; these × by 44*d.* (3s. 8*d.*) or the Value of one Piece, will give 2090088 Pence, Value of all the Pieces—then

Stated

d. Reas. *d.*

Stated thus—If 3 : 20 :: 2090088
20

3)41801760(13933920 Reas, the Answer.

(39) First $274\times4=1096s.$ or £. 54 16*s.* cost him; then—

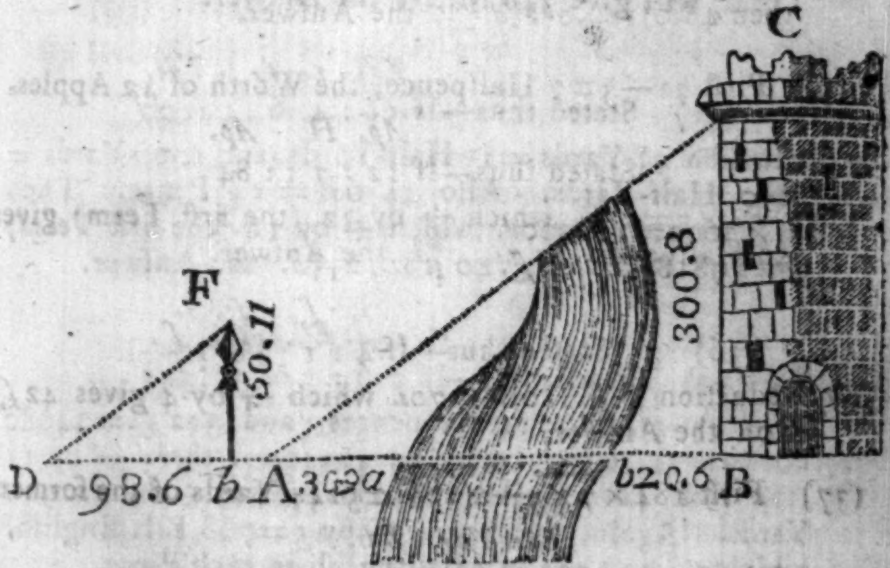
Ell E. qrs. s. d. Ell F.

Stated thus—If 1 or 5 : 7 10 :: 274

By Reduction 7*s.* 10*d.*=94 Pence, and $274\times3=822$ *qrs.* these × by 94=77268, which ÷ by 5, gives $15453\frac{1}{2}d.\frac{2}{5}$, or £. 64 7*s.* $9\frac{1}{2}d\ \frac{2}{5}$ sold for.

Then from £. 64 7*s.* $9\frac{1}{2}d.\frac{2}{5}$ take £. 54 16*s.* and there will remain 9£. 11*s.* $9\frac{1}{2}d.\frac{2}{5}$ gained, the Answer.

(40) Fig. 6.



Here F *b*=50 Feet, 11 Inches, or 611 Inches, Height of the May Pole, and D *b*=98 Feet, 6 Inches, or 1182 Inches, Length of its Shadow. Also B C=300 Feet, 8 Inches, or 3608 Inches, and A B the Length of its Shadow—then

In. *In.* *In.* *In.*

Stated thus—If 611 : 1182 :: 3608 : 6979

Ft. *In.*

Then $6979 \div 12 = 581\ 7\frac{487}{611}$ = A. B.
And 30 F. 9 In. + 20 F. 6 In. = 51 3 = A *a* + B *b*.

530 $4\frac{487}{611}$ = *a b*, the Breadth of the Stream required.

M. *s.* *M.*

(41) Stated thus—If 2 : 15 : : 7

Then $15 \times 7 = 105$*s.* which ÷ by 2 = 52*s.* 6*d.* or £. 2 12*s.* 6*d.* the Anſwer.

H. *M.* *D.* *H.*

(42) Stated thus—If 4 : 12 : : 9 8

By Reduction 9 D. 8 H. = 116 Hours, which × by 12 (the ſecond Term) = 1392 Hours; theſe ÷ by 4 (the firſt Term) will give 348 Miles, the Anſwer.

Yds *s.* *d.* *Yds.*

(43) Stated thus—If $5\frac{1}{2}$: 4 6 : : 1000

By Reduction $5\frac{1}{2}$ Yards = 11 Half-Yards, and 1000 Yards = 2000 Half-Yards. Alſo 4*s.* 6*d.* = 54 Pence. Then $54 \times 2000 = 108000$*d.* which ÷ by 11 (the firſt Term) gives $9818\frac{2}{11}$*d.* or £. 40 18*s.* $2\frac{2}{11}$*d.* the Anſwer.

s. *d.* *Ell.* *qrs.* £.

(44) Stated thus—If 5 6 : 1 or 3 : : 352

By Reduction 5*s.* 6*d.* = 11 Six-pences, and 352 £. = 14080 Six-pences, which × by 3 = 42240*qrs.* theſe ÷ by 11 (the firſt Term) gives 3840 *qrs.* theſe ÷ by 4 = 960 Yards. Again, 3840 *qrs.* ÷ by 5 = 768 Ells Engliſh, which ÷ by 64 = 12 Ells Engliſh in each Piece.

£. £. *s.* £. *s.*

(45) Stated thus—If 100 : 4 15 : : 40 10

By Reduction 100 £ = 2000*s.* and £. 4 15*s.* = 95*s.* Alſo £. 40 10*s.* = 810*s.* which × by 95 = 7,695*s.* theſe ÷ by 2,00 will give 38*s.* $5\frac{1}{2}$*d.* $\frac{16}{20}$, or £. 1 18*s.* $5\frac{1}{4}$*d.* $\frac{16}{20}$, the Anſwer.

(46)

(46) First $24 \times 20 = 480$ Yards, contained in the 24 Pieces.

Yds. Yds. Yds.

Stated thus—If 12 : 8 :: 480

Here $480 \times 8 = 3840$ Yards, which ÷ by 12, gives 320 Yds. the Answer.

A. £. s. d. A.

(47) Stated thus—If 1 : 1 19 6 :: 240

By Reduction £. 1 19*s.* 6*d.* = 79 Sixpences, which × by 4 × 60 (240) the Product is 18960 Six-pences; these ÷ by 40, will give 474 £. per Annum; this ÷ by 4 = £. 118 10*s.* per Quarter, the Answer.

£. s. d. £.

(48) Stated thus—If 1 : 10 6 :: 1000

By Reduction 10*s.* 6*d.* = 11 Six-pences, which × by 1000, the Product is 11000 Six pences, or 525 £. the Answer.

M. fur. p. ft. Min. Mile Yds.

(49) Stated thus—If 37 2 37 $5\frac{1}{2}$: 4 :: 1 or 1760

By Reduction 37 *m.* 2 *fur.* 37 *p.* $5\frac{1}{2}$ *ft.* = 394592 Half-feet, and 1760 Yards. = 10560 Half-feet, which × by 4 Minutes (the second Term) the Product will be 42240 Minutes; this × by 60, to bring the Minutes into Seconds, gives 2534400 Seconds; these ÷ by 394592 (the first Term) the Quotient is $6'' \; 25\frac{146080}{394592}'''$ past 12, the Answer.

£. s. d. E.E. £. s. d.

(50) Stated thus—If 1 7 10 : 4 :: 118 17 $7\frac{1}{2}$

By Reduction £. 1 7*s.* 10*d.* = 668 Half-pence, and £. 118 17*s.* $7\frac{1}{2}$*d.* = 57063 Half-pence. Also 4 Ells English = 80 Nails; then $57063 \times 80 = 4565040$ Nails, which ÷ by 668 = 6833 Nails, and $\frac{596}{668}$ over, bought; then to find how many Pieces, each 33 Ells Flemish, 1 *qr.* 2 *na.* or 402 Nails, are contained in the Quantity bought; thus, 402)6833(16 Pieces, and 33 *Ells.* 1 *qr.* 1 *na.* $\frac{596}{668}$ over, the Answer.

(51) First, from 100 £. take 60 £. Remains 40 £. Serge cost.

Yds. Y. Yds.

Stated thus—If 2 : 3 :: 236

Here

Here $236\times3=708$; this $\div$ by $2=354$ Yards of Shalloon, which cost 60 £. consequently 60 £. $\div$ by 354, will give 3*s.* 4*d.* $\frac{252}{2354}$, what the Shalloon cost per Yard; and 40 £. $\div$ by 236, will give 3*s.* 4*d.* $\frac{168}{2234}$, what the Serge cost per Yard.

s. *lb.* £. *s.* *d.*

(52) Stated thus—If 14 : 8 :: 4 19 $1\frac{1}{4}$

By Reduction 14*s.* = 672 Farthings, and £. 4 19*s.* $1\frac{3}{4}$*d.* = 4757 Farthings, which $\times$ by 8, (the second Term) the Product will be 38056; this, $\div$ by 672 (the first Term) will give 56 *lb.* 10 *oz.* 1 *dr.* $\frac{352}{672}$, the Answer.

C. *lb.* *s.* *C. qrs. lb.*

(53) Stated thus—If 1 or 112 : 7 :: 20 2 16.

By Reduction 20 *C.* 2 *qrs.* 16 *lb.* = 2312 *lb.* these $\times$ 7*s.* = 16184*s.* which $\div$ by 112 will give 144*s.* 6*d.* or £. 7 4*s.* 6*d.* the Answer.

(54) A Pipe = 126 Gallons, from which take 12 Gallons. Remains 114 Gallons, or 456 Quarts, at 18*d.* per Qt. Then

Qt. *d.* *Qts.*

Stated thus—If 1 : 18 :: 456

Here $456\times18=8208$ Pence, which, $\div$ by 12 and 20, gives £. 34 4*s.* sold for, from which take £. 25 4*s.* Remains 9 £. gained, the Answer.

C. qrs. lb. £. *s.* *d.* *lb.*

(55) Stated thus—If 20 2 16 : 7 4 6 :: 1

Here by Reduction 20*C.* 2*qrs.* 16*lb.* = 2312*lb.* and £. 7 4*s.* 6*d* = 6936 Farthings, which $\div$ by 2312 (the first Term) gives $\frac{3}{4}$, the Answer.

(56) First $9\times4=36$ square Inches in a Brick, and 20×20 = 400 Feet square, which $\times$ by 144 = 57600 square Inches, the Content of the Room.—Then

Inch. *B.* *Inch.*

Stated thus—If 56 : 1 :: 5760

36)57600(1600 Bricks, the Answer.

(57)

(57) First $22 \times 4 = 88$ Miles I have gone before you set out, and $32 - 22 = 10$ —— you gain of me per Day; then

Stated thus—If 10 : 1 : : 88

10)88($8\frac{4}{5}$ Days, before you overtake me.

Then $8\frac{4}{5} \times 4 \times 8(32) = 281\frac{3}{5}$ Miles you travelled before I was overtaken, consequently $350 - 281\frac{3}{5} = 68\frac{2}{5}$ Miles on this Side Edinburgh, the Answer.

(58) First, from 20 take 11. Remains 9 Days to be sunk.

Days. Yea. Days.

Then stated thus—If 3 : 400 : : 9

Here $400 \times 9 = 3600$; this $\div$ by $3 = 1200$ Years after Anno 1700; then 1200 added to $1700 = 2900$, the Year of Christ required.

(59) *d. lb. d.*

First, Beef cost per Day, $2\frac{1}{4} \times 5 = 11\frac{1}{4}$, and per Week, $11\frac{1}{4}d.$ $\times 7 = 6s.\ 6\frac{3}{4}d. = 315\ qrs.$

Biscuit cost per Day, $1\frac{1}{2}d. \times 3lb. = 4\frac{1}{2}d.$ and per Week, $4\frac{1}{2}d.$ $\times 7 = 2s.\ 7\frac{1}{2}d. = 126\ qrs.$

Meat for the Ship's Company cost per Day £. 12 12*s.* and per Week £. 12 12*s.* $\times 7 =$ £. 88 4*s.* or 84672 Farthings.

Qrs. Qrs. Qrs.

Stated thus—If 315 : 126 : : 84672

126

315)10668672($33868\frac{252}{315}$ Farthings, or £. 35 5*s.* 7*d.* $\frac{252}{315}$, the Answer.

H. M. H.

(60) Stated thus—If 8 : 135 : : 1

Here $135 \div 8 = 16$ Miles, 7 Furlongs, what both rode per Hour.

	M.	*fur.*	*p.*		*M.*	*f.*		*M.*	*fur.*	*p.*	
Then,	16	7	0	less	2	4	=	14	3	0	
And	14	3	0	÷ by	2		=	7	1	20	B. rode per Hour.
Also,	7	1	20	+ by	2	4	=	9	5	20	A. ditto.

$$\text{For } \begin{Bmatrix} 7 & 1 & 20 \\ 9 & 5 & 20 \end{Bmatrix} \times \text{by } 8 = \begin{Bmatrix} 57 & 4 & \text{B.} \\ 77 & 4 & \text{A.} \end{Bmatrix} \text{ travelled.}$$

(column heads: *M. fur. p.* and *M. fur.*)

Proof, 135 Miles.

(61) Suppose 130 to be the whole Work; then A will perform $\frac{1}{13}$th Part, and B. $\frac{1}{10}$th, which will be $13+10=23$d Part of the Work performed by them both together in one Day.

W. Day. W.

Then Stated thus—If 23 : 1 :: 130

23)130(5 Days, 7 Hours, $49\frac{1}{2}\frac{1}{3}$ Minutes, the Answer.

(62) Here, suppose 198 to be the whole Work, of which B and C will perform $\frac{1}{18}$, viz. 11, and with the Help of A they will perform $\frac{1}{11}$, viz. the 18th Part in one Day; then $18-11=7$th Part performed by A alone.

W. Day. W.

Then stated thus—If 7 : 1 :: 198

7)198(28 Days, 3 H. $25\frac{5}{7}$ M. the Answ.

Here the Reason of my supposing 130 to be the whole Work in the 61st Question, is because $\frac{1}{10}$ and $\frac{1}{13}$ reduced to a Common Denominator becomes $\frac{13}{130}$ and $\frac{10}{130}$ by (Case V. p. 197.) whose sum is $\frac{23}{130}$ by (Sect. 39. p. 202.) Likewise in the last Queston I supposed 198, for $\frac{1}{11}$ and $\frac{1}{18}$ reduced to a Common Denominator becomes $\frac{18}{198}$ and $\frac{11}{198}$, whose difference is $\frac{7}{198}$ per Sect. 40. page 203.

£. £. £.

(63) Stated thus—If 17 : 3 :: 140

£. *s.* *d.*

Here $140\times3=420$; this $\div$ by $17=24$ 14 $1\frac{1}{4}\frac{11}{17}$ taken off. Then from £. 140 take £. 24 14*s.* $14\frac{1}{4}$*d.*$\frac{11}{17}$. Remains £.115 5*s.* $10\frac{1}{2}$*d.*$\frac{6}{17}$, the Assessment, which $\div$ by 5, gives £. 23 1*s.* 2*d.*—$\frac{3}{17}$ the Taxes; this, taken from £.140, leaves £.116 18*s.* $9\frac{3}{4}$*d.*$\frac{9}{17}$, the Answer.

(64) First 1000 $\div$ by $\begin{cases} 5=200 \text{ gained by Land Trade.} \\ 8=125 \text{ ——— Sea ditto.} \end{cases}$

Sum, 325 Gain per Year.

Then

Then $1000 \div 2\frac{1}{2} = 400$£. lost in 1 Year.
And 400£.$-325=75$£. run out per Year.

J. Y. £.

Stated thus—If $75 : 1 :: 1000$
$75)1000(13\frac{1}{3}$ Years, the Anſwer.

(65) Here, from 10 in the Morning to 6 in the Evening is 8 Hours—I ſet out before you, then $2 \times 8 = 16$ Miles I travelled before you ſet out, and $170-16=154$ Miles. Alſo, $2+3=5$ Miles both travelled per hour.

M. H. M.

Stated thus—If $5 : 1 :: 154$
$5)154(30\frac{4}{5}$ Hours they will meet.

Then $30\frac{4}{5} \times 2 = 61\frac{3}{5}$, which added to $16 = 77\frac{3}{5}$ Miles, or 77 *m.* 4 *fur.* 32 *p.* from Exeter; and $30\frac{4}{5} \times 3 = 92\frac{2}{5}$ M. or 92 *m.* 3 *fur.* 8 *p.* from London.

	M.	*fu.*	*p.*		
Anſwer,	77	4	32	diſtant from	Exeter.
	92	3	8		London.
Proof,	170 Miles.				

(66) Firſt, from 13° take 1°, remains 12°, Moon gains of the Sun per Day.
And $30° \times 3 = 90°$, from the firſt of Aries to the firſt of Cancer.
Alſo, $90° + 3 = 93°$ Sun before the Moon.

D.

Then ſtated thus—If $12° : 1 :: 93°$
$12)93)7\frac{3}{4}$ Days, in which Time, the Sun will be overtaken by the Moon.
∴ $7\frac{3}{4} + 3 = 10\frac{3}{4}$ Degrees of Cancer, the Anſwer.

(67) Firſt, from 21 take 15. Remains 6 Rods, the Dogs gained in running 21 Rods, and fourſcore $=80.+16=$ 96 Rods the Hare ſtarted before the Dogs.—Then

R. R. R.

Stated thus—If $6 : 21 :: 96$

Here $96 \times 21 = 2016$; this $\div$ by 6, gives 336 Rods the Dogs ran, and $336-96=240$ Rods the Hare ran.

(68)

(68) First, $5\frac{2}{3}$ Miles $=29920$ Feet.

Feet. Sec. Feet.

Stated thus—If 1150 : 1 :: 29920

115,0)2992,0(26 Sec. $1\frac{5}{115}$ Thirds, the Answer.

(69) First, 1 Min. 3 Seconds $=63$ Seconds.

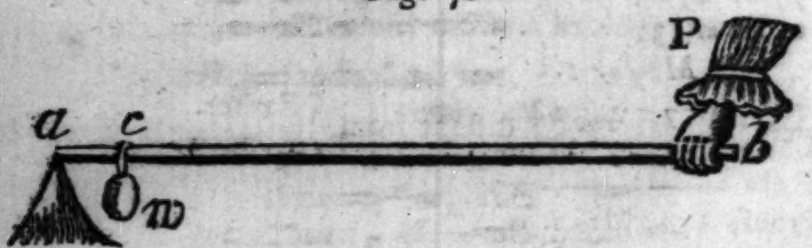
″ *Feet.* ″

Stated thus—If 1 : 1150 :: 63

63

72450 Feet, or 13 Miles, 5 Furlongs, 30 Poles, 5 Yards, the Answer.

Fig. 7.



(70) Here $ab=100$ Inches, and $ac=7\frac{1}{2}$, P the Power, or 168 *lb.* and *w.* the Weight to be moved.

Then from 100 take $7\frac{1}{2}$, remains $92\frac{1}{2}=ab-ac.=cb.$

In. lb. In.

Stated thus—If $7\frac{1}{2}$: 168 :: $92\frac{1}{2}$

Here $7\frac{1}{2}$ Inches $=15$ Half Inches, and $92\frac{1}{2}=185$ Half Inches, which $\times$ 168 $=31080$ *lb.* these $\div$ by $15=2072$ *lb.* $=w.$ the Answer.

For A's ac : P :: cb : w.

(71) First $9\times2=18$ Inches the Diameter of the Crank.

Ft. In. Ft.

Stated thus—If 6 : 18 :: 9

9

6)162(27 Inches the Answ.

(72)

(72) First $62\frac{1}{2}\times 3=187\frac{1}{2}$*lb.* Wt. of 3 solid Feet of Water;

Feet. lb. Feet.

Therefore, stated thus—If 6 : $187\frac{1}{2}$ :: 9

9

6)$1687\frac{1}{2}$($281\frac{1}{4}$*lb.* the Answ.

(73) Stated thus—As 25 : 1000 :: 1 : 40, so the less is moved with a Force 40 Times greater than the other.

(74) Stated thus—As 60 : 8 :: 100 :

6,0)80,0($13\frac{1}{3}$. Answ. as $13\frac{1}{3}$ to 1.

(75) Stated thus—As 8 : 48 :: 1 : 6

Answer, Lesser to the Greater as 1 to 6

(76) First 2 Hours $\times 60=120$ Minutes.

Then—stated thus—As 40 : 120 :: 1 : 3.

Answer, the Swifter to the Slower, as 1 to 3.

(77) First $30\times 12=360$, and 60 Inches $=5$ Feet.

Then as 1 : 5 :: 360 : 1800

Consequently $1800-5=1795$ Feet, the Answer.

(78) First $50\div 5=10$

Then—stated thus—As 5 : 10 :: 1 : 2

So that the first Body hath been in Motion double the Time of the latter.

THE RULE OF THREE INVERSE.

In. B. In. L. In. B.

(2) Stated thus—If 12 : 12 :: 4

Here $12\times 12=144$; this $\div 4=36$ Inches long, the Answer.

£. M. £.

(3) Stated thus—If 500 : 6 :: 220

Here $500\times 6=3000$, this $\div$ by 220, gives 13 Months, $19\frac{3}{11}$ Days, the Answer.

M. oz. M.

(4) Stated thus—If 3 : 14 :: 8

Here $3\times 14=42$, this $\div$ by $8=5$*oz.* 4*dwts.* the Answer.

s. d. oz. s.

(5) Stated thus—If 4 6 : 12 :: 3

Here 4*s.* 6*d.*=9 Six-pences, and 3*s.*=6; then $12\times 9=108$; this $\div$ by 6, the second Term, gives 18 *oz.* the Answ.

L (6)

Qrs. Yds. Qrs.

(6) Stated thus—If 5 : 275 :: 3

Here 275 × 5 = 1375, this ÷ by 3, gives 458 *yds.* 1 *qr.* $1\frac{1}{3}$ *na.* the Answer.

£. Yrs. £.

(7) Stated thus—If 80 : 15 :: 600

Here 15 × 80 = 12,00, this ÷ by 6,00, gives 2 Years, the Answer.

M. W. D. M.

(8) Stated thus—If 10 : 43 5 :: 6

Here 43 *w.* 5 *d.* = 306 Days, which × by 10 = 3060, this ÷ by 6, gives 510 Days, or 72 Weeks, 6 Days, the Answer.

Ft. Ft. Ft.

(9) Stated thus—If 24 : 30 :: $2\frac{1}{4}$

Here 24 *ft.* = 96 *qrs.* and $2\frac{1}{4}$ = 9 *qrs.* then 96 × 30 (the second Term) = 2880 Feet; these ÷ by 9 (the last Term) gives 320 Feet, or $106\frac{2}{3}$ Yards, the Answer.

Note, Paper is 27 Inches, or $2\frac{1}{4}$ Feet wide.

Mo. Men. Mo.

(10) Stated thus—If 3 ; 1500 :: 5

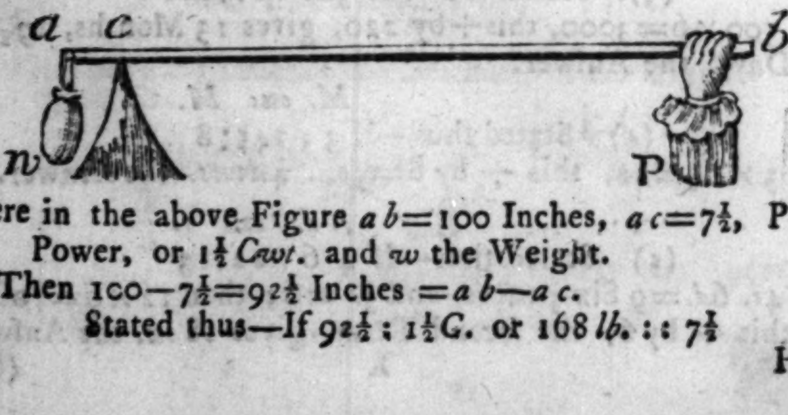
Here 1500 × 3 = 4500, this ÷ by 5, gives 900 Men, to continue in; then from 1500 take 900, remains 600 Men to depart.

H. Cock M.

(11) Stated thus—If 6 : 1 :: 15

Here 6 Hours × 60 = 360 Minutes, which ÷ by 15, (the last Term) gives 24 Cocks, the Answer.

(12) Fig. 8.



Here in the above Figure *a b* = 100 Inches, *a c* = $7\frac{1}{2}$, P the Power, or $1\frac{1}{2}$ *Cwt.* and *w* the Weight.

Then 100 − $7\frac{1}{2}$ = $92\frac{1}{2}$ Inches = *a b* − *a c*.

Stated thus—If $92\frac{1}{2}$; $1\frac{1}{2}$ *C.* or 168 *lb.* :: $7\frac{1}{2}$

Here

Here $92\frac{1}{2}=185$ Half-inches, and $7\frac{1}{2}=15$; then $185\times168=31080$; this $\div$ by 15, (the last Term) gives 2072 *lb.* or $18\frac{1}{2}$ *C.* the Answer.

For as *bc* : *w* : : *ac* : P.

(13) Fig. 9.

In the above Fig. *ce*=70 Inches, *ac*=2, H the Hogshead, and *w* the Weight.

In. *C.* *In.*

Stated thus—If 2 : $9\frac{1}{2}$: : 70

Here $9\frac{1}{2}$ *C.*=1064 *lb.* which × by 2=2128 *lb.* these $\div$ by 70, (the last Term) gives 30*lb.* 6*oz.* $6\frac{28}{70}$*drs.* the Weight required.

For as *ac* : *w* : : *ec* : H

(14) Stated thus—As 100000 : 20 : : 30 : 66666 Feet, 8 Inches per Second, the Answer.

lb. *Feet.* *lb.*

(15) Stated thus—As 200 : 100 : : 8 : 2500 Feet per Second.

CONTRACTIONS *in the* RULE *of* THREE.

(2) First $3\times3=9$ *C.* in the three Chests.

	C.	£.	s.	C.	
Stated thus—If	9 :	11	13 : :	72	Here the 1 and 3 Terms
	1		8	8	[$\div$ by 9.
Answer,	£. 93		4		

	C.	£.	C.	
(3) Stated thus—If	26 :	78 : :	156	Here 1 and 2 $\div$ by 26.
	1	3	3	
		£.	468 the Answer.	

lb. s. lb.
(4) Stated thus—If 3 : 18 : : 112 Here the 1 and 2 ÷ by 3.
1 6 6

Anſwer, 672 *s.*=£. 33 12 *s.*

(5) Firſt 6 × 8=48 Yards in the 6 Gowns.
Yds. £. *Yds.*
Then—ſtated thus—If 48 : 6 : : 64 Here the 1 & 2 ÷ by 6.
8)64(8 £. the Anſwer.

(6) Firſt $1\frac{1}{2}$ *C.*=168 *lb.*
lb. s. lb.
Then—ſtated thus—If 24 : 12 : : 168 Here 1 and 2 ÷ 12.
2)168(84*s.* or 4£. 4*s.* the Anſwer.

(7) Firſt 24 × 3=72 Yards in the 3 Pieces.
Yds. £. *s.* *Yds.*
Then—ſtated thus—If 72 : 64 14 : : 8 Here 1 and 3 ÷ by 8.
9)64 14(7 £. 3*s.* $9\frac{1}{4}$*d.* $\frac{1}{3}$ Anſwer.

(8) Firſt 6 × 3=18 *lb.* in the 6 Parcels.
lb. £. *s.* *lb.*
Stated thus—If 18 : 12 19 : : 6 Here 1 and 3 Terms ÷ by 6.
3)12 19(4 £. 6*s.* 4*d.* the Anſwer.

In. In. In.
(9) Stated thus—If 12 : 12 : : 3 Here the 1 and 3 ÷ by 3.
4 4 1

Anſwer, 48 Inches, or 4 Feet,

M. C. M.
(10) Stated thus—If 512 : 225 : : 64 Here 1 and 3 ÷ by 64.
8 8 1

Anſwer, 1800 *Cwt.*

THE

THE RULE OF FIVE:

OR,

The DOUBLE RULE of THREE.

P. *D.* *B.*
(2) Placed thus—6 : 12 : 2, then 6×12=72, the Divisor.
36 : 4 :—, and 36×4×2=288, the Dividend.
So 288÷by 72=4 Bushels, Answer.

P. *w.* *£.* Then 21×6=126 the Divisor.
(3) 6 : 21 : 120, And 46×14×120=77280£. Dividend.
14 6 ——. So 77280÷126=£.613 6*s.* 8*d.* the Answer.

(4) Here 9£. 5*s.*=8880 Farthings, and £.70 10*s.* $3\frac{3}{4}$*d.*= 67695 Farthings.
C. *C.* *qrs.* Then 8880×125=1110000 the Divisor.
40 ; 100 : 8880 And 67695×40×100=270780000 Divid.
— : 125 67695 So 27078,0000÷111,0000=243 *C.* 3 *qrs.* 21*lb.* 15*oz.* $2\frac{18}{111}$ *drs.*

(5) First, from 125£. 8*s.* take 120£. Remains 5£. 8*s.*= 108*s.* Interest.
£. *M.* *s.* Then 120×9=1080 the Divisor.
120 : 9 : 108 And 108×12×100=129600*s.* Dividend.
100 12 : —— Therefore 12960,0÷108,0=120 *s.* or 6£. the Answer.

(6) First, 100£.=24000 Pence, and £.259 13*s.* 5*d.*= 62321*d.* then
D. *W.* *£.* Here 52×24000=1248000 the Divisor.
24000 : 52 : 5 And 62321×5×20=6232100£. therefore
62321 ; 20 :—62321,00÷12480,00=£.4 19*s.* $10\frac{1}{4}$*d.* $\frac{1152}{1243}$ the Answer.

(7) First, 8 Months =32 Weeks, then
Mn. *W.* *oz.* Here 32×14=448 the Divisor.
1400 : 20 : 14 And 8×20×1400=224000 the Dividend.
—— : 32 : 8 Therefore 224000÷448=500 Men, Answ.

(8) First, 10 Guineas=210 Shillings, and 5£.=100*s.* then
M. s. D. Here 100×12=1200 the Divisor.
8 : 5 : 100 And 210×8×5=8400 the Dividend.
:12: 210 Therefore 84,00÷12,00=7 Men, the Answer.

(9) First, 24 Weeks, 4 Days =172 Days; then
M. D. £. Here 140×12=1680 the Divisor.
5 : 172 : 140 And 172×5×200=172000 the Dividend.
2 : —— : 200 Therefore 17200,0÷168,0=$102\frac{64}{168}$ Days, or 3 *Mo.* 2 *W.* $4\frac{64}{168}$ *D.* the Answer.

(10) First, 3£. 10*s.*=70*s.* and 38£. 10*s.*=770*s.* Also 1 Year =4 *qrs.* and $1\frac{1}{4}$=5 *qrs.*
£. qrs. s. Here 70×5=350 the Divisor.
100 : 4 : 70 And 770×4×100=308000 the Dividend.
—— : 5 : 770 Therefore 30800,0÷35,0=880£. Answer.

(11) First, 36*s.* ÷ by 12 (the Sacks in a Chaldron) =3*s.* what the Coals cost per Week, and 41£.=820*s.* then
P. w. s. Here 3×6=18 the Divisor.
7 : 1 : 3 And 820×7=5740 the Dividend.
: 6 : 820 Therefore 5740÷18=$318\frac{8}{9}$ Poor, the Answer.

THE RULE OF THREE REPEATED.

(2) First 18×2=36 Inches, Length of the Lever; then
In. Ft. In. Ft.
Stated thus—As 40 : 104 : 36 : $93\frac{3}{5}$
Feet. Hou. Feet.
Again,—As $93\frac{3}{5}$: 13 : : 73 : 10 Hours, 8 Min. 20 Sec. the Answer.

In. lb. In. lb.
(3) Stated thus—As 1 : $1\frac{1}{2}$: : 12 : 18 wt. 12 In. } from the
Again, As 1 : $1\frac{1}{2}$: : 28 : 42 wt. 28 — } Shoulders
Consequently 42—18=24 *lb.* the Answer.

(4) First 30 Inches =$2\frac{1}{2}$ Feet, 24=2 Feet, and 4 *C.* 70 *lb.* =518 *lb.* then 11—$2\frac{1}{2}$=$8\frac{1}{2}$ Feet; also 11—2=9.

	Feet.	*lb.*	*Feet.*	*lb.*	
Stated thus—As	11	: 518	: : $8\frac{1}{2}$	: $400\frac{3}{11}$	Force in the former Case.
And Contra, As	$8\frac{1}{2}$	: $400\frac{3}{11}$	: : $2\frac{1}{2}$	: $117\frac{8}{11}$	Pressure.
Again, As	11	: 518	: : 9	: $423\frac{9}{11}$	Force in lat. Ca.
Also, As	9	: $423\frac{9}{11}$	: : 2	: $94\frac{2}{11}$	Pressure.

(5)

H. D. H. D.

(5) Stated thus—As 12 : 7 :: 16 : $5\frac{1}{4}$ of 16 Hours each.

M. D. M. D. H. M.

Then, As 152 : $5\frac{1}{4}$ :: 576 : 19 14 $18\frac{144}{152}$ the Anſwer.

M. gall. M. gall.

(6) Stated thus—As 9 : 14 :: 31 : $48\frac{2}{9}$ fills in 31 Minutes.

Then $48\frac{2}{9}$—40=$8\frac{2}{9}$ Gallons in the Tub at the End of 31 Minutes.

And from 2 to 5=3 Hours, or 180 Minutes.

M. gal. M. sec.

Again, as 31 $8\frac{2}{9}$: : 180 : $47\frac{23}{31}$ fills in 3 Hours.

And 147—$47\frac{23}{31}$=$9\frac{8}{31}$ Gallons wants of being full.

gal. M. gal. M. sec.

Alſo, as 14 : 9 : : $99\frac{8}{31}$: 63 $48\frac{21}{31}$ the Tub will be full.

Which added to 5 o'Clock, will give 3 Min. $48\frac{21}{31}$ Sec. after 6, the Tub will be full.

(7) First $1\frac{1}{2}d.$=3 Halfpence, and 6*s.* $7\frac{1}{2}d.$=159.

£. £.

Stated thus—As 3 : 1 :: 159 : 53=$\frac{4}{5}$ of the Rent.

Then 53£.÷4=13£. 5*s.*=$\frac{1}{5}$ of the Rent, or the Anſwer.

For 53£. added to 13£. 5*s.* gives 66£. 5*s.* the whole Rent.

£. *s.* £. *s.* *s.*

Again, if 1 : 4 :: 66 5, or 265 : £.13 5*s.* the Anſwer.

Or, inſtead of the laſt Stating, thus; from £. 66 5*s.* the whole Rent, take $\frac{4}{5}$, viz. 53£. Remains £. 13 5*s.* the King's Tax, as before.

(8) Firſt, 45 + 13=58 Perſons, and 17 Guineas = 357 Shillings.

P. £. *P.*

Stated thus—If 45 : 20 :: 58 ; $25\frac{7}{9}$£. in the ſame Time — And in twice that Time the 58 Perſons will ſpend $25\frac{7}{9}$ × 2=$51\frac{5}{9}$£'s Worth, at 17 Guineas per Head.

£. *G.* *s.* £. *s.* *d.*

Then—If $51\frac{5}{9}$: 17 or 357 : : 63 : 436 $2\frac{1}{2}.\frac{416}{464}$=£. 21 16*s.* $5\frac{1}{4}d.\frac{416}{464}$ per Hhd. which × by 2=£. 43 12*s.* $5\frac{3}{4}d.\frac{365}{464}$ per Pipe, the Anſwer.

(9) Firſt 8—5=3 Mile B. } gained of A, in one Day,
And 10—5=5 — C. }

Then

M. D. M. D.

Then ſtated thus—If $3 : 1 :: 73 : 24\frac{1}{3}$ when A and B meet.

Again, $5 : 1 :: 73 : 14\frac{3}{5}$ ——A and C——.

So that B nor C can never meet with A but at the End of theſe Periods, when A and C will have travelled 219 Miles.

D. D. D. M.

$\because$ Then, if $14\frac{3}{5} : 219 :: 24\frac{1}{3} : 365$

$\left.\begin{matrix}219\\365\end{matrix}\right\} \times$ by $\left\{\begin{matrix}24\frac{1}{3}\\14\frac{3}{5}\end{matrix}\right\} = 5329$ Days, the 73d Time of their general Meeting $\because 5329 \div 73 = 73$ Days, their firſt general Meeting.

M. D. M.

For as $73 : \left\{\begin{matrix}24\frac{1}{3}\\14\frac{2}{3}\end{matrix}\right\} :: \left\{\begin{matrix}219\\365\end{matrix}\right\} : 73$ Days.

(10) Firſt, for every Day he worked he received 12 Pence.
And for every Day he played he paid - 8 ——

Sum 20

Likewiſe, as his idle Days came to the ſame Money as thoſe he worked, therefore the Proportion will be as follows.

D. D. D.

As $20 : 390 : 8 : 156$ Days he worked.

And $20 : 390 : 12 : 234$ ——— played.

For 156, at 12*d.* per Day, comes to the ſame Money as 234 at 8*d.* per Day, *viz.* £.7 16*s.* the Proof.

(11) Firſt, fourſcore and eleven, or 91 Guineas $=5733$ Four-pences.

And from December 11th to May 20th $=150$ Days.

Alſo, 100 Marks $=4000$ Four-pences.

And from September 3d to Chriſtmas $=113$ Days.

Then ſtated thus—If $5733 : 150 :: 4000 ; 214\frac{305}{400}$, or rather 215 Days, from which take $113=102$ Days, and 40£. $=2400$ Four-pences.

Again, Recep. If $4000 : 102 :: 2400 : 170$ Days, the Anſwer nearly.

(12) Firſt £.41 8*d.*$=500$ Pence, and $500 \div 20 = 25$ Day's Wages; then $40-25=15$ Days more.

For every Day he worked he had - 20 Pence.
And for every Day he played, - 10

Sum 30 Pence.

Then

d. *D.* *d.* *Days.*

Then—If 30 : 15 :: $\left\{\begin{array}{l}10 : 5 \text{ worked.} \\ 20 : 10 \text{ idle.}\end{array}\right.$

∵ He was idle 10 Days, and worked $(5+25)=30$ Days.

P R A C T I C E.

CASE I.

(1)

$\frac{1}{4}$	$\frac{1}{4}$	2107 at $\frac{1}{4}$*d.*
	12	526$\frac{3}{4}$
	2,0	4,3 10
		£.2 3 10$\frac{3}{4}$

(2)

$\frac{1}{2}$	$\frac{1}{2}$	1470 at $\frac{1}{2}$*d.*
	12	735
	2,0	6,1 3
		£.3 1 3

(3)

$\frac{1}{2}$	$\frac{1}{2}$	1276 at $\frac{3}{4}$*d.*
$\frac{1}{4}$	$\frac{1}{2}$	638
		319
	12	957
	2,0	7,9 9
		£.3 19 9

CASE II.

(4)

12)1762 at 1*d.*

2,0) 14,6 10

£. 7 6 10

(5)

$\frac{1}{4}$	$\frac{1}{4}$	1400 at 1$\frac{1}{4}$*d.*
		350
	12	1750
	2,0	145 10
		£.7 5 10

(6)

(6)

d.		
$1\frac{1}{2}$	$\frac{1}{8}$	2462 at $1\frac{1}{2}d.$
	2,0	30,7 6
		£.15 7 6

(7)

d.		
$1\frac{1}{2}$	$\frac{1}{8}$	1041 at $1\frac{3}{4}d.$
$\frac{1}{4}$	$\frac{1}{6}$	130 $1\frac{1}{2}$
		21 $8\frac{1}{4}$
	2,0	15,1 $9\frac{3}{4}$
		£.7 11 $9\frac{3}{4}$

(8)

2	$\frac{1}{6}$	2490 at $2d.$
	2,0	41,5
		£.20 15

(9)

2	$\frac{1}{6}$	2408 at $2\frac{1}{4}d.$
$\frac{1}{4}$	$\frac{1}{8}$	401 4
		50 2
	2,0	45,1 6
		£.22 11 6

(10)

2	$\frac{1}{6}$	640 at $2\frac{1}{2}d.$
$\frac{1}{2}$	$\frac{1}{4}$	106 8
		26 8
	2,0	13,3 4
		£.6 13 4

(11)

2	$\frac{1}{6}$	1740 at $2\frac{3}{4}d.$
$\frac{1}{2}$	$\frac{1}{4}$	290
$\frac{1}{4}$	$\frac{1}{2}$	72 6
		36 3
	2,0	39,8 9
		£.19 18 9

(12)

3	$\frac{1}{4}$	746 at $3d.$
	2,0	18,6 6
		£.9 6 6

(13)

3		1417 at $3\frac{1}{4}d.$
$\frac{1}{4}$	$\frac{1}{12}$	354 3
		29 $6\frac{1}{4}$
	2,0	38,3 $9\frac{1}{4}$
		£.19 3 $9\frac{1}{4}$

(14)

3	$\frac{1}{4}$	3091 at $3\frac{1}{2}d.$
$\frac{1}{2}$	$\frac{1}{6}$	772 9
		128 9
	2,0	90,1 $6\frac{1}{2}$
		£.45 1 $6\frac{1}{2}$

d.		
(15) 3	¼	214 at 3¾*d.*
¾	¼	53 6
		13 4½
	2,0	6,6 10½
		£.3 6 10½

d.		
(16) 4	⅓	2000 at 4*d.*
	2,0	66,6 8
		£.33 6 8

(17) 3	¼	569 at 4¼*d.*
1	⅓	142 3
		47 5
	2,0	11 10¼
		20,1 6¼
		£.10 1 6¼

(18) 3	¼	1246 at 4½*d.*
1½	½	311 6
		155 9
	2,0	46,7 3
		£.23 7 3

(19) 3	¼	1426 at 4¾*d.*
1	⅓	356 6
¾	¼	118 10
		89 1½
	2,0	56,4 5½
		£.28 4 5½

(20) 4	⅓	2740 at 5*d.*
1	¼	913 4
		228 4
	2,0	114,1 8
		£.57 1 8

(21) 4	⅓	2147 at 5¼*d.*
1	¼	715 8
¼	¼	178 11
		44 8¾
	2,0	9339 3¾
		£.46 19 3¾

(22) 4	⅓	674 at 5½*d.*
		224 8
1½	⅛	84 3
	2,0	30,8 11
		£.15 8 11

(23)

d.		
(23) 3	$\frac{1}{4}$	1746 at $5\frac{1}{4}d.$
2	$\frac{1}{6}$	436 6
$\frac{1}{4}$	$\frac{1}{4}$	295 0
		109 1 = $\frac{1}{4}$ of
	2,0	83,6 $7\frac{1}{2}$
		£.41 16 $7\frac{1}{2}$

d.		
(24) 6	$\frac{1}{2}$	1741 at 6d.
	2,0	87,0 6
		£.43 10 6

(25) 4	$\frac{1}{3}$	2142 at $6\frac{1}{4}$
2	$\frac{1}{2}$	714
$\frac{1}{4}$	$\frac{1}{8}$	357
		44 $7\frac{1}{2}$
	2,0	111,5 $7\frac{1}{2}$
		£.55 15 $7\frac{1}{2}$

(26) 6	$\frac{1}{2}$	1040 at $6\frac{1}{2}d.$
$\frac{1}{2}$	$\frac{1}{12}$	520
		43 4
	2,0	56,3 4
		£.28 3 4

(27) 6	$\frac{1}{2}$	1746 at $6\frac{3}{4}d.$
$\frac{3}{4}$	$\frac{1}{8}$	873
		109 $1\frac{1}{2}$
	2,0	98,2 $1\frac{1}{2}$
		£.49 2 $1\frac{1}{2}$

(28) 4	$\frac{1}{3}$	1000 at 7d.
3	$\frac{1}{4}$	333 4
		250 0
	2,0	58,3 4
		£.29 3 4

(29) 6	$\frac{1}{2}$	1656 at $7\frac{1}{4}d.$
1	$\frac{1}{6}$	828
$\frac{1}{4}$	$\frac{1}{4}$	138
		34 6
	2,0	100,0 6
		£.50 0 6

(30) 6	$\frac{1}{2}$	1420 at $7\frac{1}{2}d.$
$1\frac{1}{2}$	$\frac{1}{4}$	710
		177 6
	2,0	88,7 6
		£.44 7 6

	d.		
(31)	4	⅓	674 at 7¾*d.*
	3		224 8
		¼	168 6
	¾	¼	42 1½=¼ of 3*d.*
		2,0	43,5 3½
			£.21 15 3½

	d.		
(32)	6	½	2170 at 8*d.*
	2	⅓	1085
			361 8
		2,0	144,6 8
			£.72 6 8

(33)	6	½	1700 at 8¼*d.*
	2	⅓	850
	¼	⅛	283 4
			35 5
		2,0	116,8 9
			£. 58 8 9

(34)	6	⅓	1765 at 8½*d.*
	2	⅓	882 6
	½	¼	294 2
			73 6½
		2,0	125,0 2½
			£.62 10 2½

(35)	6	½	749 at 8¾*d.*
	2	⅓	374 6
			124 10
	¾	⅛	46 9¾=⅛ of 6*d.*
		2,0	54,6 1¾
			£.27 6 1¾

(36)	6	½	1417 at 9*d.*
	3	½	708 6
			354 3
		2,0	106,2 9
			£.53 2 9

(37)	6	½	2373 at 9¼*d.*
	3	½	1186 6
	¼	1/12	593 3
			49 5¼
		2,0	182,9 2¼
			£.91 9 2¼

(38)	6	½	1476 at 9½*d.*
	2	⅓	738
			246
	1½	¼	184 6=¼ of 6*d.*
		2,0	116,8 6
			£.58 8 6

M

(39)

(39)

d.		
6	$\frac{1}{2}$	1760 at $9\frac{3}{4}d.$
3	$\frac{1}{2}$	880
$\frac{3}{4}$	$\frac{1}{4}$	440
		110
	2,0	143,0
		£.71 10

(40)

d.		
6	$\frac{1}{2}$	6000 at $10d.$
		3000
4	$\frac{1}{3}$	2000
	2,0	500,0
		£.250

(41)

6	$\frac{1}{2}$	4652 at $10\frac{1}{4}d.$
3	$\frac{1}{2}$	2326
1	$\frac{1}{3}$	1163
$\frac{1}{4}$	$\frac{1}{4}$	387 8
		96 11
	2,0	397,3 7
		£.198 13 7

(42)

6	$\frac{1}{2}$	2476 at $10\frac{1}{2}d.$
3	$\frac{1}{2}$	1238
$1\frac{1}{2}$	$\frac{1}{2}$	619
		309 6
	2,0	216,6 6
		£.108 6 6

(43)

6	$\frac{1}{2}$	2176 at $10\frac{3}{4}d.$
		1088
4	$\frac{1}{3}$	725 4 = $\frac{1}{3}$ of 1*s.*
$\frac{3}{4}$	$\frac{1}{8}$	136 0 = $\frac{1}{8}$ of 6*d.*
	2,0	194,9 4
		£.97 9 4

(14)

6	$\frac{1}{2}$	1276 at $11d.$
3	$\frac{1}{2}$	638
		319
2	$\frac{1}{3}$	212 8 = $\frac{1}{3}$ of 6*d.*
	2,0	116,9 8
		£.58 9 8

(45)

6	$\frac{1}{2}$	2142 at $11\frac{1}{4}d.$
		1071
4	$\frac{1}{3}$	714
1	$\frac{1}{4}$	178 6
$\frac{1}{4}$	$\frac{1}{4}$	44 $7\frac{1}{2}$
	2,0	200,8 $1\frac{1}{2}$
		£.100 8 $1\frac{1}{2}$

(46)

6	$\frac{1}{2}$	4760 at $11\frac{1}{2}d.$
		2380
4	$\frac{1}{3}$	1586 8
$1\frac{1}{2}$	$\frac{1}{4}$	595 0 = $\frac{1}{4}$ of 6*d.*
	2,0	456,1 8
		£.228 1 8

d.			
(47) 6	$\frac{1}{2}$	640 at 11$\frac{3}{4}$*d.*	
3	$\frac{1}{2}$	320	
		160	
2	$\frac{1}{3}$	106 8	$=\frac{1}{3}$ of 6*d.*
$\frac{3}{4}$	$\frac{1}{4}$	40 0	$=\frac{1}{4}$ of 3*d.*
	2,0	62,6 8	
		£.31 6 8	

When the Price is 2, 3, 4, 6, or 8*d.*—Some Arithmeticians make Uſe of this Method.—Thus; for 2*d.* they divide the given Quantity by 120; for 3*d.* by 80; for 4*d.* by 60; for 6*d.* by 40, and for 8*d.* by 30, which gives the Anſwer in Pounds. But this Method I do not approve of, for young Beginners, becauſe the Remainder may ſometimes be very large, and therefore to value it, will be a greater Taſk to them, and take more Time than in doing the whole Work; otherwiſe.

CASE III.

d.			
(49) $\frac{1}{4}$	$\frac{1}{4}$	2140 at 1*s.* $\frac{1}{4}$*d.*	
	12	535	
		44 7	added to [2140
	2,0	218,4 7	
		£.109 4 7	

d.		
(50) 6	$\frac{1}{2}$	1749 at 1*s.* 11$\frac{1}{2}$*d.*
1$\frac{1}{2}$	$\frac{1}{4}$	874 6
4	$\frac{1}{3}$	583 0
		218 7$\frac{1}{2}$
	2,0	342,5 1$\frac{1}{2}$
		£.171 5 1$\frac{1}{2}$

(51) 4	$\frac{1}{3}$	2140 at 1*s.* 5*d.*
1	$\frac{1}{4}$	713 4
		178 4
	2,0	303,1 8
		£.151 11 8

(52) 6	$\frac{1}{2}$	1453 at 1*s.* 7$\frac{1}{2}$*d.*
1$\frac{1}{2}$	$\frac{1}{4}$	726 6
		181 7$\frac{1}{2}$
	2,0	236,1 1$\frac{1}{2}$
		£.118 1 1$\frac{1}{2}$

(53)

6	$\frac{1}{2}$	1614 at 1*s.* 10*d.*
		807
4	$\frac{1}{3}$	538
	2,0	295,9
		£.147 19

(54)

6	$\frac{1}{2}$	2647 at 1*s.* 11$\frac{3}{4}$*d.*
3	$\frac{1}{2}$	1323 6
		661 9
2	$\frac{1}{3}$	441 2 $= \frac{1}{2}$ of 6*d*
$\frac{3}{4}$	$\frac{1}{4}$	165 5$\frac{1}{4}$
	2,0	523,8 10$\frac{1}{4}$
		£.261 18 10$\frac{1}{4}$

The 53d Example may be performed thus —1*s.* 8*d.* is $\frac{1}{12}$ of a Pound, 2*d.* is $\frac{1}{10}$ of 1*s.* 8*d.*

CASE IV.

(55) 2476 at 2*s.*
1
£. 247 12

(56) 1476 at 4*s.*
2
£. 295 4

(57) 276 at 6*s.*
3
£. 82 16

(58) 2100 at 8*s.*
4
£. 840 0

(59) 274 at 10*s.*
5
£. 137 0

(60) 674 at 12*s.*
6
£. 404 8

(61) 2680 at 14*s.*
7
£. 1876 0

(62) 267 at 16*s.*
8
£. 213 12

(63) 1267 at 18*s.*
9
£. 1140 6

The 55, 56, and 59th Examples may be performed thus; 2*s.* is $\frac{1}{10}$, 4*s.* is $\frac{1}{5}$, and 10*s.* is $\frac{1}{2}$ of a Pound, therefore the given Quantities being divided by 10, 5, and 2, will give the Answer as above.

CASE

CASE V.

(66) 2174 at 7s.
7
2,0)1521,8
£. 760 18

(67) 1427 at 9s.
9
2,0)1284,3
£. 642 3

(68) 647 at 11s.
11
2,0)711,7
£. 355 17

(69) 267 at 13s.
13
2,0)347,1
£. 173 11

(70) 274 at 17s.
17
2,0)465,8
£. 232 18

(71) 1260 at 19s.
19
2,0)2394,0
£. 1197 0

CASE VI.

(72) 4s. $\frac{1}{5}$)2420 at 4s.
£. 484

(73) 5s. $\frac{1}{4}$)1764 at 5s.
£. 441

(74) 1s. 8d. $\frac{1}{12}$)4762 at 1s. 8d.
£. 396 16 8

(75) 2s. 6d. $\frac{1}{8}$)467 at 2s. 6d.
£. 58 7 6

(76) 3s. 4d. $\frac{1}{6}$)1760 at 3s. 4d.
£. 293 6 8

(77) 6s. 8d. $\frac{1}{3}$)176 at 6s. 8d.
£. 58 13 4

CASE VII.

(78)

d.		
3	$\frac{1}{4}$	1420 at 3s. 3d.
		3
		4260
		355
2,0		461,5
		£. 230 15

(79)

d.		
6	$\frac{1}{2}$	427 at 5s. 9d.
		5
		2135
3	$\frac{1}{2}$	213 6
		106 9
2,0		245,5 3
		£. 122 15 3

(80)

(80)

d.		
6	$\frac{1}{2}$	402 at 10*s.* 8$\frac{3}{4}$*d.*
		10
		4020
2	$\frac{1}{3}$	201
		67
$\frac{3}{4}$	$\frac{1}{8}$	25 1$\frac{1}{2}$ = $\frac{1}{8}$ of 6*d.*
	2,0	431,3 1$\frac{1}{2}$
		£.215 13 1$\frac{1}{2}$

(81)

d.		
6	$\frac{1}{2}$	174 at 17*s.* 9$\frac{1}{2}$*d.*
		17
		2958
3	$\frac{1}{2}$	87
$\frac{1}{2}$	$\frac{1}{6}$	43 6
		7 3
	2,0	309,5 9
		£.154 15 9

(82)

3*d*	$\frac{1}{4}$	273 at 19*s.* 4$\frac{3}{4}$*d.*
		19
		5187
1	$\frac{1}{3}$	68 3
		22 9
$\frac{3}{4}$	$\frac{1}{4}$	17 0$\frac{3}{4}$ = $\frac{1}{4}$ of 3*d.*
	2,0	5295 0$\frac{3}{4}$
		£.264 15 0$\frac{3}{4}$

(83)

6*d*	$\frac{1}{2}$	260 at 14*s.* 11$\frac{3}{4}$*d.*
		14
		3640 = 14*s.*
3	$\frac{1}{2}$	130 = 6*d.*
		65 = 3
2	$\frac{1}{3}$	43 4 = 2
$\frac{3}{4}$	$\frac{1}{4}$	16 3 = $\frac{3}{4}$ of 3*d.*
	2,0	3894 7
		£.194 14 7

Example 78. may be performed thus—2*s.* is $\frac{1}{10}$, 1*s.* is $\frac{1}{2}$ of 2*s.* and 3*d.* is $\frac{1}{4}$ of 1*s.*—Example 79. thus; 5*s.* are $\frac{1}{4}$, 6*d.* is $\frac{1}{10}$ of 5*s.* and 3*d.* is $\frac{1}{2}$ of 6*d.* Likewise, Example 80. thus; 10*s.* is $\frac{1}{2}$ of a Pound, 6*d.* is $\frac{1}{20}$ of 10*s.* 2*d.* is $\frac{1}{3}$ of 6*d.* and $\frac{3}{4}$ is $\frac{1}{8}$ of 6*d.* which Parts added together, will give the Anſwers as before.

CASE VIII.

(84) 120 at 4 £.
4
£. 480

(85) 96 at 17 £.
17
£. 1632

(86)

(86) 100 at 3 £.
3
£. 300

(87) 142 at 42 £.
42=6×7
£. 5964

CASE IX.

+ (88) 649 at 2 £. 6*s.*
2
1298
194 14
£. 1492 14

× (89) 526 at 7 £. 16*s.*
7
3682
420 16
£ 4102 16

(90) 142 at 1 £. 17*s.*
17
2,0)241,4
120 14 added to 142 £.
£. 262 14

× (91) 164 at 24 £. 19*s.*
20
24 £. 19=499*s.*
164
1996
7984
2,0)8183,6
£. 4091 16

(92) 271 at 5 £. 7*s.*
5
1355 £.
2,0)189,7=271×7
94 17 added to 1355 £.
£. 1449 17

(93) 604 at 20 £. 9*s.*
20
12080
5 | $\frac{1}{4}$ | 151
4 | $\frac{1}{5}$ | 120 16
£. 12351 16

(94)

(94) 10s. | ½ | 914 at 10£. 15s.
10
9140
5 | ½ | 457
228 10
£. 9825 10

(95) 737 at 1£. 14s.
7
512 18 added to [737£.
£. 1249 18

CASE X.

(96)
6s. 8d | ⅓ | 274 at 7£. 6s. 8d.
7
1918
91 6 8
£. 2009 6 8

(97)
3s. 4d. | ⅙ | 120 at 12£. 3 4
12
1440
20
£. 1460

(98)
1s. 8d. | $\frac{1}{12}$ | 97 at 9£. 1s. 8d.
9
873
8 1 8
£. 881 1 8

(99)
5s. | ¼ | 512 at 42£. 5s.
42
21504
128
£. 21632

CASE XI.

(100)
1472 at £. 4 6 7½
4
6d | $\frac{1}{12}$ | 5888
441 12
1½ | ¼ | 36 16
9 4
£. 6375 12

(101)
10s. | ½ | 279 at £. 6 11 9¾
6
1674 0
1 | $\frac{1}{10}$ | 139 10
6d | ½ | 13 19
3 | ½ | 6 19 6
¾ | ¼ | 3 9 9
0 17 5¼
£. 1837 15 8¼

(102)

```
10s|½|  1420 at £.19 14 11¾
   | |    19
   | |  -----
   | |  26980
   | |    710
  4|⅕|    284
 6d|⅛|     35 10=⅛ of 4s.
  3|½|     17 15=½ of 6
  2|⅓|     11 16 8
  ¾|¼|      4  8 9
   | |  ---------------
     £.28043 10 5
```

(103)

```
4d| ⅓|  2074 at £.1 17 5½
  |  |    37        20
  |  |  -----       --
  |  |  76738       37
  |  |    691 4
 1½| ⅛|   259 3
  |  |  ---------
  2,0|  7768,8 7
      ---------
  £.3884 8 7
```

(104)

```
        £.  s.  d.
27 at   4  11  8¼
              9×3=27
      ----------
       41   5  6¼
               3
      ----------
   £.123  16  8¾
```

(105)

```
       £.   s.  d.
64 at 12   13  7¼
              8×8=64
      ----------
      101   8  10
               8
      ----------
   £.811  10  8
```

CASE XII.

(106)

```
                              £.  s.  d.
10C. 3qrs. 14lb. at           2  11  10½
                                     10
                           -----------
qrs. lb.
 2          | ½|          25  18  9
 1          | ½|           1   5  11¼
      14    | ½|              12  11½
                               6   5¾
                           -----------
                        £.28   4   1½
```

(107)

(107)

qrs. lb. 72 *C.* 3 *qrs.* 19 *lb.* at

		£.	s.	d.
2	$\frac{1}{2}$	3	17	4½
				9×8=72
		34	16	4½
				8
		278	11	0
1	$\frac{1}{2}$	1	18	8¼
16	$\frac{1}{7}$		19	4
2			11	0½
1	$\frac{1}{2}$		1	4½
				8¼
		£. 282	2	1½

(108)

lb. 12 *C.* 1 *qr.* 17 *lb.* at

		£.	s.	d.
1	$\frac{1}{4}$	4	4	4
				12
		0	12	0
14	$\frac{1}{2}$	1	1	1
2	$\frac{1}{7}$		10	6½
1	$\frac{1}{2}$		1	6
				9
		£. 52	5	10½

(109)

lb. 23 *C.* 18 *lb.* at

		£.	s.	d.
16	$\frac{1}{7}$	4	14	1½
				11×2+1=23
		49	15	4½
				2
		99	10	9
2	$\frac{1}{8}$	4	14	1½
			13	5¼
			1	8
		£. 104	19	11¾

(110)

lb. 94 *C.* 27 *lb.* at

		£.	s.	d.
16	$\frac{1}{7}$	6	19	6
				10×9+4=94
		69	15	0
			9	
		627	15	0
		27	18	0
8	$\frac{1}{2}$		19	11
2	$\frac{1}{4}$		9	11½
1	$\frac{1}{2}$		2	5¾
			1	2¾
		£. 657	6	7

(1

(111)

(111)

16*C.* 2*qrs.* at

qrs.		*£.*	*s.*	*d.*
2	$\frac{1}{2}$	2	6	11
				$4\times4=16$
		9	7	8
				4
		37	10	8
		1	3	$5\frac{1}{2}$
		£. 38	14	$1\frac{1}{2}$

(112)

48*C.* 2*qrs.* 7*lb.* at

		£.	*s.*	*d.*
2*qrs.*	$\frac{1}{2}$	74	16	6
				$8\times6=48$
		598	12	0
			6	
		3691	12	0
7*lb.*	$\frac{1}{8}$	37	8	3
		4	13	$6\frac{1}{4}$
		£. 3733	13	$9\frac{1}{4}$

Wrong

(113) *lb.* 24*lb.* at

lb.		*£.*	*s.*	*d.*
16	$\frac{1}{7}$	4	17 per *C.*	
8	$\frac{1}{2}$	0	13	$10\frac{1}{4}$
		0	6	11
		£. 1	0	$9\frac{1}{4}$

(114) 17*lb.* at

lb.		*£.*	*s.*	*d.*
14	$\frac{1}{8}$	3	5	4
2	$\frac{1}{7}$	0	8	2
1	$\frac{1}{2}$	0	1	2
		0	0	7
		£. 0	9	11

When the Pupil is perfect in all the Rules to the foregoing Cases, he may then learn the Contractions, some of which are as follow.

Contractions to Case I. and II.

2	$\frac{1}{80}$	1276 at $\frac{3}{4}d.$		
$\frac{3}{4}$	$\frac{1}{4}$	15	19	
		£. 3	19	9

See Example 3.

3	$\frac{2}{80}$	174,0 at $2\frac{1}{4}d.$			
$\frac{1}{4}$	$\frac{1}{12}$	21	15	0	subtract.
		1	16	3	
		£. 19	18	9	

See Example 11.

s. d.
2 6 | $\frac{1}{8}$ | 1420 at $7\frac{1}{2}d.$
$7\frac{1}{2}$ | $\frac{1}{4}$ | 177 10
£. 44 7 6
See Ex. 30.

s. d.
6 0 | $\frac{3}{10}$ | 1417 at 9*d.*
3—See Case 4.
9 | $\frac{1}{8}$ | 425 2
£. 53 2 9
See Ex. 36.

Contractions to Case III. VII. and IX.

s. d.
5 | $\frac{1}{4}$ | 246 at 15*d.*
1 3 | $\frac{1}{4}$ | 61 10
£. 15 7 6

s. d.
1 8 | $\frac{1}{12}$ | 3162 at 1*s.* 7*d.*
1 | $\frac{1}{20}$ | 263 10 0 } subtract.
13 3 6
£. 250 6 6

s. 6*d.*
6 | $\frac{3}{10}$ | 427 at 5*s.* 9*d.*
3
128,2 0 } subtract.
3 | $\frac{1}{80}$ | 5 6 6
£. 122 15 6

18*s* | $\frac{9}{10}$ | 246 at 17*s.* 10*d.*
9
221 8 } subtract.
$\frac{1}{120}$ | 2 1
£. 219 7

2*d* | $\frac{1}{120}$ | 241 at £. 6 19 10
7
1687 } subtract.
2 0 2
£. 1684 19 10

5*s.* | $\frac{1}{4}$ | 641 at £. 1 15
2
1282 0 } subtract.
160 5
£. 1121 15

18. TARE

18. TARE and TRETT.

(2) First, 6*C.* 2*qrs.* 17*lb.*×6×4 (24 the No. of Hhds.)= *C.* 159 2*qrs.* 16*lb.* Grofs, from which take *C.*17 3*qrs.* 27*lb.* Remains *C.*141 2*qrs.* 17*lb.* Neat, the Anfwer.

(3) Grofs *C.*23 2*qrs.* 19*lb*; Tare *C.*1 12*lb.*; Difference, *C.* 22 2*qrs.* 7*lb.* the Neat required.

(5) First, *C.*2 3*qrs.*×8=22*C.* Grofs; and 22*lb.*×8=*C.*1 2*qrs.* 8*lb.*; then from 22*C.* take *C.*1 2*qrs.* 8*lb.* Remains *C.*20 1*qr.* 20*lb.* Neat, the Anfwer.

(6) First, *C.*3 0*qr.* 27*lb.*×8×5 (40 the Number of Bales)= *C.*129 2*qrs.* 16*lb.* Grofs, and 18*lb.*×8×5=*C.*6. 1*qr.* 20*lb.* Tare; then from *C.*129 2*qrs.* 26*lb.* take *C.*6 1*qr.* 20*lb.* Remains *C.*123 0*qr.* 24*lb.* Neat, the Anfwer.

(8) First, *C.*2 1*qr.* 17*lb.*×2×11 (22 Barrels)=*C.*52 3*qrs.* 10*lb.* Grofs, which÷by 8, (becaufe 14*lb.* is $\frac{1}{8}$ of 1*Cwt.*) gives *C.*6 2*qrs.* 11*lb.* 12*oz.* Tare, which take from the Grofs, will remain *C.*46 0*qr.* 14*lb.* 4*oz.* Neat. X

(9) First, *C.*6 2*qrs.* 12*lb.*×9=*C.*59 1*qr.* 24*lb.* Grofs; then 14*lb.* is $\frac{1}{8}$, viz. *C.*7 1*qr.* 20*lb.* 8*oz.* and 2*lb.* $\frac{1}{7}$ of 14*lb.* viz. *C.*1 0*qr.* 6*lb.* 14*oz.* Alfo, 1*lb.* is $\frac{1}{2}$ of 2*lb.* viz. 2*qrs.* 3*lb.* 7*oz.* which Parts, added together=*C.*9 0*qr.* 2*lb.* 13*oz.* Tare, which taken from the Grofs, viz. *C.*59 1*qr.* 24*lb.*=*C.*50 1*qr.* 21*lb.* 3*oz.* Neat, the Anfwer.

(11) First, from *C.*16 3*qrs.* take *C.*1 1 *qr.* 12*lb*; remains *C.*15 1*qr.* 16*lb.* Suttle, which, ÷ by 26, (4*lb.* being $\frac{1}{26}$ of 104) gives 2*qrs.* 10*lb.* 4*oz.* Trett; this, taken from the Suttle, leaves *C.*14 3*qrs.* 5*lb.* 12*oz.* Neat, the Anfwer.

(12) First, *C.*2 3*qrs.* 27*lb.*×9×3 (27 Bags)=*C.*80 3*qrs.* 1*lb.* Grofs, then as 13*lb.* cannot be conveniently divided into aliquot Parts of 112, I take for 16*lb.* which is $\frac{1}{7}$; therefore I ÷ the Grofs by 7, which gives *C.*11 2*qrs.* 4*lb.* 2*oz.* Tare, fuppofing it had been 16*lb.* per *C.* from which I take Parts with the 13*lb.* viz. 8*lb.* is $\frac{1}{2}$ of 16*lb.* viz. *C.*5 3*qrs.* 2*lb.* 1*oz.* and 4 is $\frac{1}{2}$ of 8, viz. *C.*2 2*qrs.* 3*lb.* 15*oz.* Alfo, 1*lb.* is $\frac{1}{4}$ of 4, viz. 2*qrs.* 24*lb.* 12*oz*; thefe, added together,=*C.*9 1*qr.* 13*lb.* 13*oz.* Tare, which, taken from the Grofs,=*C.*71 1*qr.* 15*lb.* 3*oz.* Suttle; this÷by 26=*C.*2. 2*qrs.* 27*lb.* 8*oz.* Trett, which taken from the Suttle, leaves *C.*68 2*qrs.* 15*lb.* 11*oz.* Neat, the Anfwer.

14) First, C.42 1*qr.*=172*qrs.* these ÷ by 168=1*qr.* $\frac{1}{2}$*lb.* Cloff, which taken from C.42 3*qrs.*=C.42 1*qr.* 27$\frac{1}{2}$*lb.* Neat, the Anfwer.

(16) First, the 4 Hogfheads added together=C.20 3*qrs.* 13*lb.* Grofs, which ÷ by 7=C.2 3*qrs.* 25$\frac{6}{7}$*lb.* Tare, fuppof-ing it was at 16*lb.* per 112; but it is only 8*lb.* which is $\frac{1}{2}$ of 16, viz. C.1 1*qr.* 26$\frac{1}{2}$*lb.* Tare, taken from the Grofs, leaves C.19 1*qr.* 14*lb.* 8*oz.* Suttle; this ÷ by 26=C.0 2*qrs.* 27*lb.* 7$\frac{9}{13}$*oz.* Trett, this taken from the Suttle, leaves C.18 2*qr.* 15*lb.* 0$\frac{4}{13}$*oz.* Suttle; this, ÷ by 168, =12*lb.* 6*oz.* Cloff, which take from the laft Suttle, leaves C.18 2*qrs.* 3*lb.* 10$\frac{4}{13}$*oz.* Neat, the Anfwer.

QUESTION I.

	£.	s.	d.		C.	qr.	lb.	
Neat Proceeds,	4	14	6	$\frac{1}{10}$	9	3	10	Grofs.
Cuftom, &c.	2	8	6		0	3	26$\frac{1}{3}$	Tare.
Freight,	1	2	8					
Factorage,	0	4	9	C.	8	3	11$\frac{4}{5}$	Net=4959 Fifths.

And 1 *Cwt.* =560 Ditto.

£. 8 10 5=2045 Pence.

d.

Stated thus—If 4959 : 2045 :: 560
560

4959)1145200(230$\frac{3}{4}$*d.*$\frac{3643}{4959}$, or £.0 19*s.* 2$\frac{3}{4}$*d.* $\frac{3643}{4959}$ the Anfwer.

Quest. 2.—First, 1180×80=94400 Inches, which, ÷ by 231, (the Number of Inches in a Gallon) gives 408$\frac{152}{231}$ Gallons; thefe, ×7$\frac{1}{2}$*lb.* (the Number of *lb.* in a Gallon) gives 3064$\frac{216}{231}$*lb.* Grofs, which, ÷ by 10, =306$\frac{114}{231}$*lb.* Tare; this, taken from the Grofs,=2758$\frac{102}{231}$*lb.* Neat.

Then, ftated thus—If 112*lb.* : 4*s.* 6*d.* or 54*d.* :: 2758$\frac{102}{231}$*lb.* : 1329$\frac{3}{4}$*d.*$\frac{22032}{25872}$, or £.5 10*s.* 9$\frac{3}{4}$*d.*$\frac{22032}{25872}$, the Anfwer.

19. SIMPLE

19. SIMPLE INTEREST.

(2) First, £.824 18*s.* 2*d.* Principal, × by 4, Rate per Cent. =£.3299 12*s.* 8*d.* which, ÷ by 100, gives £.32 19*s.* 11*d* $\frac{12}{100}$, the Answer. ✗

(3) First, 500£. × 5 = 2500£. which, ÷ by 100, = 25£. Interest for 1 Year; this, × by 4, (the Number of Years) = 100£. the Answer. ✗

(4) First, £.526 18*s.* 8*d.* × 4, the Rate = £.2107 14*s.* 8*d.* this × 9, (the Number of Years) = £.18969 12*s.* which ÷ by 100 = £.189 13*s.* 11*d.* $\frac{4}{100}$, the Interest; this added to £.526 18*s.* 8*d.* Principal, gives £.716 12*s.* 7*d.* $\frac{4}{100}$ the Amount required.

(5) First, £.264 0*s.* 4*d.* × by 5 = £.1320 1*s.* 8*d.* this, × by 12 (the Time) = 15841£. which, ÷ by 100, = £.158 8*s.* 2¼*d.* $\frac{6}{10}$ Interest; which, added to the Principal, gives £.422 8*s.* 6¼*d.* $\frac{6}{10}$, the Amount required.

(7) First, £.520 10*s.* 10*d.* × by 3½ = £.1821 17*s.* 11*d.* which, ÷ by 100 = £.18 4*s.* 4½*d.* Interest for a Year; this × by 3 = £.54 13*s.* 1½*d.* Interest for 3 Years, added to the Principal, gives £.575 3*s.* 11½*d.* the Amount.

(8) First, £.140 10*s.* × 4¾ = £.667 7*s.* 6*d.* ÷ 100 = £.6 13*s.* 5½*d.* ⅗ Interest for a Year, which, × by 7, = £.46 14*s.* 3¼*d.* Interest for 7 Years, the Answer.

(9) First, £.470 × 3¼ = £.1527 10*s.* ÷ 100 = £.15 5*s.* 6*d.* Interest for a Year; which, × 5, = £.78 7*s.* 6*d.* Interest for 5 Years; this, added to the Principal, viz. £.470 = £.548 7*s.* 6*d.* the Amount required.

(11) First, £.742 12*s.* 6*d.* × 2½ = £.1856 11*s.* 3*d.* this, ÷ by 100 = £.18 11*s.* 3¼*d.* the Answer.

(12) First, £.374 19*s.* 10¾*d.* × 2 = £.749 19*s.* 9½*d.* which ÷ by 100 = £.7 9*s.* 11¼*d.* $\frac{9}{10}$, the Answer.

BROKAGE.

(14) First, 1000£. ÷ 100 = 10£. then, by Practice,

s.	d.		£.
4	0	⅕	10 at 4*s.* 6*d.*
	6	⅛	2 0
			0 5
Answer,			£. 2 5

Errors in No. 8 and 9

(15) First, £.540 10*s.* ÷ 100 = £.5 8*s.* 1*d.* then

s.	*d.*		£.	*s.*	*d.*	
10	0	½	5	8	1	at 13 10½
3	4	¾	2	14	0½	
			0	18	0	
	6	$\frac{1}{20}$	0	2	8¼	
Answ. £.			3	14	8¾	

(16) First, £.2474 15 ÷ 100 = £.24 14*s.* 11¼*d.* then

s.	*d.*		£.	*s.*	*d.*	
10	0	½	24	14	11¼	at 19 9½
			12	7	5½	
6	8	⅓	8	4	11¾	
2	6	⅛	3	1	10¼	
	7½	¼	0	15	5½	
Answ. £.			24	9	9	

INSURANCE.

(18) First, £.2460 × 10¾ = £.26445, which ÷ 100 = £.264 9*s.* the Insurance required.

(19) First, £.2500 × 6⅞ = £.17187 10*s.* which, ÷ by 100 = £. 171 17*s.* 6*d.* the Insurance required.

(20) First, £.7406 17*s.* 6*d.* × 15¾ = £. 116658 5*s.* 7¾*d.* ½ which ÷ 100 = £.1166 11*s.* 7¾*d.* the Insurance required.

PURCHASING of STOCKS.

(22) First, £. 460 × 87¾ = 403,65£. which ÷ by 100 = £. 403 13*s.* the Purchase required.

(23) First, £.2470 17*s.* 10*d.* × 3½ (the Excess) = £ 8648 2*s.* 5*d.* which ÷ 100 = £.86 9*s.* 7¼*d.* $\frac{96}{100}$; this, added to £.2470 17*s.* 10*d.* the Stock, gives £.2557 7*s.* 5¼*d.* $\frac{96}{100}$ the Answer.

(24) First, 876£. × 14⅝ (the Excess) = £. 12811 10*s.* which ÷ by 100 = £.128 2*s.* 3½*d.* $\frac{4}{10}$; this added to 876£. the Stock, gives £. 1004 2*s.* 3½*d.* $\frac{4}{10}$ the Purchase required.

(26)

		£.	*s.*	*d.*	
		246	12	6	
Year.				5	
¼	¼	1233	2	6	= 1 Year.
		308	5	7½	= ¼
		£.15,41	8	1½	; this ÷ by 100 = £. 15

8*s.* 3¼*d.* the Answer.

(27)

	£.	s.	d.	
½ ½	298	11	0	
		×4		
	1195	12	0	
¼ ½	149	9	0	
	74	14	6	
Yr. ½ ½	1419	15	6	=1 Year.
		×4		
	5679	2	0	
	709	17	9	

£. 63,88 19 9; this ÷ by 100 £ 63 17*s.* 8½*d.* $\frac{84}{100}$, the Interest, which, added to the Principal, gives £.362 15*s.* 8½*d.* $\frac{84}{100}$, what is due, the Answer.

(28)

	£.	s.	d.	
½ ½	604	17	6	
			×5	
	3024	7	6	
	302	8	9	
Mo. 6 ½	3326	16	3	=1 Year.
			×3	=3 Years.
	9980	8	9	
3 ½	1663	8	1½	
	831	14	0¾	

£. 124,75 10 11¼=3½ Years, this ÷ by 1,00 = £.124 15 *s.* 1 *d.* the Interest, which added to the Principal gives £. 729 12*s.* 7*d.* the Amount required.

(30) First, £.600 × 3¾=22,50£. this ÷ by 1,00 = £.22 10*s.* the Interest for 1 Year; then, by the Rule of Three,

If 52 *W.* : 22£. 10*s.* :: 26 *W.*

Here £.22 10*s.*=450*s.* these × 26 = 11700*s.* which ÷ by 52=225*s.* or £.11 5*s.* the Interest for 26 Weeks; this added to the 600£. gives £.611 5*s.* the Amount required.

(31) First, 740£.×5=37,00£.÷1,00=37£. Interest for a Year; then

If 52 *W.* : 37£. :: 42 *W.*

Here 37×42=1554£. these ÷ by 52=£.29 17*s.* 8¼*d.* $\frac{12}{52}$, the Interest for 42 Weeks, which, added to 37£.×4, viz. 148£. the Interest for 4 Years, gives £.177 17*s.* 8¼*d.* $\frac{12}{452}$, the Interest for 4 Years, 42 Weeks, the Answer.

(32) First, 200£.×4½=9,00£.÷1,00=9£. Interest for a Year; then

If 51 *W.* : 9£. :: 50 *W.*

Here $9 \times 50 = 450 \div 52 =$ £.8 13*s.* $0\frac{3}{4}$*d.* $\frac{36}{52}$, Interest for 50 Weeks; which added to the Interest for a Year gives £.17 13*s.* $0\frac{3}{4}$*d.* $\frac{36}{52}$, the whole Interest; this added to 200£.=£.217 13*s.* $0\frac{3}{4}$*d.* $\frac{36}{52}$, the Amount required.

(34) First, £.340 10*s.* × 5 = £.17,02 10*s.* ÷ 1,00 = £.17 0*s.* 6*d.* the Interest for a Year.

And from Jan. the 1st to July the 18th, = 199 Days by the Table, then,

	D.	£.	*s.* *d.*	*D.*
If	365 :	17	0 6 ::	199

Here £.17 0*s.* 6*d.* = 4068 Pence; these × 199 = 813114*d.* which ÷ 365 = $2227\frac{1}{2}$*d.* $\frac{306}{365}$, or £.9 5*s.* $7\frac{1}{2}$*d.* $\frac{306}{365}$ Answ.

(35) First, 500 £. × 4 = 20,00£. ÷ 1,00 = 20£. Interest for a Year.

And from December the 4th, 1772, to March the 10th, 1774 = 1 Year, 96 Days; then,

	D.	£.	*D.*
If	365 :	20 ::	96

Here 96 × 20 = 1920£. this, ÷ 365 £.5 5*s.* $2\frac{1}{4}$*d.* $\frac{315}{365}$ Interest for 96 Days; which, added to 20£. (Interest for a Year) = £.25 5*s.* $2\frac{1}{4}$*d.* $\frac{315}{365}$, the Interest required.

(37) First, £.4 10*s.* Rate × $9\frac{1}{2}$ Years, Time = £.42 15*s.* Interest of 100£. for $9\frac{1}{2}$ Years; which Interest added to 100£. = £.142 15*s.* the Amount; then

	£. *s.*	£.	£.	*s.* *d.*
If	142 15 :	100 ::	85	6 10

Here £.142 15*s.* = 2855*s.* and £.856 10*s* = 17130*s.* which × 100 = 1713000£. these ÷ 2855 = 600£. the Answer.

(38) First, 3£. Rate, × $7\frac{1}{2}$, Time, = 13.20£. ÷ 1,00 = £.23 5*s.* Interest; which, added to 100, Principal, = £.123 5*s.* the Amount; then,

	£. *s.*	£.	£.	*s.* *d.*
If	123 5 :	100 ::	614	3 11

Here £.123 5*s.* = 29580*d.* and £.614 3*s.* 11*d* = 147407*d.* × 100 = 1474070,0£. ÷ 2958,0 = £.498 6*s.* 8*d.* the Answer.

(40) First, 600£. × $4\frac{1}{2}$ = 2700£. ÷ 100 = 27£. Interest of 600£. for a Year; and from £.856 10*s.* take 600£. remains £.256 10*s.* Interest of 600£. for the whole Time; then,

If

$$\begin{array}{l} \quad £.\ \ Y.\ \ £.\ \ s. \\ \text{If } 27 : 1 :: 256\ 10 \\ \quad 20 \qquad\quad 20 \\ \hline 54,0\ )513,0(9\tfrac{1}{2} \text{ Years, Anſwer.} \end{array}$$

(41) Firſt, £. 498 6*s.* 8*d.* × 3 = £. 1459 ÷ 100 = £. 14 19*s.* Intereſt of the given Principal for a Year; and £. 614 3*s.* 11*d.*—£. 498 6*s.* 8*d.* = £. 115 17*s.* 3*d.* Intereſt of the Principal for the whole Time; then,

$$\begin{array}{l} £.\ \ s.\ \ Y.\ \ £.\ \ s.\ \ d. \\ \text{If } 14\ 19 : 1 :: 115\ 17\ 3 \end{array}$$

Here £. 14 19*s.* = 2588*d.* and £. 115 17*s.* 3*d.* = 27807*d.* which ÷ 2588 = $7\frac{1}{4}$ Years, the Anſwer.

(43) Firſt, from £. 856 10*s.* take 600£. Remains £. 256 10*s.* Intereſt of the 600£. for $9\frac{1}{2}$ Years.

$$\begin{array}{l} £.\ \ \ £.\ \ s.\ \ \ \ s.\ \ \ £. \\ \text{If } 600 : 256\ 10 \text{ or, } 5130 :: 100 \\ \qquad\qquad\qquad\qquad 100 \\ \hline \end{array}$$

6,00)5130,00(855*s.* Intereſt of 160£. for $9\frac{1}{2}$ Years, which ÷ $9\frac{1}{2}$, viz. 19 Half-Years, gives 45*s.* or 2£. 5*s.* Intereſt of 100£. for Half a Year, which × 2 = £. 4 10*s.* per Cent. the Anſwer.

(44) Firſt, from £. 614 3*s.* 11*d.* take £. 498 6*s.* 8*d.* Remains £. 115 17*s.* 3*d.* Intereſt; then,

$$\begin{array}{l} £.\ \ s.\ \ d.\ \ £.\ \ s.\ \ d.\ \ £. \\ \text{If } 498\ 6\ 8 : 115\ 17\ 3 :: 100 \\ \text{Reduced, } 119600d. : 27807d. :: 2400d. \\ \qquad\qquad\qquad\qquad\qquad 2400 \\ \hline \end{array}$$

1196,00)6673680,00(5580, or 465*s.*

And $7\frac{3}{4}$ = 31 Qrs. then, 31)465(15*s.* × 4 = 3£. per Cent. the Anſwer.

(45) Firſt, 5000£. × $4\frac{1}{2}$ = 225,00£. ÷ 1,00 = 225£. Intereſt for a Year; which ÷ 4 = £. 56 5*s.* Intereſt due to Lady-Day, which is a Quarter, and in this Manner proceed with each new Principal for the Intereſt,

	£.			
1771.				
Christmas, lent -	5000 at $4\frac{1}{2}$ per Cent.			
1772. Inter. due to Lady-D.	56	5		
Amount, - -	5056	5		
Drew out, - -	194	5=185 Guineas.		
Remains - -	4862	0 New Principal.		
Interest of which to Midſ.	54	13	$11\frac{1}{4}$	
Amount, - -	4916	13	$11\frac{1}{4}$	
Paid in 500 Moidores,=	675	0	0	
Sum, - -	5591	13	$11\frac{1}{4}$	
Drew out, - -	700	0	0	
Remains - -	4891	13	$11\frac{1}{4}$	New Principal.
Interest to Michaelmas,	55	0	$7\frac{1}{2}$	
Amount, - -	4946	14	$6\frac{3}{4}$	
Paid in, - -	569	17	0	
Answer, £.	5516	11	$6\frac{3}{4}$	

(56) Here multiply each new Principal by the Number of Days it has continued, and divide the Sum of all the Products by 7300, viz. 100×365, the Quotient will be the Interest required.

Mr. Ralph Newlands, Dr.

		£.	Days	Products.
1771.				
May 1.	Lent per Bill at one Day's Date,	500	13	6500
13.	Received in Part, - - -	50		
	Balance, - - - - -	450	22	9900
June 4.	Received in Part, - -	56		
	Balance, - - - -	394	40	15760
July 14.	Received in Part, - -	44		
	Balance, - - - - -	350	9	3150
	Carried over, Sum			35310

		£.	Days	Products.
	Brought over,	350		35310
July 23.	Received in Part, - -	50		
	Balance, - - - -	300	26	7800
Aug. 18.	Received in Part, - - -	87		
	Balance - - - -	213	12	2556
30.	Received in Part, - - -	13		
	Balance, - - - -	200	22	4400
Sep. 21.	Received in Part, - -	30		
	Balance, - - - -	170	27	4590
Oct. 18.	Received in Part, - -	30		
	Balance, - - - -	140	11	1540
29.	Received in Part, - -	40		
	Balance, - - - -	100	13	1300
Nov. 11.	Received in Part, - -	50		
	Balance, - - - -	50	47	2350
Dec. 28.	Received in full of Principal, -	50		
	Sum of the Products,			59846

Then, 73,00)598,46(8£. 3s. $11\frac{1}{2}d.\frac{116}{730}$ Interest due on this Account.

(47) JOHN JAMESON, Dr.

	£.	s.	d.	Days	Products £.	s.	d.
To a Bill at one Day's Date,	878	19	10	40	35159	13	4
Feb. 27. Received in Part, -	57	15	7				
Ballance, - -	821	4	3	19	15603	0	9
Mar. 18. Received in Part, -	37	14	0				
Ballance, - -	783	10	3	42	32907	10	6
April 29. Received in Part, -	34	11	0				
Carried over,	748	19	3	13	9736	10	3

		£.	s.	d.	Days	Products £.	s.	d.
	Balance, -	748	19	3		93406	14	10
May 12.	Received in Part,	136	15	7				
	Balance, -	612	3	8	38	23292	19	4
June 19.	Received in Part, -	67	13	4				
	Balance, -	544	10	4	26	14157	8	8
July 15.	Received in Part,	15	15	6				
	Balance, -	528	14	10	10	5207	8	4
25.	Received in Part,	111	11	11				
	Balance, -	417	2	11	70	29200	4	2
Oct. 3.	Received in Part, -	78	7	4				
	Balance, -	338	15	7	47	15922	12	5
23.	Received in Part,	100	0	0				
	Balance, -	238	15	7	4	955	2	4
	Received in Part,	100	0	0				
		138	15	7	37	5134	16	7
Dec. 30.	Received in full of the Principal,	138	15	7				
	Sum of the Products, £.					187327	6	8

Then 73,00)187327£. 6s. 8d.(25£. 13s. $2\frac{1}{2}d.\frac{604}{730}$, the Interest required.

		£.	s.	d.
(48)	First, from 109 Moidores take 2s. 6d. Rem. Amount of the Bond,	147	0	6
	And 109 Guineas - - - Value of Ditto,	=114	9	0
	Difference or Interest, £.	32	11	6

Also, £. 114 9s. Principal, × 4 Rate, =£ 4,57 16s.; this ÷1,00,=£.4 11s. $6\frac{1}{4}d.$ Interest for a Year.

Then

£. s. d. Yea. £. s. d.
Then—If 4 11 6½ : 1 :: 32 11 6
Reduced 2197 Halfpence : 1 :: 15636 Halfpence; therefore
2197)15636(7 Years, 42 Days, the whole Time.

July hath 31 Days } Sum 49—42=7th of July, the An-
Till Aug. 18th, } swer.

(49) First, £.39 19s. 8d.=9596 Pence; then, per Sect. XV.

Prin. Yrs. Interest.
Placed thus { 100 : 12 : 9596
1 : 0 : 240

Then 240×12×100=288000 Dividend, which ÷ by 9596 =30 Years, $4\frac{5116}{9596}$ Days, the Answer.

(50) First, from Aug. 7, 1766, to May 11, 1771=4 Years, 277 Days.

	£.	s.	d.	
1st Bond was made for -	1114	10	0	at 6 per Ct.
Interest of which for 4Y. 277 D. is	318	4	6¼	
Sum,	1432	14	6¼	Amount.
May 11. Paid off, - -	140	0	0	
2d Bond, - - -	1272	14	6¼	at 5¼
Due Sept. 5, (See next Page).	21	16	8	Interest.
Sum,	1394	11	1¼	Amount.
Sept. 5. Paid off, - -	87	11	9	
3d Bond, - - -	1226	19	5¼	} subtract.
Sept. 11. Received in full, -	1409	16	8	}
Interest,	182	17	2¾	Difference.

Then, from September 5, 1776, to September 11, 1777 = 6 Years, 6 Days, or 2196 Days, 3d Bond or Principal, viz. £.1226 19s. 5¼d.=1177893 Farthings. £.182 17s. 2¾d.=175547 Farthings. £.100=96000 Farthings, and a Year =365 Days; then, by Case 7, or rather by Sect. 15, thus—

Prin.

	Prin.	*Time.*	*Interest.*
	1177893	2196	175547
	96000	365	

Then $1177893 \times 2196 = 2586658028$ the Divisor.

And $175547 \times 365 \times 96000 = 6151166880000$ the Dividend.

Therefore, $2586658028)6151166880000(2378\frac{94089416}{2586658028}$ Farthings, or £. 2 9*s*. 6½*d*. the Answer required.

The Time of the 2d Bond's Continuance is found, thus—

$$\begin{array}{llllll} \textit{P.} & & \textit{Time.} & \textit{R.} & & \\ 100 & & 365 & 5 & 5 & 0 \\ .1292 & 14\ 6\tfrac{1}{4} & \text{—} & 21 & 16 & 8 \end{array}\Bigg\}\text{Red.}\quad \begin{array}{lll} 96000 & 365 & 1260 \\ 1241017 & \text{—} & 5240 \end{array}$$

Then $1241017 \times 1260 = 1563681420$ the Divisor.

$5240 \times 365 \times 96000 = 1836096000000$ the Dividend.

$1563681420)1836096000000(117$ Days, since May 11, which answers to Sept. 5, as before mentioned.

(51)

$$\text{If he puts in } \left\{\begin{array}{c} £. \\ 100 \\ 200 \\ 300 \end{array}\right\} \text{ to have } \left\{\begin{array}{c} £. \\ 40 \\ 55 \\ 70 \end{array}\right\} \text{ per Year.}$$

$$\text{Then from } \left\{\begin{array}{c} £. \\ 55 \\ 70 \end{array}\right\} \text{ take } \left\{\begin{array}{c} £. \\ 40 \\ 55 \end{array}\right\} = 15 \text{ per C. for his Money.}$$

$$\text{And } \left\{\begin{array}{c} £. \\ 40 \\ 55 \\ 70 \end{array}\right\} - \left\{\begin{array}{c} £. \\ 15 \\ 30 \\ 45 \end{array}\right\} = 25£. \text{ per Year for his Attendance.}$$

(52)

	£.	*s.*	*d.*
£.900 at 111⅜ per Cent. per Case II. -	1002	7	6
Brokerage of 900£. at 2*s*. 6*d*. per Cent. -	1	2	6
	£. 1003	10	0
Midsummer Dividend, at 2 per Cent. -	18	0	0
	£. 985	10	0
Interest of £.1003 10*s*. for 45 Days, at 5 per Cent. - - - - -	6	14	8
Brokerage of £. 400 at 2*s*. 6*d*. per Cent.	0	10	0
Carried over,	£. 992	14	8

	£.	s.	d.
Brought over,	£. 992	14	8
Sold 400£. at 92½ per Cent, - -	370	0	0
	£. 622	14	8
Interest for ½ Year, due Feb. 10, 1746,	15	11	4¼
	£. 638	6	0¼
Dividend received at that Time, - -	10	0	0
	£. 628	6	0¼
Interest due to Aug. 10, - - -	15	14	1¾
	644	0	2
Dividend received at that Time, - -	10	0	0
	£. 634	0	2
Interest due to Feb. 1747, - - -	15	17	0
	£. 649	17	2
Dividend received then, - - -	10	0	0
	£. 639	17	2
Interest to the 10th of August, - -	15	19	11
	£. 655	17	1
Midsummer Dividend, received August 10,	10	0	0
	£. 645	17	1
Sold off 500£. at 102⅝ per Cent. - -	512	2	6
	£. 133	14	7
Brokerage, - - - - - -	0	12	6
To my Damage in the Whole, -	£. 133	2	1

O 20. COM-

20. COMPOUND INTEREST.

(2) Firſt, 150£. ×4=6,00£. ÷100=6£. Intereſt, which added to 150=156£. ſecond Year's Principal; then 156£. ×4=624£. ÷100=6£. 4*s.* 9½*d.* Intereſt, + 156£.=£.162 4*s.* 9½*d.* third Year's Principal, which ×4,=£.648 19*s.* 2*d.* ÷ 1,00=£.6 9*s.* 9½*d.* Intereſt, which added to £. 162 4*s.* 9½*d.*=£. 168 14*s.* 7*d.*= fourth Year's Principal, which × 4,=£. 674 18*s.* 4*d.* ÷100=£. 6 14*s.* 11¼*d.* fourth Year's Intereſt, which added to £.168 14*s.* 7*d.*=£.175 9*s.* 6¼*d.*= fifth Year's Principal, which ×4,=£. 701 18*s.* 3*d.* this ÷100= £.7 0*s.* 4½*d.* fifth Year's Intereſt, which added to £.175 9*s.* 6¼*d.*=£.182 9*s.* 11¼*d.* the Amount.

(3)	£.	£.	*s.*	*d.*	
5	$\frac{1}{20}$	440	16	0	=1ſt Year's Principal.
		22	0	9½	———— Intereſt.
	$\frac{1}{20}$	462	16	9½	= ſecond Year's Principal.
		23	2	10	= ———— Intereſt.
	$\frac{1}{20}$	485	19	7½	= third Year's Principal.
		24	5	11¾	= ———— Intereſt.
	$\frac{1}{20}$	510	5	7¼	= fourth Year's Principal.
		25	10	3¼	= ———— Intereſt.
Mo.	$\frac{1}{20}$	535	15	10½	= fifth Year's Principal, or Amount of 4 Years.
6	½	26	15	9½	= ———— Intereſt.
1	$\frac{1}{6}$	13	7	10¾	
15 *D.*	½	2	4	7¾	
		1	2	3¾	
		16	14	10¼	= Intereſt for 7 Mo. 15 Days.
		535	15	10½	= 4 Year's Amount.
		552	10	8¾	= Amount for 4 Y. 7 Mo. 15 D.
		440	16	0	= Principal.
Anſwer	£	111	14	8¾	Compound Intereſt.

21. RE-

21. REBATE OR DISCOUNT.

(2) First, 100+6=106£. Amount of 100£. for a Year.
Then—If 106£. : 100£. :: 100£.

100

106)10000(£.94 6s. $9\frac{1}{2}d.\frac{4}{106}$ Answ.

(3) *Mo.* £. | £. s. d. £. £. s. d.

6 | $\frac{1}{3}$ | 4 then—If 103 6 8 : 100 :: 600 10 6

| | 2 24800d. : 100 :: 144126d.

4 | $\frac{1}{3}$ | 1 6 8

248,00)144126,00(£.581 3s. 1d. present Worth, which, taken from the Principal, leaves £.19 7s. 5d. Answer.

Int. for 10 M.	3	6	8
Principal,	100	0	0
Amount,	103	6	8

(4) First, from December 12, to July 27,=227 Days.
Then—If 365D. : 5£. :: 227 Days.

227

365)1135(£.3 2s. $2\frac{1}{4}d.$ Rebate of 100£. for 227 Days.

£. s. d. £. s. d. £. s.
Again,—If 103 2 $2\frac{1}{4}$: 3 2 $2\frac{1}{4}$ :: 890 16
98985 *qrs.* : 2985 *qrs.* :: 855168

855168

98985)2552676480(25787*qrs.* or £. 26 17s. 3d. the Answer.

(5) First, one-half of 430£.=215£.

Mo. £. | £. s. d. £. s. d. £.

4 $\frac{1}{3}$ 5 | If 101 13 4 : 1 13 4 :: 215

Interest,	1	13	4
Prin.	100	0	0
Amount,	101	13	4

24400d. : 400d. :: 51600d.

51600

244,00)206400,00(845$\frac{3}{4}$d. or £.3 10s. $5\frac{3}{4}d.$ Disc. of one-half for 4 Months.

 Again,

Again, £. 1 13*s.* 4*d.* × 2 = £. 3 6*s.* 8*d.* Interest of 100£. for 8 Months.

£. s. d. £. s. d. £.

Then—If 103 6 8 : 3 6 8 :: 215

or, 24800*d.* : 800*d.* :: 51600*d.*

51600

248.00)41280000(1664½*d.* or £. 6 18*s.* 8½*d.*

Discount of Half for 8 Months, which, added to that for 4 Months, gives £. 10 9*s.* 2¼*d.* Answer.

(6) First, from May 21, to Christmas, = 218 Days; then—

If 365*D.* : 5£. :: 218 Days.

218

365)1090(£. 2 19*s.* 8¼*d.* Interest of 100£. for 218 Days.

£. s. d. £. £.

Again,—If 102 19 8 : 100 :: 550

or, 98866*d.* : 100 :: 52800*d.*

52800

98866)5280000(£. 534 1*s.* 1½*d.* Answer.

(7) *Mo.* £. — 3 ¼ | 6 — then—If 101 10 : 100 :: 100 (£. s. £. £.)

Interest,	1 10	2030*s.* : 100 :: 2000*s.*
Prin.	100 0	2000
Amount,	101 10	203,0)20000,0(£. 98 10*s.* 10¼*d.* present Worth of 100£. for 3 Months.—Again,

Mo. £. — 4 | ⅓ | 6 — then—If 102 10 : 100 :: 60 (£. s. £. £.)

1 | ¾ | 2 0

	0 10	2050*s.* : 100 :: 1200
		1200
Interest,	2 10	
Princip.	100 0	205,0)1200,0(£. 58 10*s.* 8¾*d.* present Worth of 60£. for 5 Months.
Amount,	102 10	

Now,

Now, $100+60=160$ £. And 3220 £.-160 £.$=3060$ £. for 9 Months.

Mo.	£.	s.	
6 $\frac{1}{2}$	6		If 104 10 : 100 :: 3060 (£. s. : £. :: £.)
3 $\frac{1}{2}$	3		2090 : 100 :: 61200s.
	1	10	612
Interest,	4	10	209,0)6120,0(£. 2928 4s. 7d, present Worth of 3060£. for 9 Months; therefore all the present Worths, added together, will be £. 3085 5s. 9d. the Answer.
Prin.	100	0	
Amt.	104	10	

(8) First $\frac{1}{5}$th of 400£.=80£. for three Months.

Mo.	£.	s.	d.	
3 $\frac{1}{4}$	4	10		then – If 101 2 6 : 100 :: 80 (£. s. d. : £. :: s.)
Interest,	1	2	6	4045 : 100 :: 3200 Six pences.
Princip,	100	0	6	4045)320000(£.79 2s. $2\frac{1}{4}$d. the present Worth of 80£. for 3 Months.
Amount	101	2	6	

Again, $\frac{1}{2}$ of 400£.=200£. for 6 Months.

Mo.	£.	s.	
6 $\frac{1}{2}$	4	10	then—If 102 5 : 100 :: 200 (£. s. : £. :: £.)
Interest,	2	5	2045s. : 100 :: 4000s.
Princip.	100	0	2045)400000(£. 195 11s. $11\frac{3}{4}$d. the present Worth of 200£. for 6 Months.
Amount	102	5	

Now, $80+200=280$£. And $400-280=120$£. to continue for three times 3 Months, which is 9 Months.

Mo.	£.	s.	d.	
6 $\frac{1}{2}$	4	10		then—If 103 7 6 : 100 :: 120 (£. s. d. : £. :: £.)
3 $\frac{1}{2}$	2	5		4135 : 100 :: 4800 Six-pences.
	1	2	6	4135)480000(£.116 2s. $7\frac{1}{2}$d. the present Worth of 120£. for 9 Months.
Interest,	3	7	6	
Principal	100	0	0	
£.	103	7	6	

Now

Now all the present Worths, added together, gives £. 390 15*s*. 9½*d*. which, taken from 400 £. Value of the Goods, leaves £. 9 4*s*. 1½*d*. the Rebate.

(9) First, ⅓ of 360£.=120£. to continue 5 Months.

Mo.	£.		£. *s*. £. £.
4 ⅓	3		then—If 101 5 : 100 :: 120
1 ¼	1	0	2025*s*. : 100 :: 240*s*.
	0	5	2025)24000(£. 118 10*s*. 4¼*d*. the present Worth of 120£. for 5 Months.
Interest,	1	5	
Princip.	100	0	
Amount,	101	5	

Again, 360—120=240£. to continue 10 Months.

Mo.	£.		£. *s*. £. £.
6 ½	3		then—If 102 10 : 100 :: 240
	1	10	2050*s*. : 100 :: 4800*s*.
4 ⅓		10	205,0)480000(£. 234 2*s*. 11*d*. present Worth of 240£. for 10 Mo. which, added to that for 5 Mo. will make £. 352 13*s*. 3¼*d*. the present Worth required.
Interest,	2	10	
Princip.	100	0	
Amount,	102	10	

(10) 5£.×1/20 500£. Principal.

25 Interest of 500£. for one Year; then 25£.×12=300£. Interest of 12 Years.

Again, 5×12=60£. Interest of 100£. for 12 Years.

If 160£. : 60£. :: 500£. : 187£. 10*s*.

∵ 300—£.187 10*s*.=112£. 10*s*. Advantage to Interest.

(11)

(11)

Mo.		£.	s.	d.
6	$\frac{1}{2}$	4	7	6 = $4\frac{3}{8}$£.
				2
		8	15	0
2	$\frac{1}{3}$	2	3	9
1	$\frac{1}{3}$	0	14	7
15 D.	$\frac{1}{2}$	0	7	$3\frac{1}{2}$
		0	3	$7\frac{3}{4}$
10	$\frac{1}{3}$	0	2	5
Inter.		12	6	$8\frac{1}{4}$
Prin.		100	0	0
Amt.		112	6	$8\frac{1}{4}$

then — If 112£. 6*s.* $8\frac{1}{4}$*d.* : 100£. :: 13377£. 13*s.* 4*d.* or, 107841 *qrs.* : 100 :: 12842560 *qrs.* 107841)1284256000(£. 11908 15*s.* $9\frac{3}{4}$*d.* $\frac{105801}{107841}$, the ready Money required.

22. EQUATION OF PAYMENTS.

(2)

£.	Mo.		*Prod.*
200	× 3 =		600
100	× 4 =		400
300	× 6 =		1800
6,00			28,00)

Answer, 4 Months, 20 Days.

(3)

£.	Mo.	*Prod.*
200	× 7 =	1400
260	× 5 =	1300
46,0)		270,0(5 *M.* $26\frac{4}{46}$ *D.*

the Answer.

(4) Here suppose 120£. to be the Sum owed.

£. *Mo.* *Prod.*

$$\text{then } \left.\begin{matrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{6}\end{matrix}\right\} \text{of } 120 = \left\{\begin{matrix}60, \times 3 = 180\\ 40, \times 4 = 160\\ 20, \times 9 = 180\end{matrix}\right.$$

12,0)520(4 Mo. 12 D. Answer.

(5) Here, as the Debt is to be paid at four equal Payments, and $\frac{1}{4}$ being paid down, there remains $\frac{3}{4}$ to be paid, at three equal Payments; consequently, the Sum of the different

different Times that each Payment is to be made, being divided by 3, will give the Answer; thus

$4+5+6=15$, this, $\div 3=5$ Months, the Answer.

(6)	£. Mo. *Prod.*
Owed,	$240\times5=1200$
Paid down,	40
Remains,	2,00)12,00(6 Months, the Answer.

23. SINGLE FELLOWSHIP.

(2) First, $320+340=660$£. C and D put in; then, from 824£. take 660£. remains 164£. E's Stock, and $824+70=894$£. their whole Gain; therefore,

$$\begin{array}{l} \text{£.} \quad \text{£.} \\ \text{As } 824 : 894 :: \end{array} \left\{ \begin{array}{rrrrrl} \text{£.} & \text{£.} & \textit{s.} & \textit{d.} & \textit{Rem.} & \\ 320 : & 347 & 3 & 8\frac{1}{4} & 128 & \text{C's} \\ 320 : & 368 & 17 & 8 & 72 & \text{D's} \\ 164 : & 177 & 18 & 7\frac{1}{2} & 642 & \text{E's} \end{array} \right\} \text{Gain.}$$

(3) Here, suppose 600£. to be their Stock; then

$$\left.\begin{array}{c} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{5} \end{array}\right\} \text{of } 600\text{£.} = \left\{\begin{array}{l} \text{£.} \\ 300 \text{ A's} \\ 200 \text{ B's} \\ 150 \text{ C's} \\ 120 \text{ D's} \end{array}\right\} \text{Stock.} \qquad \begin{array}{l} \text{Stated thus,} \\ \quad \text{£.} \quad \text{£.} \\ \text{As } 770 : 120 :: \end{array} \left\{\begin{array}{r} \text{£.} \\ 300 \\ 200 \\ 150 \\ 120 \end{array}\right.$$

770 Sum. Answ. as below.

By Fractions thus—$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, $=\frac{60}{120}$, $\frac{40}{120}$, $\frac{30}{120}$, and $\frac{24}{120}$—Then, $60+40+30+24=154$.

$$\begin{array}{l} \quad\quad\quad \text{£.} \\ \therefore \text{As } 154 : 120 :: \end{array} \left\{ \begin{array}{rrrrrl} \text{£.} & \text{£.} & \textit{s.} & \textit{d.} & \textit{Rem.} & \\ 60 : & 46 & 15 & 0\frac{3}{4} & 9 & \text{A's} \\ 40 : & 31 & 3 & 4\frac{1}{2} & 6 & \text{B's} \\ 30 : & 23 & 7 & 6\frac{1}{4} & 43 & \text{C's} \\ 24 : & 18 & 14 & 0\frac{1}{4} & 19 & \text{D's} \end{array} \right\} \text{Loss.}$$

(4) First, $30+48+42=120$£. their whole Gain.—Then,

$$\begin{array}{l} \quad\ \text{£.} \quad\ \text{£.} \\ \text{As } 120 : 400 :: \end{array} \left\{ \begin{array}{l} 30 : 100 \text{ D's} \\ 48 : 160 \text{ E's} \\ 42 : 140 \text{ F's} \end{array} \right\} \text{Stock.}$$

(5)

(5)

	£.		s.		
First,	1000		=20000	A's	Debt.
	640		=12800	B's	
	900		=18000	C's	
	842	16	=16856	D's	

Sum, 3382 16=67656 the whole Debt.

And £. 2420 17*s.* 6*d.*=581010 Pence, his whole Worth.

Then,—

		s.		d.	£.	s.	d.
s. *d.*		20000	:	171754	=715	12	10
As 67656 : 581010 : :		12800	:	$109922\frac{1}{2}$	=458	0	$2\frac{1}{2}$
		18000	:	$154578\frac{1}{4}$	=644	1	$6\frac{1}{4}$
		16856	:	$144754\frac{1}{4}$	=603	2	$10\frac{1}{4}$

Rem.

45504	A	Received.
360	B	
360	C	
41088	D	

(6) Here, suppose 420, as it will divide by 3, 4, 5, 6, and by 7, and have no Remainder, then $\frac{1}{3}$ of 420=140 A $\frac{1}{4}$=105 B, $\frac{1}{5}$=84 C, $\frac{1}{6}$=70 D, and $\frac{1}{7}$=60 E, therefore, added together, $=\frac{459}{420}$, then, neglecting the Denominator, the Statings will stand thus;

	£.	£.	s.	d.			
£.	140 :	152	10	$1\frac{1}{4}$	105	A's	Share.
	105 :	114	7	$6\frac{3}{4}$	423	B's	
As 459 : 500 : :	84 :	91	10	$0\frac{3}{4}$	63	C's	
	70 :	76	5	$0\frac{1}{2}$	282	D's	
	60 :	65	7	$2\frac{1}{4}$	45	E's	

Or by Fractions, thus, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$, $=\frac{840}{2520}$, $\frac{630}{2520}$, $\frac{504}{2520}$, $\frac{420}{2520}$, and $\frac{360}{2520}$. Here neglecting the Denominator, and the Sum of the Numerators will be the first Term, and each Numerator the third; then proceed as above, will give the Answer.

(7) First, $\frac{3}{8}+\frac{3}{7}=\frac{21}{56}+\frac{24}{56}=\frac{45}{56}$=A+B's=Part. Then, $\frac{56}{56}-\frac{45}{56}=\frac{11}{56}$ C's Part.

£.		£.	s.	d.			
As 11 : 140 : :	21 :	267	5	$5\frac{1}{4}$	$\frac{7}{11}$	A	paid.
	24 :	305	9	1	$\frac{4}{11}$	B	

(8)

(8) First, 10+7=17; then—If 17 : 52 10 :: 10

	£.	£. s.	£.
First, 10+7=17; then—If	17 :	52 10 ::	10
			10

17)525 0(£.30 17s. $7\frac{3}{4}$d. $\frac{1}{17}$, A's Gain.

	£.	£.	£. s. d.
And,—If	45 :	100 ::	30 17 $7\frac{3}{4}\frac{1}{17}$

734400 : 100 :: 504000 Seventeenths.

7344,00)504000,00(£ 68 12s. $6\frac{1}{2}$d. $\frac{2502}{7344}$ A's. Adventure.

	£. s.	
Again,—If 17 :	52 10 ::	7
		7

17)367 10(£.21 12s. 4d. $\frac{16}{17}$ B's Gain.

	£.	£.	£. s. d.
Also—If	45 :	100 ::	21 12 $4\frac{16}{17}$

734400 : 100 :: 352800

7344,00)352800,00(£.48 0s. $9\frac{1}{4}$d $\frac{4572}{7344}$ B's. Adventure.

(9) First, 50 Guineas =£.52 10s. B's Gain, which take from £.74 11s. A's Gain, leaves £.22 1s. Difference. And £.52 10s.+£.74 11s.=£.127 1s. the whole Gain.

	£. s.	£. s. d.	£. s.
Then,—If	22 1 :	19 19 8 ::	127 1
	441s. :	4796d. ::	2541s.
		2541	

441)12186636(27634d. or £.115 2s. 10d. $\frac{42}{441}$ Cost, which added to the Gain, viz. £.127 1s. = £.242 3s. 10d. or 58126 Pence, which ÷ by 55s. viz. 660 Pence, will give 88 Anchors, and 3s. 10d. over; then, £.127 1s.÷88=£.1 8s. $10\frac{3}{4}$d. gained per Anchor.

(10) First, from 14s 6d. take 8s. 6d. remains 6s. A gains more than B; then,

If

	s.	s.	s.	d.
If	6 :	35 : :	14	6
	12 :	35 : :	29	
		29		

12)1015(84*s.* 7*d.* or
£.4 4*s.* 7*d.* A's Stock.

	s.	s.	s.	d.
Again, If	6 :	35 : :	8	6
	12 :	35 : :	17	
		17		

12)595(49 *s.* 7 *d.* or
£.2 9*s.* 7*d.* B's Stock.

(11) First, 210+312=522£. Sum of A and B's Stock, and 140—£.37 10*s.*=£.102 10*s.* their Gain, then,

	£.	£.	s.	£.	£.	s.	d.	Rem.		
If	522 :	102	10 : :	310 :	41	4	$8\frac{1}{2}$	108	A's	Gain.
	522 :	102	10 : :	312 :	61	5	$3\frac{1}{4}$	414	B's	

IF £. 102 10*s.* : 522 : : £. 37 10*s.* : £. 190 19*s.* 6*d.*$\frac{20}{205}$ C's Stock.

(12) First, 8+5=13£. their Gain per Cent.

	£.	£.	£.	£.	s.	d.			
Then—If	13 :	154 : :	8 :	94	15	$4\frac{1}{2}$	$\frac{6}{13}$	A's	Gain.
	13 :	154 : :	5 :	59	4	$7\frac{1}{4}$	$\frac{7}{13}$	B's	
Ans. A had for his Trouble,				35	10	9	$\frac{12}{13}$		

(13) First, $\overset{AB,}{\frac{2}{7}}$, $\overset{AC,}{\frac{3}{8}}$, $\overset{BC,}{\frac{3}{10}}$, $=\overset{AB,}{\frac{160}{560}}$, $\overset{AC,}{\frac{210}{560}}$, $\overset{BC,}{\frac{168}{560}}$, by Case 5. Sect. 38; then rejecting the Denominator, we shall have,

First, 210—160= 50 C's Share more than B's.
And 210—168= 42 A's ———————.
Also, 160— 42=118 by A and B equally.
∵ 118÷ 2= 59 B's Share.
And 59+ 42=101 A's
Also, 59+ 50=109 C's
(the last three) = 269 their Sum.

			s.	d.	Rem.		
If 269 : 30*s.* : :	59 :	6	$6\frac{3}{4}$	225	B's	Share of the 30*s.*
	101 :	11	3	180	A's	
	109 :	12	$1\frac{3}{4}$	$\frac{133}{269}$	C's	

(14)

Meal. Malt. Meal. Malt.

(14) First,—If $3 : 5 :: 8 : 13\frac{1}{3} = \frac{40}{3}$

Then, $8 + 7 + 13\frac{1}{3} = 28\frac{1}{3} = \frac{85}{3}$, their Sum.

Qrs. Bush.

$\frac{85}{3} : \frac{40}{3} :: \frac{100}{1} : \frac{800}{17} = 47\frac{1}{17} = 376\frac{8}{17} = \frac{6400}{17}$ Malt.

$\frac{85}{3} : \frac{8}{1} :: \frac{100}{1} : \frac{480}{17} = 28\frac{4}{17} = 225\frac{15}{17} = \frac{3840}{17}$ Meal.

$\frac{85}{3} : \frac{7}{1} :: \frac{100}{1} : \frac{420}{17} = 24\frac{12}{17} = 197\frac{11}{17} = \frac{3360}{17}$ Oatmeal.

$$\left.\begin{array}{r} \frac{800}{17} \times \frac{2}{3} = \frac{1200}{17} \\ \frac{480}{17} \\ \frac{420}{17} \times \frac{1}{2} = \frac{210}{17} \end{array}\right\} \text{Price of the} \left\{\begin{array}{l} \text{Malt.} \\ \text{Meal.} \\ \text{Oatmeal.} \end{array}\right.$$

Here the Denominator (17) may be omitted, and the Numerators divide by 30, the Quotient will still retain the same Proportion.

$3,0)120,0 + 48,0 + 21,0(= 40 + 16 + 7 = 63$ Sum.

$$\begin{array}{l} \text{£.} \\ \text{As } 63 : 142 :: \end{array} \left\{\begin{array}{rcrrrl} & & \text{£.} & s. & d. & \\ 40 : & \frac{5680}{63} = & 90 & 3 & 2\frac{2}{21} & \text{Malt.} \\ 16 : & \frac{2272}{63} = & 36 & 1 & 3\frac{5}{21} & \text{Meal.} \\ 7 : & \frac{994}{63} = & 15 & 15 & 6\frac{14}{21} & \text{Oatmeal.} \end{array}\right\} \text{Cost.}$$

$$\left.\begin{array}{lrrl} & & s. & d. \\ \text{Then } \frac{5680}{63} \div \frac{6400}{17} = \frac{96560}{403200}\text{£.} = & 4 & 9\frac{30}{63} & \text{Malt.} \\ \text{And } \frac{2272}{63} \div \frac{3840}{17} = \frac{38624}{241920} = & 3 & 2\frac{20}{63} & \text{Meal.} \\ \text{Also } \frac{994}{63} \div \frac{3360}{17} = \frac{16898}{211680} = & 1 & 7\frac{10}{63} & \text{Oatmeal.} \end{array}\right\} \text{per Bush.}$$

(15) First, $\frac{12}{11}$ of $\frac{3}{8} = \frac{36}{88}$, or $\frac{9}{22}$, B's Part.

And $\frac{4}{13}, \frac{9}{22}, \frac{1}{6} = \frac{528}{1716}, \frac{702}{1716}, \frac{286}{1716}$, or $\frac{264}{858}, \frac{351}{858}, \frac{143}{858}$.

Then $\frac{351}{858} - \frac{264}{858} = \frac{86}{858} =$ Difference betwixt A's and B's.

And $\frac{143}{858} + \frac{87}{858} = \frac{230}{858}$ C's Part.

Also $\frac{264}{858} + \frac{351}{858} + \frac{230}{858} = \frac{845}{858} =$ A's + B's + C's.

$\frac{858}{858} - \frac{845}{858} = \frac{13}{858}$ D's Part.

$$\begin{array}{l} \text{£.} \\ \text{As } 858 : 400 :: \end{array} \left\{\begin{array}{rrrrrl} & \text{£.} & s. & d. & \text{Rem.} & \\ 264 : & 123 & 1 & 6\frac{1}{4} & 726 & \text{A's} \\ 351 : & 163 & 12 & 8\frac{1}{2} & 780 & \text{B's} \\ 230 : & 107 & 4 & 6\frac{1}{4} & 54 & \text{C's} \\ 13 : & 6 & 1 & 2\frac{1}{2} & 156 & \text{D's} \end{array}\right\} \text{Share.}$$

(16) See Question IX. in Exercise for Fractions.

DOUBLE

24. DOUBLE FELLOWSHIP;

OR,

FELLOWSHIP WITH TIME.

$$
\begin{array}{lrl}
(2) & 4\times4\times50= & 800 \text{ Officers Pay and Time.}\\
 & 8\times4\times40= & 1280 \text{ Midshipmen's ditto.}\\
 & 120\times3\times28= & 10080 \text{ Sailors ditto.}\\
\hline
 & & 12160 \text{ Sum.}
\end{array}
$$

$$
\begin{array}{r}
\pounds. \\
\text{As } 12160 : 4000 ::
\end{array}
\left\{
\begin{array}{rrrrrl}
 & \pounds. & s. & d. & \textit{Rem.} & \\
800 : & 263 & 3 & 1\tfrac{3}{4} & 704 & \text{Officers.}\\
1280 : & 421 & 1 & 0\tfrac{1}{2} & 640 & \text{Midship.}\\
10080 : & 3315 & 15 & 9\tfrac{1}{4} & 1088 & \text{Sailors.}
\end{array}
\right.
$$

$$
\left.
\begin{array}{rrrlrrrl}
\pounds. & s. & d. & & \pounds. & s. & d. & \\
\because 263 & 3 & 1\tfrac{3}{4} & \div\ 4= & 65 & 15 & 9\tfrac{1}{4} & \text{Officers.}\\
421 & 1 & 0\tfrac{1}{2} & \div\ 8= & 52 & 12 & 7\tfrac{1}{2} & \text{Midsh.}\\
3315 & 15 & 9\tfrac{1}{4} & \div 120= & 27 & 12 & 7\tfrac{1}{2} & \text{Sailors.}
\end{array}
\right\}
\begin{array}{l}
\text{each, Fractions}\\
\text{rejected.}
\end{array}
$$

$$
\begin{array}{lrl}
(3) & 60\times4= & 240 \text{ A's Stock and Time.}\\
 & 40\times5= & 200 \text{ B's ditto.}\\
 & 30\times3= & 90 \text{ C's ditto.}\\
\hline
 & & 530 \text{ Sum of their Stocks and Times.}
\end{array}
$$

$$
\begin{array}{r}
\pounds. \\
\text{As } 530 : 40 ::
\end{array}
\left\{
\begin{array}{rrrrr}
 & \pounds. & s. & d. & \textit{Rem.}\\
240 : & 18 & 2 & 3 & 36 \text{ A's}\\
200 : & 15 & 1 & 10\tfrac{1}{2} & 30 \text{ B's}\\
90 : & 6 & 15 & 10 & 40 \text{ C's}
\end{array}
\right\}
\text{Part.}
$$

$$
\begin{array}{ll}
 & \pounds.\ \textit{Mo.}\\
(4) & \left.\begin{array}{l} 400\times6=2400\\ 200\times6=1200 \end{array}\right\} =3600 \text{ A's Stock and Time.}\\
 & \left.\begin{array}{l} 360\times7=2520\\ 460\times2=\ 920\\ 340\times3=1020 \end{array}\right\} =4460 \text{ B's ditto.}
\end{array}
$$

£. *Mo.*

$$\left.\begin{array}{r} 190\times 8=1520 \\ 300\times 2=\ \ 600 \\ 200\times 2=\ \ 400 \end{array}\right\}=2520 \text{ C's ditto.}$$

then $3600+4460+2520=10580$ their Sum.

		£.	s.	d.	Rem.		
£.	3600 :	156	10	5	920	A's	
As 10580 : 460 : :	4460 :	193	18	3	552	B's	Share.
	2520 :	109	11	$3\frac{1}{2}$	$\frac{644}{1058}$	C's	

	£.	s.	d.
(5) Reciprocally, As 19 :	84	12	6 : : 7
			6×3+ 119
7)	1607	17	6
Answer, B's Adventure, £.	229	13	$11\frac{1}{7}$

(6) First, 20+£. 26 5*s.*+32=£. 78 5*s.* Sum of their Gain.

		£.	s.	d.	Rem.		
£. s. £.	20 0 :	163	11	$6\frac{3}{4}$	225	A's	Stock
As 78 5 : 640 : :	26 5 :	214	13	11	980	B's	and
	32 0 :	261	14	6	$\frac{360}{1565}$	C's	Time.

Therefore,—

£.	s.	d.		£.	s.	d.		
163	11	$6\frac{3}{4}$	÷9=	18	3	6	A's	
214	13	11	÷7=	30	13	5	B's	Stock, Fract. rejected.
261	14	6	÷5=	52	6	$10\frac{3}{4}$	C's	

(7) First, £. 72 10*s.*=1450*s.* C's Stock; then by Sect. 15.

S. T. G.
1450 9 23 Here $23\times 6=138$ Divisor.
—— 6 13 And $1450\times 9\times 13=169650$ Dividend.

138)169650(1229*s.* 4*d.* $\frac{24}{138}$=61 £. 9*s.* 4*d.* A's Stock.

Again,—

Again,—

S.	*T.*	*G.*	
1450	9	23	Here $23 \times 5 = 115$ Divisor.
——	5	18	And $1450 \times 9 \times 18 = 234900$ Dividend.

	£.	s.	d.	
$115)234900(2042s.\ 7\frac{1}{4}d.\ \frac{25}{115} =$	102	2	$7\frac{1}{4}$	B's Stock.
	74	10	0	C's ——.
	61	9	4	A's ——.
Answer, the whole Stock, £.	236	1	$11\frac{1}{4}$	

(8) Here suppose X's Gain to be 2
Then by the Question Y's will be 3 } =9 their Sum.
And C's - - - 4

		£.	s.	d.	Pence.	
£.						
Now as 9 : 420 : :	2 :	93	6	8	=22400 X's	Gain.
	3 :	140	0	0	=33600 Y's	
	4 :	186	13	4	=44800 Z's	

Therefore, $22400 \times 4 = 89600$ X's
$33600 \times 6 = 201600$ Y's } Gain and Time,
$44800 \times 9 = 403200$ Z's

694400 their Sum.

Now rejecting the 0's we shall have the following Proportion.

		£.	s.	d.	Rem.		
£.							
As 6944 : 4262 : :	896 :	549	18	$8\frac{1}{2}$	448	X's	Stock.
	2016 :	1237	7	1	4480	Y's	
	4032 :	2474	14	$2\frac{1}{4}$	2016	Z's	

(9) First, $\frac{1}{5}, \frac{2}{3}, = \frac{3}{15}, \frac{10}{15} = \frac{13}{15}$, then $\frac{15}{15} - \frac{13}{15} = \frac{2}{15}$ B's Gain; and rejecting the Denominator, we shall have A's Gain =3, B's 2, and C's 10; then by Sect. 15.

S.	*T.*	*G.*	
400	7	10	Here $10 \times 5 = 50$ Divisor.
——	5	3	And $400 \times 7 \times 3 = 8400$ Dividend.

5,0)840,0(168 £. A's Stock.

 Again,

Again,

S.	*T.*	*G.*	
400	7	10	Here $10 \times 8 = 80$ Divisor.
——	8	2	And $7 \times 2 \times 400 = 5600$ Dividend.
			8,0)560,0(70£. B's Stock.

(10)

	Days.	*Days.*
From Feb. 10, to June 10,	=122	=208 A's Time.
Jan. 17, to April 30,	= 61	
July 14, to 14 after St. James's,	= 24	
Aug. 2, to Nov. 13,	=104	=179 B's.
May 1, to July 24,	= 55	
Sept. 30, to October 19,	= 20	——
		387 Horse in Use.

D.	*£.*	*s.*		*D.*	*£.*	*s.*	*d.*	*Rem.*		
As 387 :	7	10	::	208 :	4	0	$7\frac{1}{4}$	$\frac{297}{387}$	A's	Share.
				179 :	3	9	$4\frac{1}{2}$	$\frac{90}{387}$	B's	

25. BARTER.

(2) $4d.\frac{1}{3}$ 45 at 1*s.* 4*d.* then per Sect. 10. $60 \div 18 = 3\frac{1}{3}$ Yds.

15

——

60*s.*

(3) First, 30*Cwt.* = 360*lb.* at $7\frac{1}{2}d.$ per *lb.*

$6\frac{1}{2}d.$		3360	*d. d. s. s.*
		——	then—If 6 : $7\frac{1}{2}$ :: 36 : 45
$1\frac{1}{2}d.$	$\frac{1}{4}$	1680	
		420	Again,—If 45*s.* : 1*Cwt.* :: 2100*s.*
		——	45)2100(46*Cwt.* 2*qrs.* $18\frac{2}{3}$*lb.* the Answer.
		2,0)210,0	
		——	
		105£.=2100*s.*	

(4) If 8 6 : 10 :: 1 6 (*s. d. s. s. d.*)

17   3   3

17)30(1*s.* 9*d.*$\frac{3}{17}$ per *lb.* Answer.

(5) First, 20£.=4800 Pence dealt for.

d. *d.* *d.*
Then—If 6 : 4 :: 4800
4800

6)19200(3200*d.*=£ 13 6*s.* 8*d.* real Value of B's Currants. Now 20£.÷2=10£. or 200*s.* and 6*s.* 8*d.* =80*d.*

If 7*s.* : 80*d.* :: 200*s.*
200

7)16000(

	£.	*s.*	*d.*	
$2285\frac{1}{2}d\frac{6}{7}$=	9	10	5*d.*$\frac{1}{2}\frac{6}{7}$	real Value of B's Candles.
	10	0	0	gave in ready Money.
A received	19	10	5*d.*$\frac{1}{2}\frac{6}{7}$	
For	13	6	8	
Answer,	£. 6	3	9*d.*$\frac{1}{2}\frac{6}{7}$	A got of B.

(6) *Cwt.* £. *s.* £. *s.* £. *s.*
First, 18 at 1 11 per *C.*=1 11×3×6=27 18 real Value.
And ditto at 2 2 ——— =2 2×3×6=37 16 Adv. Value.

∵ A advanced his Sugar, 9 18

Also, 3)37£. 16*s.*(=12£. 12*s.* A received in Cash, and £. 37 16*s.*—£.12 12*s.*=25£. 4*s.* in Paper.
Again, 15*s.* 6*d.*—14*s.*=1*s.* 6*d.* B advanced his Paper. Therefore,

s. *d.* *d.* £. *s.*
If 15 6 : 18 :: 25 4
×2 ×40 Six-pences in a £.

31 1008
×18

31)18144(585*d.* or £. 2 8*s.* $9\frac{1}{4}d.\frac{5}{31}$ B's Advance on his Paper.

Then from £. 9 18*s*. take £. 2 8*s*. 9¼*d*. $\frac{5}{31}$. Rem. £. 7 9*s*. 2½*d*. $\frac{26}{31}$ in A's Favour, the Answer.

£. *s*. £. *s*. £. *s*.
(7) First, 5 6—4 5 = 1 1 Gain per Piece.
And 5 6 ÷ 2 = 2 13 required down.

Also, 4 5—2 13 = 1 12 Value of the Half remaining.
+1 1

Made £. 2 13 of the Half remaining.

£. *s*. £. *s*. *d*.
Then,—If 1 12 : 2 13 :: 3
or 384*d*. : 53*s*. :: 3*d*.
× 3 × 12

384)1908(4¾*d*. $\frac{336}{384}$ per *lb*. Answer.

£. *s*.
(8) First, 13 0 } × 50 = { 650 advanced Val. of the Cloth.
11 10 } × 50 = { 575 real Value.

75 £. Gain by the Cloth.

s. *d*.
Again, 2*s*. 6*d*. × 2 = 5 0 adv. Value of the Wool per Tod.
—4 2 real Value.

0 10 Gain per Tod, or 5*d*. per Stone.

s. *d*. *St*. £.
If 2 6 : 1 :: 650
× 2 × 40

5 5)26000

4*d*. $\frac{1}{60}$ 5200 Stone at 5*d*. per Stone.

1¼ 86 13 4
21 13 4

108 6 8 gained by the Wool.

A Sack =26 Stone; therefore 5200÷26=200 Sacks, which will pay for the Cloth.

∵ £.108 6*s.* 8*d.*—75£.=£.33 6*s.* 8*d.* B's Gain by this Affair.

(9) If 10 : 16 : : 20 or 240 (*d.* *d.* *s.* *d.*)

240

10)3840

12)384

32*s.* advanced Value of the Malt.
20 real Value.

12*s.* B's Gain per Quarter.

Here 30 Guineas ×21=630 Shillings, which ÷ by 12= $52\frac{1}{2}$ Quarters, or 420 Bushels, the Answer.

(10) 720 Ells at 5*s.* per E. or $\frac{1}{4}$=180£. real. } Value of the
Ditto ——6*s.* 8*d.* or $\frac{1}{3}$=240 adv. } Holland.
240£. at 10£. per C. or $\frac{1}{10}$= 24 Discount.
Then 240—24=216£.
And 216÷ 2=108 paid in ready Money.

Then,—If 216 : 180 : : 36 (£. £. *s.*)

20 180

4320)6480(£.1 5*s.* real Value per *lb.*

Now 108£.=2160*s.* which ÷30=72 *lb.* the Quantity delivered.

(11) If 8 or 96 : 10 : : 6 (*s.* *d.* *s.* *d.*)

6

60

12

96)720($7\frac{1}{2}d.$ Barter Price of the Pamphlets.

£.
100 Reams at 8*s*.=40 real } Value of the
Ditto, at 10*s*.=50 advanced } Paper.
$\frac{1}{4}$=50£.÷4=2£. 10*s*. B to have in Cash.
40£. Value of B's Pamphlets.
×40 Six-pences in a £.

1600 Pamphlets to be delivered.

From 40£. take 12£. 10*s*. Rem. 27£. 10*s*. what they then stood him in, so that the Advantage to B is 27£. 10*s*.

(12) First, 140*lb*. 11*oz* =1691*oz*.

5*s*. | $\frac{1}{4}$ | 1691 at 6*s*. 4*d*.
1 | $\frac{1}{5}$ | 422 15
4*d*. | $\frac{1}{3}$ | 84 11
28 3 8

£. 535 9 8 real Value of the Plate.

6 8 | $\frac{1}{3}$ | 1691 at 7*s*. 2*d*.
563 13 4
6 | $\frac{1}{40}$ | 42 5 6

£. 605 18 10 adv. V.

£.	s.	s.	£.	s. d.

From 605 18 10 take 535 9 8 Remains £. 70 9*s*. 2*d*. the whole Advantage of A's Plate.

$\frac{1}{7}$ £. 605 18 10—
86 11 $3\frac{1}{7}$ received in ready Specie.

£. 519 7 $6\frac{6}{7}$

7*C*. 2*qrs*. 18*lb*.=858*lb*. at 11*s*. 2*d*.=

10*s*. | $\frac{1}{2}$ | 858 at 11*s*. 2*d*.
1 | $\frac{1}{10}$ | 429
2*d*. | $\frac{1}{6}$ | 42 18
7 3

479 1 adv. Val. of B's Tea.

5*s*. | $\frac{1}{4}$ | 858 at 9*s*. 6*d*.
214 10
4 | $\frac{1}{5}$ | 171 12
6*d*. | $\frac{1}{8}$ | 21 9

407 11 real Value of B's Tea.

£.	s.	d.		£.	s.	d.
519	7	$6\frac{6}{7}$		519	7	$6\frac{6}{7}$
—479	1	0		407	11	0
£. 40	6	$6\frac{6}{7}$	Dis. allowed by A.	111	16	$6\frac{6}{7}$

	£.	s.	d.
A's Advantage by the Rise of his Plate,	70	9	2
B's whole Advantage, - -	£. 41	7	$4\frac{6}{7}$

(13) 14*C.* 2*qrs.* 25*lb.* at 3£. 3*s.* per *C.*=46£. 7*s.* $6\frac{1}{4}$*d.* advance Value of A's Hops.

s. s. s.
If 59 : 63 :: 6
6

59)378(6*s.* $4\frac{3}{4}$*d* $\frac{31}{59}$ advanced Price of B's Wine per Gallon. Now $1\frac{1}{2}$ *hhd.*=$94\frac{1}{2}$ *gal.* at 6*s.* $4\frac{3}{4}$*d.* $\frac{31}{59}$

$10\times9+4\frac{1}{2}=$ $94\frac{1}{2}$

B's Wine comes to	39	5	$5\frac{1}{4}$*d.* $\frac{8}{59}$
A's Hops -	46	7	$6\frac{1}{4}$
Answ. £.	16	2	$1\frac{1}{4}$*d.* $\frac{50}{59}$

26. LOSS and GAIN.

(2) First, 9*C.* 2*qrs.* 18*lb.*=1082*lb.* and 46£.+12£. 12*s.* =£. 58 12*s.* or 1172*s.* sold for; therefore,
1082)1172(1*s.* $0\frac{3}{4}$*d.* $\frac{107}{1082}$ per *lb.* the Answer.

(3) First, 10*s.* 6*d.*—8*s.* 6*d.*=2*s.* Gain by 8*s.* 6*d.*—then

s. d. s. £.
If 8 6 : 2 :: 100
2 40
17 4000
2

17)8000(470*s.* 7*d.* $\frac{1}{17}$=23£. 10*s.* 7*d.* $\frac{1}{17}$ Answ.

(4)

(4) First, $100£.+8=108£.$ Amount; then

s.		*s. d.*	
If 5 : 108 : :	6 3	then $135-100=35£.$ Answer.	
20*d.*	25		

Mr. WEBSTER's Answer is 10£.

2,0)2700(135£.

(5) First, $100+7£.\ 10s.=107£.\ 10s.$ Amount.

s.	*£. s.*	*s. d.*
If 5 :	107 10 : :	5 9
4	20	4
20	2150	23
	×23	

2,0)49450(2472*s.* 6*d.* = £.123 12*s.* 6*d.* Amount per C.

Then £.123 12*s.* 6*d.*—100 = £.23 12*s.* 6*d.* the Answer.

Mr. STONEHOUSE's Answer is £.8 12*s.* 6*d.*

(6) First, $100+15=115£.$ Amount; then

s. d.		*s.*	
If 11 6 :	115 : :	12	Here $115\times24=2760\div23=$
2		2	
23		24	£. 120 Amount per Cent.

Therefore $120-100=20£.$ per Cent. Answer.

Mr. HILL makes the Answer £.15 13*s.* 0½*d.* $\frac{2}{23}$.

(7) 3*d* | ¼ | 500 at 1*s.* 3*d.* Now $100-9=91£.$ or 1820*s.* then

125

£. *s.* If 1820 : 100 : : 625 (*s.* £. *s.*)

$625=31\ 5$

1820)62500(£. 34 6*s.* 9¾*d.* $\frac{6}{182}$ Amt.

Therefore £. 34 6*s.* 9¾*d.* $\frac{6}{182}$—31£. 5*s.* = £. 3 1*s.* 9¾*d.* $\frac{6}{182}$ the Answer.

Mr. DILWORTH's Answer is £. 2 16*s.* 3*d.*

(8) First, $100+25=125£.$ Amount; then

£. s.	*£.*	*£.*	*£. s. d.*	
If 6 15 :	125 : :	8 :	148 2 11½ $\frac{30}{135}$	Amount per Cent.

Then

Then £. 148 2*s.* 11$\frac{1}{2}$*d.* $\frac{30}{135}$—100=£. 48 2*s.* 11$\frac{1}{2}$*d.* $\frac{30}{135}$ the Anſwer.

Mr. WALKINGHAM's Anſwer is £. 29 12*s.* 7*d.* $\frac{1}{9}$

(9) Firſt 100+30=130£. Amount; then

£. *s.* £. £. *s.* £. *s.* *d.*
If 3 10 : 130 : : 4 5 : 157 17 1$\frac{1}{2}$ $\frac{6}{6}$ Amount per Cent.
Then £. 157 17*s.* 1$\frac{1}{2}$*d.* $\frac{6}{7}$—100=£. 57 17*s.* 1$\frac{1}{2}$*d.* Anſwer.

(10) Firſt, 100−17=83£. and 100+20=120£. then

£. £. £. *s.* £. *s.*
If 83 : 100 : : 52 10 : 63 5$\frac{5}{83}$
£. £. £. *s.* £. *s.* *d.*
Again, If 100 : 120 : : 63 5$\frac{5}{83}$: 75 18 0$\frac{3}{4}$ Worth.
Sold for 52 10 0

Anſwer, £. 23 8 0$\frac{3}{4}$ $\frac{78}{166}$

(11) 112 *lb.* at 2*s.* 11*d.* $\frac{2}{3}$=£. 16 12*s.* 10*d.* $\frac{2}{3}$ ſold for at Turky.
Then, £. 16 12*s.* 10*d.* $\frac{2}{3}$÷2=£. 8 6*s.* 5*d.* $\frac{1}{3}$ coſt him.
Therefore, from £ 8 6*s.* 8*d.* $\frac{1}{3}$, take 8£. Rem. 6*s.* 5*d.* $\frac{1}{3}$ Loſs, the Anſwer.

(12) Firſt, 4*s.* 3*d* +2*d.*=4*s.* 5*d.* prime Coſt and Charges.

s. *d.* *s.* £. *s.* *d.*
Then—If 4 5 : 6 : : 100 : 2716 11$\frac{3}{4}$ $\frac{5}{51}$
Again, 100—12=88£.
£. £. *s.* *d.*
Alſo—If 100 : 88 : : 2716 11$\frac{3}{4}$ $\frac{5}{51}$
£. *s.* *d.*
Or, as 5088000 : 88 : : 6912000 : 119 10 11$\frac{1}{4}$ $\frac{1440}{5088}$ Amt.
Then, £. 119 10*s.* 11$\frac{1}{4}$*d.* $\frac{1440}{5088}$—100=£. 19 10*s.* 11$\frac{1}{4}$*d.* $\frac{1140}{5080}$, gained per *lb.* the Anſwer.

(13) Firſt, 800×14=11200*lb.* at 12$\frac{1}{2}$*d.* per *lb.*
Then 11200×12$\frac{1}{2}$ = 140000*d.* Value of the Anchovies; which ÷ by 749,0, gives 18$\frac{1}{2}$*d.* $\frac{574}{749}$ Amount of 12$\frac{1}{2}$*d.*
Again, 100+17=117£. Amount; then

If

	£.		£.		d.
If	117	:	100	::	$18\frac{1}{2}\frac{574}{749}$

Or, as 84127680 : 100 :: 5542600 : 1*s.* $3\frac{3}{4}$*d.* $\frac{20853216}{84127680}$, the Answer.

(14) First, £.41 3*s.* 4*d.*=9880*d.* and 3*s.* 1*d.*=37*d.*
Then, 9880÷37=$267\frac{1}{37}$ *lb.* at 3*s.* 1*d.* per *lb.* bought.
Again, £.34 2*s.* 6*d.*=8190*d.* and 4*s.* 6*d.*=54*d.*
Then, 8190÷54=$151\frac{2}{3}$ *lb.* sold at 4*s.* 6*d.* per *lb.*
Therefore, $267\frac{1}{37}-151\frac{2}{3}=267\frac{3}{111}-151\frac{74}{111}=115\frac{40}{111}$ *lb.*
Spoiled at 3*s.* 1*d.*=37*d.* so that $115\frac{40}{111}\times 37=4268\frac{37}{111}$ *d.* or £.17 15*s.* $8\frac{1}{4}$*d.* $\frac{37}{111}$ prime Cost of the Goods spoiled.

(15) First, $\frac{3}{8}$ of 11*s.*=$\frac{33}{8}$, or 4*s.* $1\frac{1}{2}$*d.* gained per Thousand when he sold them at 11*s.* ∵ 11*s.*—4*s.* $1\frac{1}{2}$*d.*=6*s.* $10\frac{1}{2}$*d.* per Thousand prime Cost; then
As 6*s.* $10\frac{1}{2}$*d.* : 11*s.* :: 100£. : 160£. Amount of 100£.
Again, as 11*s.* : 160£. :: 13*s.* 6*d.* : 196£. 7*s.* $3\frac{1}{4}$*d* $\frac{1}{11}$ Amount per Cent. at 13*s.* 6*d.*
Then, £.196 7*s.* $3\frac{1}{4}$*d.* $\frac{1}{11}$—100=£.96 7*s.* $3\frac{1}{4}$*d.* $\frac{1}{11}$ gained per Cent. the Answer.

	£.		£.	s.	d.	
(16)	5	$\frac{1}{20}$	500	16	8	at $6\frac{1}{2}$£. per Cent.
	1	$\frac{1}{5}$	25	0	10	} C's Profit.
	$\frac{1}{2}$	$\frac{1}{2}$	5	0	2	}
			2	10	1	}

	£.	s.	d.		£.	s.	d.		£.	s.	d.	
Then	500	16	8	—	32	11	1	=	468	5	7	Wine cost C.
Also,	468	5	7	—	38	11	6	=	429	14	1	—— cost B.

	£.		£.	s.	d.	
Again,	£.20	$\frac{1}{5}$	429	14	1	at $4\frac{1}{3}$ per Cent.
	4	$\frac{1}{5}$	85	18	$9\frac{3}{4}$	
	$\frac{1}{3}$	$\frac{1}{12}$	17	3	9	} A's Profit.
			1	8	$7\frac{3}{4}$	}

Then
£.429 14 1—18 12 $4\frac{3}{4}$=411 1 $8\frac{1}{4}$ cost A.

Also,

Alſo, £.411 1*s*. 8*d*.=8221*s*. 8$\frac{1}{2}$*d*. and 15 Pipes × 126=190 1890 Gallons; then 1890)8221*s*. 8$\frac{1}{2}$*d*.(4*s*. 4*d*.$\frac{13}{189}$ per Gallon, the Anſwer.

(17) Firſt, $\frac{2}{7}$ of 480£. 12*s*.=£.961 4*s*.=£.137 6*s*. 3*d*.$\frac{3}{7}$ Coſt of the damaged Goods; then £.137 6*s*. 3*d*.$\frac{3}{7}$—£.48 18*s*.=£. 88 8*s*. 3*d*.$\frac{3}{7}$ made of the damaged Goods.

s. d. *Yd.* *£. s. d.*

∵ As 5 6 :: 1 :: 88 8 3$\frac{3}{4}$: 321$\frac{234}{462}$ Yards damaged.

321$\frac{234}{462}$ × 7=2250$\frac{252}{462}$ Yards, which ÷ 2=1125$\frac{126}{462}$ Yards, bought in all.

Then 1125$\frac{126}{462}$—321$\frac{234}{462}$=803$\frac{354}{462}$ Yards undamaged.

Again,—From £.480 12*s*. take £.88 8*s*. 3*d*.$\frac{3}{7}$. Rem. £.392 3*s*. 8*d*.$\frac{4}{7}$ to be made of the undamaged Goods.

Yds. *£. s. d.* *Ell.*

∵ If 803$\frac{325}{462}$: 392 3 8$\frac{4}{7}$ :: 1

Or, as $\frac{371311}{462}$: $\frac{658872}{7}$ :: $\frac{5}{4}$: $\frac{152190432}{10396708}$=12*s*. 2$\frac{1}{4}$*d*. $\frac{1475775}{2588677}$ per Ell.

27. ALLIGATION MEDIAL.

	Gal.	at	*s.*		*s.*
(2)	14	×	8	=	112
	12	×	6	=	72
	10	×	7	=	70
	20	×	4	=	80
	8	×	9	=	72

As 64 : 406 :: 1 *Gal.* : 6*s*. 4$\frac{1}{2}$*d*. the Anſwer.

	lb.	at	*s.*		*s.*
(3)	20	×	12	=	240
	12	×	8	=	96
	16	×	6	=	96
	12	×	4	=	48

As 60 : 480 :: 1 *lb.* : 8*s*. per *lb.* Anſwer.

	Car.		*lb.*	*Car.*
(4)	22	×	4	=88
	20	×	3	=60
	18	×	3	=54
	—		—	—
			10	

As 10 : 202 :: : $120\frac{1}{5}$ Carrats fine, the Anſwer.

		£. £. d. d.
(5)	13×80=1040	If 100 : 110 :: 2600 : 11 $10\frac{1}{4}$ or 2860
	20×60=1200	then 2850—2600=260. Alſo 2600
	10×36= 360	+260=2860.
	— *qrs.* ——	∵ As 172 : 2860 :: 1 : $16\frac{1}{2}d.\frac{88}{172}$, the
Sum	43= 172,2600	Anſwer.

28. ALLIGATION ALTERNATE.

(2)

			Buſh.		*s. d.*	
60*d.*	36	- -	12	at	3 0	Anſwer.
	48	- -	12		4 0	
	54	- -	12		4 6	
	72	24+12+6=	42		6 0	

(3)

	Car.		*Car.*		*Car.*	
20 *Car.*	18	-	3	at	8	Carrats fine, the Anſwer.
	13	2+1+4=	7		23	
	19	-	3		19	
	16	-	3		16	

(4)

			lb.		*d.*	
6*d.*	10	- -	1	of	10	per *lb.* Anſwer.
	$7\frac{1}{2}$	- -	$1\frac{1}{2}$		$7\frac{1}{2}$	
	5	- -	4		5	
	$4\frac{1}{2}$	- -	$1\frac{1}{2}$		$4\frac{1}{2}$	

As Examples of this Nature will admit of as many different Anſwers as there are different Ways of linking together a larger Price and a leſſer than middle or mean Rate propoſed, ſo conſequently the laſt will admit of ſeven different Ways or Anſwers.

29. ALLIGATION PARTIAL.

(2) *d.* — *Diff.* — *Diff. Gal. Diff. Gal.*

$$d.\ 72\left\{\begin{array}{l}84\\48\\78\\96\end{array}\right\}\quad\begin{array}{r}-\ \ -\ \ 24\\2+6+24=42\\-\ \ -\ \ 24\\-\ \ -\ \ 24\end{array}\Bigg|$$

If 24 : 28 :: 42 : 49 at 4*s.* per Gal. and 28 Gal. of each Sort of the other.

(3) *d.* — *Diff.* — *Diff. lb.* — *s. d.*

$$96\left\{\begin{array}{l}144\\108\\90\\78\end{array}\right\}\quad\begin{array}{r}6\\18\\48\\12\end{array}\Bigg|\ \begin{array}{l}\textit{Diff. lb.}\\ \text{If } 6 : 36 ::\end{array}\left\{\begin{array}{l}18 : 108\\48 : 288\\12 : 172\end{array}\right\}\text{of}\left\{\begin{array}{l}9\ \ 0\\7\ \ 6\\6\ \ 6\end{array}\right\}\begin{array}{l}\text{per}\\ \textit{lb.}\\ \text{An.}\end{array}$$

(4) *d.* — *Diff.* — *Diff. lb. Diff. lb.*

$$d.\ 24\left\{\begin{array}{l}30\\20\\18\\15\end{array}\right\}\quad\begin{array}{r}4+6+9=19\\-\ \ 6\\-\ \ 6\\-\ \ 6\end{array}\Bigg|$$

If 19 : 120 :: 6 : $37\frac{17}{19}$, so that with 120 *lb.* of the given Quantity, there must be 37 $\frac{17}{19}$ *lb.* of each Sort of the other, the Answer.

30. ALLIGATION TOTAL.

(2) *d.* — *Diff. lb.* — *d.*

$$6d.\left\{\begin{array}{l}3\\4\\7\\8\end{array}\right\}\quad\begin{array}{r}1\\2\\3\\2\end{array}\Bigg|\ \begin{array}{l}\textit{Sum. lb.}\\ \text{If } 8 : 112 ::\end{array}\left\{\begin{array}{l}1 : 14\\2 : 28\\3 : 42\\2 : 28\end{array}\right\}\text{at}\left\{\begin{array}{l}3\\4\\7\\8\end{array}\right\}$$

per *lb.* the Answer.

Sum, 8 the whole Quan. 112 *lb.*

(3) — *Diff. oz.* — *Car fine.*

$$20\left\{\begin{array}{l}16\\18\\23\end{array}\right\}\quad\begin{array}{r}3\\3\\4+2=6\end{array}\Bigg|\ \begin{array}{l}\textit{Sum. oz.}\\ \text{If } 12 : 60 ::\end{array}\left\{\begin{array}{l}3 : 15\\3 : 15\\6 : 30\end{array}\right\}\text{of}\left\{\begin{array}{l}16\\18\\23\end{array}\right\}\text{An.}$$

Sum 12 the whole Quan. 60 *oz.*

(4) *s.* *Diff.* — *Diff. lb.* — *s.*

$$7s.\left\{\begin{array}{l}5\\6\\8\\9\end{array}\right\}\quad\begin{array}{r}1\\2\\2\\1\end{array}\Bigg|\ \begin{array}{l}\textit{Sum. lb.}\\ \text{As } 6 : 168 ::\end{array}\left\{\begin{array}{l}1 : 28\\2 : 56\\2 : 56\\1 : 28\end{array}\right\}\text{at}\left\{\begin{array}{l}5\\6\\8\\9\end{array}\right\}$$

per *lb.* the Answer.

Sum 6

31. EXCHANGE.

1. With FRANCE.

Cro. d. *Cro. sol. din.*

(3) If 1 : $54\frac{1}{2}$ :: 644 12 8

or, as 720 : 109 :: 460952 : $69783\frac{8}{720}$, or £. 145 7*s.* $7\frac{1}{4}$*d.* $\frac{1}{90}$ the Anſwer.

Or thus, by Practice.

640 at 4*s.* $6\frac{1}{2}$*d.* and 12 Sols. 12 Din. at 4*s.* $6\frac{1}{2}$*d.* per

4 *s.*	$\frac{1}{5}$	640		
6 *d.*	$\frac{1}{8}$	128		
$\frac{1}{2}$	$\frac{1}{12}$	16		
		1	6	8
		0	0	$11\frac{1}{2}$
An. £.		145	7	$7\frac{1}{2}$ as before.

		s.	*d.*	
10 *so.*	$\frac{1}{6}$	4	$6\frac{1}{2}$	
2	$\frac{1}{5}$	0	9	—2 Rem.
8 *din*	$\frac{1}{3}$	0	$1\frac{3}{4}$	—1
		0	$0\frac{1}{2}$	—1
			$11\frac{1}{2}$	

d. *Cro.* £. *s.* *d.*

(4) If $54\frac{1}{2}$: 1 :: 145 7 $7\frac{1}{2}\frac{8}{720}$

or, as 78480 : 1 :: 50243768 : £. 640 12*s.* 8*d.* the Anſwer.

2. With SPAIN.

d. *Piece.* £. *s.* *d.*

(5) If 56 : 1 :: 856 6 8

or, as 56 : 1 :: 205690 : 3670 Pieces, Anſwer.

P. *d.* *Piece. re. mar.*

(6) If 1 : $54\frac{1}{4}$ :: 1426 4 26

or, as 272 : 217 :: 84203378 : $309571\frac{6}{272}$ *qrs.* or £. 322 9*s.* $4\frac{3}{4}$*d.* $\frac{66}{272}$, the Anſwer.

By Practice, thus,

1426 at 4*s.* $6\frac{1}{4}$. —— then for 4 *R.* 26 *Mar.* thus,

4*s.*	$\frac{1}{5}$	1426		
4*d*	$\frac{1}{12}$	285	4	0
2	$\frac{1}{2}$	23	15	4
$\frac{1}{4}$	$\frac{1}{8}$	11	17	8
		1	9	$8\frac{1}{2}$
		0	2	$8\frac{1}{4}$
£.		322	9	$4\frac{3}{4}$ as before.

		s.	*d.*	*Rem.*
4 *R*	$\frac{1}{2}$	4	$6\frac{1}{4}$	
17 *M.*	$\frac{1}{8}$	2	3	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$		$3\frac{1}{4}$	$\frac{1}{2}$
			$1\frac{1}{2}$	$\frac{13}{17}$
		2	$8\frac{1}{4}$ nearly.	

3. With ITALY.

(7)

4*s.*	$\frac{1}{5}$	640 at 4*s.* 4*d.* per Dollar.		
4*d.*	$\frac{1}{12}$	128	0	0
		10	13	4
	Answer, £.	138	13	4

(8) First, £.138 13*s.* 4*d.*=33280 Pence.
Then, as 52*d.* : 1 *Dol.* :: 33280 : 640 Dollars, the Answer.

(9)

2*s.*	$\frac{1}{10}$	8644 at 3*s.* 11$\frac{1}{2}$*d.* per Pez.		
1	$\frac{1}{2}$	864	8	
6*d.*	$\frac{1}{2}$	432	4	
		216	2	
4	$\frac{1}{3}$	144	1	4
1$\frac{1}{2}$	$\frac{1}{4}$	54	0	6
2*s.* 6*d.*	$\frac{1}{8}$	0	0	5$\frac{3}{4}$ $\frac{6}{8}$=$\frac{1}{8}$ of 47$\frac{1}{2}$*d.*
	Answer, £.	1710	16	3$\frac{1}{4}$ $\frac{6}{8}$

(10) First, £.1710 16*s.* 4*d.*=821192 Half-pence, and 47$\frac{1}{2}$*d.*=95 ditto.
Then—If 95*d.* : 1 *Pez.* :: 821192*d.* : 8944 *Pez.* Answer.

4. With PORTUGAL.

(11)

5*s.*	$\frac{1}{4}$	4760 at 5*s.* 4*d.*		
4*d.*	$\frac{1}{60}$	1190	0	0
		79	6	8
	Answer, £.	1269	6	8

(12) First, £.1566 6*s.* 8*d.*=375920*d.*
Then—If 64*d.* : 1 *Mil.* :: 375920*d.* : 5873 *Mil.* 720 *Reas*, the Answer.

5. With HOLLAND, FLANDERS, and GERMANY.

(13) First, £. 564 10*s*. 6*d*. = 135486 Pence, and 34*s*. 4*d*. = 412*d*.

Then, as 412*d*. : 1£. :: 135486*d*. : £. 328 16*s*. 11$\frac{3}{4}$*d*. $\frac{220}{412}$ the Answer.

(14) If £. 1 : *s*. 34 *d*. 4 :: £. 328 *s*. 16 *d*. 11$\frac{3}{4}$

or, as 960 *qrs*. : 412 *d*. :: 315695 *qrs*. : 135485$\frac{1}{4}$ *d*. $\frac{1}{12}$, or £. 564 10*s*. 5$\frac{1}{4}$*d*. $\frac{1}{12}$ the Answer.

By Practice, thus,

10*s*.	$\frac{1}{2}$	328 at £. 1 14*s*. 4*d*.		
4	$\frac{1}{5}$	164	0	
4*d*.	$\frac{1}{12}$	65	12	
		5	9	4
		1	9	1$\frac{1}{4}$
Answ. £.		564	10	5$\frac{1}{4}$

		s.	*d*.
10*s*.	$\frac{1}{2}$	34	4
5	$\frac{1}{2}$	17	2
1	$\frac{1}{5}$	8	7
6*d*.	$\frac{1}{2}$	1	8$\frac{1}{2}$
3	$\frac{1}{2}$	0	10$\frac{1}{4}$
2	$\frac{1}{3}$	0	5
$\frac{3}{4}$	$\frac{1}{4}$	0	3$\frac{1}{4}$
		0	1$\frac{1}{4}$
£. 1		9	1$\frac{1}{4}$

(15) If *s*. 33 *d*. 3 : £. 1 :: *Guil*. 4200 *Stiv*. 12 *Pen*. 8

12   20

399   84012

2

399)168025(421£. 2*s*. 3$\frac{1}{2}$*d*. $\frac{260}{399}$ Answer.

(16) If £. 1 : *s*. 33 *d*. 3 :: £. 421 *s*. 2 *d*. 3

or, as 240 : 399*d*. :: 101067*d*.

×399

24,0)4032573 3(168023*d*.

213*d*.

8

24,0)1704(7$\frac{24}{240}$ *Pen*.

Now 168023÷40=4200 Guil. 11 Stiv. $15\frac{34}{240}$ Pen. the Answer.

(17) First, £. 242 13*s.* 6*d.*=58242 Pence; then 4,0)5824,2(1456 Guil. 1 Stiv. the Answer.

6. To Change CURRENT MONEY into BANCO.

```
          G.    £.        G.  St.
(18) If 105 : 100 : : 495  18
           20               20
        -----      --------   G. St. Pin.
        21,00     )9918,00(472  5  11 9/21, Answer.
        -----      --------
              Rem.  6
                   20
                  ---
              21)120(5 Stivers.
                  ---
              Rem.  15d.
                   ×16
                   ---
              21)240(11 9/21 Pin.
                   ---
           G.     G. St.   G.  St.
(19) If 100  : 470  8 : : 105 12 1/2
            20      20         20
          ----    ----       ----
          2000    9408       2112
             2    ----          2
          ----               ----
          4000               4225
          ----               4908
                   -----------
           4,000)39748,800(
                 -----------
               2,0)993,7 Stiv. 3 1/5 Pin.
                   -------
     Answer, Guild. 496 17 3 1/5
```

7. With V E N I C E.

	Du.	*d.*	*Du.*	*sol.*	*den.*
(20) If 1 :	$47\frac{6}{8}$: :		4700	10	8

Or, as 240 : 382 : : 1128128 : $224450\frac{1}{4}d.\frac{1}{8}$, or, £. 935 4*s.* $2\frac{1}{4}d.\frac{1}{8}$ the Answer.

Or by Practice thus,—

		Du.		d.		then for 10 Sol. 8 Den. at
$\frac{4}{6}$	$\frac{1}{2}$	4700 at	$47\frac{6}{8}$;		St.	d.
		×47			5 $\frac{1}{4}$	$47\frac{6}{8}$
		220900				$11\frac{7}{8}$
$\frac{2}{8}$	$\frac{1}{2}$	2350			4 $\frac{1}{5}$	$9\frac{4}{8}$
		1175			1 $\frac{1}{4}$	$2\frac{3}{8}$
		$25\frac{2}{8}$			4d. $\frac{1}{2}$	$\frac{6}{8}$
		12)$224450\frac{2}{8}$			4d. $\frac{1}{3}$	$\frac{6}{8}$
		2,0)1870,4—2				$25\frac{2}{8}$
		Ans. £. 935 4 $2\frac{2}{8}$				

8. With POLAND and PRUSSIA.

G.P. £. *Flor.*

(21) If 270 : 1 :: 4684

Or, as 270 : 1 :: 140520 : 520£. 8*s.* 10*d.*$\frac{2}{3}$ Flemish.
Again, If 34*s.* 4*d.* : 1£. : : 520£. 8*s.* 10*d.*$\frac{2}{3}$
Or, as 1236 Thirds : 1 :: 374720 Thirds : 303£. 3*s.* 5*d.*$\frac{204}{1236}$ $\frac{1080}{1236}$ the Answer.

(22) d.		s. d.	
4 $\frac{1}{3}$	390 at	33 4	Now 270 Gros. = 9 Florins ;
	×33		Then—If 1£. : 9 *Fl.* :: 650£. ;
	12870		5850 Florins, the Answer.
	130		
	2,0)1300,0		
	650£. Flemish.		

9. With

9. With RUSSIA.

Rub. cop.
(23) 6420 42
$\times 100$

$122)642042(5262\frac{78}{122}$ Rix-dollars.
$\times 2\frac{1}{2}$

$\left.\begin{array}{r} 10525\frac{34}{122} \\ 2631\frac{39}{122} \end{array}\right\}$ Florins Current.

Fl. Cur. Fl. B.
If 103 : 100 : : $13156\frac{73}{122}$ *Flor. Banco.*
Or, as 12566 : 100 : : 1605105 : $12773\frac{2491}{6283}$.
Now $12773\frac{2491}{6283}\times 40=510935\frac{5395}{6283}$ Pence, and 34*s.* 6*d.* = 414*d.*
Then—If 414*d.* : 1 £. : : $510935\frac{5395}{6283}$*d.*
Or, as 2601162 : 1 £. : : 3210210000 : £. 1234 2*s.* $10\frac{1}{2}$*d.* $\frac{2087964}{2601162}$, the Answer.

10. With IRELAND.

£. £. £. *s.* *d.*
(24) If 112 : 100 : : 740 14 6
Or, as 4480*d.* : 100 : : 29629*d.* : 661 £. 7*s.* $2\frac{1}{4}$*d.* $\frac{14}{448}$ the Answer.

(25)

		£.	*s.*	*d.*	
10	$\frac{1}{10}$	651	14	$11\frac{3}{4}$	at 112 per 100
2	$\frac{1}{5}$	65	3	$5\frac{3}{4}$	
		13	0	$8\frac{1}{4}$	
Answer,		£. 729	19	$1\frac{3}{4}$	

11. With AMERICA and the WEST-INDIES.

£. £. £. *s.*
(26) If 164 : 100 : : 1474 16
Or, as 3280*s.* : 100 : : 29496*s.* : £. 899 5*s.* $4\frac{1}{4}$*d.* $\frac{184}{328}$ the Sterl. required.

(27)

(27)

£.		£.	s.	d.	
50	$\frac{1}{2}$	943	17	$5\frac{1}{4}$	at 164 per 100
10	$\frac{1}{5}$	471	18	$8\frac{1}{2}$	
2	$\frac{1}{5}$	94	7	$8\frac{3}{4}$	
2	$\frac{1}{5}$	18	17	$6\frac{1}{2}$	
		18	17	$6\frac{1}{2}$	

Anſwer, £. 1547 18 $11\frac{1}{2}$ Currency.

(28) Firſt, $100+30+5=135$ Amount.

Then—If 135£. : 100£. :: 987£. 12*s*.

Or, as 2700*s*. : 100 :: 19752*s*. : £.731 11*s*. $1\frac{1}{4}d.\frac{1}{3}$ remitted.

Conſigned	-	640	16	9
Gained	-	£. 90	14	$4\frac{1}{4}\,\frac{1}{3}$ Sterling.

£. s. d. £. s. d. £.

Therefore,—If 640 16 9 : 90 14 $4\frac{1}{4}\frac{1}{3}$: : 100

Or, as 153792*d*. : 261268 Thirds : : 24000*d*. : 6£. 12*s*. $2\frac{3}{4}d.\frac{1}{3}$ gained per Cent.

(29)

£.		£.	s.	d.	
25	$\frac{1}{4}$	1470	12	8	at $136\frac{1}{2}$ per 100
		367	13	2	
10	$\frac{1}{10}$	147	1	$3\frac{1}{5}$	
1	$\frac{1}{10}$	14	14	$1\frac{2}{5}$	
$\frac{1}{2}$	$\frac{1}{2}$	7	7	$0\frac{3}{5}$	
		2007	8	$3\frac{1}{5}$	

s. d. £. d.

(30) If 34 4 : 1 : : 52

Or, as 412*d*. : 240*d*. : : 52*d*. : 30*d*.$\frac{30}{103}$ for 400 Reas,

∵ $30\frac{30}{103}\times 2\frac{1}{2}=75\frac{75}{103}$ Pence Sterling for 1000 Reas.

(31)

(31) *Crowns.*

4*s.*	$\frac{1}{5}$	1200 at 4*s.* 7*d.* per Crown,
6*d.*	$\frac{1}{8}$	240
1	$\frac{1}{6}$	30
		5
		£. 275 0 0

£. *s.* £.
If 100 : 10 :: 275 :: 27*s.* 6*d.* Comm'ſſion.
Therefore, 275£. + 1£. 7*s.* 6*d.* = £. 276 7 6 = 66330*d.*
Then, as 56*d.* : 1*Cr.* :: 66330*d.* : $1184\frac{13}{28}$ *Cr.*
∵ 1200 — $1184\frac{13}{28} = 15\frac{15}{28}$ A's Gain.

d. *Sols.* *d.* *Sols.*
(32) As 67 : 32 :: 70 : $30\frac{29}{67}$ Lubeck, per Florin.

(33) Recip.—If 54*d.* : 33*s.* 6*d.* :: $54\frac{1}{2}$*d.*
Or, as 108 : 402 :: 109 : $398\frac{34}{109}$*d.* = 33*s.* 2*d.* $\frac{24}{109}$ Flemiſh, the Anſwer.

£. £. *s.* *d.*
(34) If 100 : 102 :: 33 4
Or, as 6000 : 102 : : 100 : 1£. 14*s.* Flem. per £. Sterling.

(35) If $100\frac{3}{20}$: 100 :: 91 : $90\frac{1730}{2003} = 90\frac{520730}{602903}$.
Alſo, $100\frac{1}{3}$: 100 :: 93 : $92\frac{208}{301} = 92\frac{416624}{602903}$.
Then $92\frac{416624}{602903} - 90\frac{528730}{602903} = 1\frac{438357}{602903}$, Gained per Cent. for two Months; therefore $1\frac{438357}{602903} \times 6 = 10\frac{218530}{602903}$ = 10£. 19*s.* $1\frac{3}{4}$*d.* Gain per Cent. per Ann. the Anſwer.

(36) Firſt, 2000 × 40 = 80000 Flemiſh Pence.
Or, 160000 Half-pence, $90\frac{1}{2}$*d.* = 181 Ditto, and $89\frac{1}{2}$*d.* = 179.

$$\left.\begin{array}{l} 181)160000(883\frac{171}{187} \\ 179)160000(893\frac{153}{179} \end{array}\right\} \begin{array}{l} \text{Crowns.} \\ \text{Tournois.} \end{array}$$

Or, 104 : 100 : : $883\frac{177}{181}$: $849\frac{2303}{2353} = 849\frac{8656077}{8844927}$.
Alſo, 105 : 100 : : $893\frac{153}{179}$: $851\frac{1061}{3759} = 851\frac{4473123}{8844927}$.
$851\frac{4473123}{8844927} - 849\frac{8656077}{8844927} = 1\frac{4661073}{8844927}$ = 1 *Cro.* 18 *ſols.* 9 *din.* in Favour of B.

32. COMPARISON of WEIGHTS and MEASURES.

(2)	*lb.*	*lb.*	
First,	104	$84\frac{1}{2}$	Then $104\times100\times64=665600$
	100	108	And $108\times84\frac{1}{2}=9126$
	64	——	9126)665600(72 *lb.* $14\frac{8084}{9126}$ *oz.* the Anſ.

	Yds.	*Ells.*	
(3)	100	78	Then $100\times78\times100=7800{,}00$
	78	$133\frac{1}{3}$	And $133\frac{1}{3}\times78=104{,}00$
	100	——	

Alſo, 104)7800(75 Yards, the Anſwer.

	Canes.	*Ells.*	
(4)	100	$191\frac{1}{3}$	Then $78\times100\times100=780000$
	78	$131\frac{2}{3}$	And $191\frac{1}{3}\times131\frac{2}{3}=\frac{126730}{9}$
	100	——	Alſo $780000\div\frac{226730}{9}=\frac{7020000}{226730}=30\frac{4}{5}$

Canes, nearly the Anſwer.

	lb.	*lb.*	
(5)	100	92	Then $92\times110\times60=607200$
	100	110	And $100\times100=10000$
	——	60	

Alſo, 1,0000)60,7200($60\frac{72}{100}$ *lb.* the Anſwer.

	Yds.	*Bra.*	
(6)	74	100	Then $100\times100\times30=300000$
	100	30	And $74\times100=7400$
	——	100	Alſo 74,00)300,00($40\frac{40}{74}$ Canes, the Anſw.

33. SINGLE POSITION.

(2)	Suppoſe ſhe had	10	Then $40-10=30$
	Then as many	10	If 25 : 10 : : 30 : 12 her Flock,
	One half as many	5	the Anſwer.
	Sum,	25,	for $12+12+6+10=40$, Proof.

(3) Suppose 10 to be C's Age.

∵	$10+4=14$ A's.	Then $4+4+9=17$
And	$10+14+9=33$ B's.	$45-17=28$

Sum $47-17=40 : 10 : 28 :$

$40)280(7$ C's Age.
And $7+4=11$ A's.
Also $7+11+9=27$ B's.

Proof, 45 D's.

(4) *Miles.*

Suppose Andrew goes - -	30	469
Then Ben. will go $\overline{30\times3}+3=$	93	-25
And Christopher $\overline{93\times2}+16=$	202	
		444
Sum	325	

Then $3+\overline{3\times2}+16=25$

M. *M.* *M.* *Miles.* 300 Diff.

∵ 300 : 30 : : 444 : $44\frac{2}{5}$ Andrew.
Also $\overline{44\frac{2}{5}\times3}+3=136\frac{1}{5}$ Benjamin.
And $\overline{133\frac{1}{5}\times2}+16=288\frac{2}{5}$ D. Christopher.

Proof, 469

(5)

	£.	s.	£.	s.
Suppose A paid - -	200	0	45	15
Then B $\overline{200\times2\frac{1}{2}}-45$£. 10s.$=$	654	5		×2
And C $200+654\ 5+26\ 10=$	880	15		
			91	10
Sum	1735	0	26	10
Difference			65	0

R Now

Now $2000+65=2065$, and $65+1735=1800$

	£.	£.	£.	£. s.	
Then—If	1800 :	200 ::	2065 :	229 $8\frac{16}{18}$	A } paid.
∵ 229 $8\frac{10}{18}\times 3\frac{1}{2}-45$ 15 =				757 $6\frac{2}{18}$	B
And 229 $8\frac{16}{18}+757$ $6\frac{2}{18}+26$ 10 =				1013 5	C
			Proof, £.	2000 0	

(6) Suppose they will empty the Cistern in 30 Minutes, then the first will empty $\frac{1}{2}$ or 30 Gallons.

The second	-	$\frac{1}{4}$ or 15
And the third	-	$\frac{1}{6}$ or 10
	Sum,	55

Gal. Min. G. M. Sec.

∵ If 55 : 30 :: 60 : 32 $43\frac{7}{11}$, the Time sought.

(7) First, $\frac{3}{4}$ of $\frac{1}{2}=\frac{3}{8}$—

Suppose he had 240, then $240\div 8\times 3=90$

∵ If 90 : 240 : : 900 : 2400 Men, the Answer.

(8) See Question the 28th in Double Position.

(9)

Suppose he had	12	If 37 : 12 :: 333
Then as many	12	333
$\frac{1}{2}$ as many	6	
$\frac{1}{3}$ Ditto	4	37)3996(108 Scholars, the Answer.
$\frac{1}{4}$ Ditto	3	
Sum,	37	

4. DOUBLE

34. DOUBLE POSITION.

(11) Suppose he had 8 *d.*	Again, suppose he had 7 *d.*
Then $8+8=16$	Then $7+7=14$
Also, $16-6=10$	$14-6=8$
$10+10=20$	$8+8=16$
$20-6=14$	$16-6=10$
$14+14=28$	$10+10=20$
$28-6=22$ too much.	$20-6=14$ too much.

Sup. Errors.

$$\begin{array}{ccc} 8 & & 22 \\ & \times & \\ 7 & & 14 \\ \hline 154 & & 112 \end{array}$$

Then $22-14=8$, and $154-112=42$
∴ $42 \div 8 = 5\frac{1}{4}$*d.* the Answer.

(12) *Crs.*

Suppose 28 A's	Again, suppose A had 34
Then $28+16=44$ B's	Then B $34+16=50$
And $28+44 \div 6=12$ C's	C $84 \div 6=14$
Sum, $84=28$ too little.	Sum, $98=14$ too little.

Sup. Errors.

$$\begin{array}{ccc} 28 & & 28 \\ & \times & \\ 34 & & 14 \\ \hline 952 & & 392 \end{array}$$

Then $952-392=560$, and $28-14=14$ Diff. of Errors.
∴ 14)560(- 40 A's
And $40+16$ - $=56$ B's
Also $40+56(96) \div 6=16$ C's } Crowns.

(13) Suppose he had 60	Again, suppose he had 76
Tom took $30-10=20$	Then $38-10=28$
Rem. 40	Rem. 48
Ned took $20-4=16$	$24-4=20$
Rem. 24	Rem. 28
Jack took $12-1=11$	$14-1=13$
Rem. $13=5$ too little.	Rem. $15=3$ too little.

Sup.	Er.	
60 ×	5	Then 580—180=200, and 5—3=2
76	3	∵ 200÷2=100, what he had at first.
580	180	

(14) Suppose the Father's Age to be 40
Then the Son will be 40÷5= 8
And 4 Years ago was 8—4= 4
which is 12 too little.
Again, suppose the Father was 60
Son will be 60÷5= - 12
And 4 Years since was 12—4=8, which is 4 too little.

Sup.	Er.	
40 ×	12	Then 720—160=560, and 12—4=8 ∵ 560 ÷8=70 Years, the Father's Age; and 70 ÷5=14 the Son's.
60	4	
720	160	For 14—4=10=$\frac{1}{7}$ of 70 (the Age of the Son 4 Year's ago) and 14×5=70, the Proof.

(15) Suppose the Body to be 24 Inches.
Then 24÷2+9=21 Tail; also 21+9=30, which is 6 too much.
Again, suppose the Body to be 26, then 26÷2+9=22 Tail; also 22+9=31, which is 5 too much.

Sup.	Er.	
24 ×	6	Then 156—120=36, and 6—5=1
26	5	∵ 36 Inches is the Length of the Body,
		And 36÷2+9=27 Tail.
156	120	Therefore 36+27+9=72 Inches, the Answer.

(16)

Suppose the No. to be	4	Again, suppose the No. to be	5
Then 4+4= -	8	Then 5+5= -	10
And 8×4= -	32	And 10×5= -	50
Also 32—4= -	28	Also 50—5= -	45
Likewise 28÷4=	7	45÷5= -	9
Therefore 13—7=	6	13—9= -	4
too little.		too little.	

Brought over,

Sup.	*Er.*	
4	6	Then $30-16=14$, and $6-4=2$.
5	4	$\because 14 \div 2=7$, the Number sought. For
		$7+7\times 7-7=91$, which $\div 7=13$, the Proof.
30	16	

(17)

Suppose she	14
Then he $14\times 3=$	42
$14+10+5=$	29
$42+10+5=$	57
$29\times 2 =$	58
too little by	1

Again, suppose she	16
Then he will be $16\times 3=$	48
$16+10+5=$	31
$48+10+5=$	63
31×2	62
too much by	1

$\left.\begin{matrix}63\\62\end{matrix}\right\}$ sub.

Sup.	*Er.*	
14	1	Then $16+14=30$, and $1+1=2$
16	1	$\because 30\div 2=15$ Years, her Age.
		And $15\times 3=45$ his, Answer. For
16	14	$15+10+5=30$ her, when married 15 Years.
		And $45+10+5=60$ his,

For as $8 : 16 :: 30 : 60$ Proof.

(18)

	Beg.	*Beg.*
Sup.	8	8
	$\times 4$	$\times 6$
	32	48
	$+16$	-12
	48 —	36=
12 too lit.		

	Beg.	*Beg.*
Again, suppose	10	10
	$\times 4$	$\times 6$
	40	60
	$+16$	-12
	56 —	$48=8$ too little.

Sup.	*Er.*	
8	12	Then $120-64=56$, and $12-8=4$ $\because 56\div 4=$
10	8	14, the Number of Beggars. For $14\times 4+16$
120	64	$=14\times 6-12=72d.$ what he gave, Proof.

(19) Proceed with this as with Example 11, and the Answer will be the same as that, viz. $5\frac{1}{4}d.$

(20)

Suppose the Number to be 15	Again, suppose 25
Then $15\times 3=45$	Then $25\times 3=75$
And $45-5=40$	And $75-5=70$
Also $40\div 2=20$	Also $70\div 2=35$
$20+15=35$	$35+25=60$
$40-35=5$	$60-40=20$
too little.	too much.

Sup. Er.

15 × 5		Then $125+300=425$, and $20+5=25$
25	20	∴ $425\div 25=17$, the Number required.
125	300	For $17\times 3-5=46$, and $46\div 2+17=40$, Proof.

(21)

Suppose B had 6	Again, suppose B had 8
Then A must have 4	Then A must have 6.
For $6-1=4+1=5$	For $8-2=6+1=7$
Again $4-1=3$	Again, $6-1=5$
And $6+1=7$	And $8+1=9$
Also, $3\times 2=6$	Also, $5\times 2=10$
$7-6=1$ too lit.	$10-9=1$, too much.

Sup. Er.

6 × 1		Then $8+6=14$, and $1+1=2$ ∴ $14\div 2=7$ B's Guineas, and 5 A's; for $7-2=5+1=6$ A's, when he had received 1 of B. Also, $\overline{5-1}\times 2=7+1=8$ B's, when he had received 1 of A's, Proof.
8	1	
8	6	

(22)

Suppose Father's Age 37	Again, suppose Father's 45
Then $37-5$ - $=32$	Then $45-5$ - $=40$
And $32\div 8$ - $=4$	And $40\div 8$ - $=5$
$4\times 3=12$ Son's Age.	∴ $5\times 3=15$ Son's Age.
Then $12+2$ - $=14$	Then $15+2$ - $=17$
And 14×3 - $=42$	And 17×3 - $=51$
Also, $42-7$ - $=35$	Also, $51-7$ - $=44$
∴ $37-35=2$ too little.	∴ $45-44=1$ too little.

Sup.

Sup.	*Er.*	
37 ×	2	Then $90-37=53$, and $2-1=1$ ∵ 53 is the Father's Age; then $53-5=48$, and $48 \div 8=6=\frac{1}{3}$ of the Son's Age. ∵ $6 \times 3=$ 18 the Son's Age.
45	1	
90	37	

(23)

		£.
Suppose B had		360
Then	$\overline{360 \div 3} \times 2=$	240
And	$1200-240=$	960 A's.
Then	$\overline{960 \div 4} \times 3=$	720
And	$360+720=$	1080 A's and B's.
∵	$1200-1080=$	120 too little.

	£.
Again, suppose B had	420
Then $\overline{420 \div 3} \times 2$	$=280$
And $1200-280$	$=920$ A's.
Then $\overline{920 \div 4} \times 2$	$=690$
And $420+690$	$=1110$
∵ $1200-1110$	$=$ 90 too little.

Sup.	*Er.*	
360 ×	120	Then $50400-32400=18000$ And $120-90=30$ ∵ $11000 \div 30=600$£. B's Money; and $\overline{600 \div 3} \times 2=400$. Also $1200-400=800$£. A's Money, the Answer.
420	90	
50400	32400	

For $600+\overline{800 \div 4} \times 3=800+\overline{600 \div 3} \times 2=1200$, the Proof.

greater. *lesser.*	Again, *gr.* *less.*
(24) Suppose 28 and 22	Supppose 42 and 8
$28 \div 7=4$. Also $22 \times 3=66$	$42 \div 7=6$; also $8 \times 3=24$
$4+66=70$, then $70-50=20$ too much.	$6+24=30$ ∵ $50-30=20$ too little.

Sup.

Sup.	*Er.*	
28 ×	20	Then $840+560=1400$, and $20+20=40$ $\because$ $1400 \div 40=35$, the greater Number: and $50-35=15$ the lesser; for $35 \div 7+15 \times 3=50$, the Proof.
42	20	
840	560	

(25) Suppose he worked 140 Days, $\because$ $390-140=250$ Days he was idle.

Then $140 \times 12=1680$
And $250 \times 8=2000$ } Subt. $=320$ too little.

Again, suppose he worked 150 Days, $\because$ $390-150=240$ Days idle.

Then $150 \times 12=1800$
And $240 \times 8=1920$ } Subt. $=120$ too little.

Sup.	*Er.*	
140 ×	320	Then $48000-16800=31200$
150	120	And $320-120=200$ $\because$ $31200 \div 200=156$ Days worked, and $390-156=234$ Days idle, the Answer.
48000	16800	For $156 \times 12=234 \times 8=1872$ Proof.

(26)

Suppose his Age was	24	Suppose he was	30
Then $24 \div 3 \times 2 \times 4$	$=63$	Then $30 \div 3 \times 2 \times 4$	$=80$
And $64+12+50$	$=126$	And $80+15+50$	$=145$
Then $126-100$	$=26$	Then $145-100$	$=45$
And $100-24$	$=76$	And $100-30$	$=70$
$\because$ $76-26=50$ too little.		$\because$ $70-45=25$ too little.	

Sup.	*Er.*	
24 ×	50	Then $1500-600=900$, and $50-25=25$ $\because$ $900 \div 25=36$ Years, his Age required.
30	25	
1500	600	For $36 \div 3 \times 2 \times 4+18+50-100=100-36=64$, Proof.

(27) Suppose the first Horse to be worth £. 24
Then the Trappings must be $50-24$ $=26$
And the second Horse must be $24+26$ $=50$

Now $50+26=76$, and $24 \times 2=48$; also $76-48=28$, too little.

Again,

Again, suppose the first Horse to be worth £. 28
Then the Trappings must be 50—20 =22
And the second Horse must be 28+22 =50
Now 50+22=72, and 28×2=56; also, 72—56=16, too little.

Sup.		*Er.*
24	×	28
28		16
784		384

Then 784—384=400
And 28—16=12 ∵ 400÷12=$33\frac{1}{3}$ £. Value of the first Horse; and 50—$33\frac{1}{3}$=$16\frac{2}{3}$ £. Value of the Trappings.
Also, $33\frac{1}{3}+16\frac{2}{3}$= 50 £. value of the second Horse, for 50—$16\frac{2}{3}$=$33\frac{1}{3}$×2=$66\frac{2}{3}$ the Proof.

(28)

	£.
Suppose he left	5000
Interest for 11 Years	=2700
Amount	7700
Education, &c. 100×11=	1100
Neat	6600

∵ 7400—1600=750 too little.

	£.
Again, suppose he left	6000
Interest for 11 Years	3300
Amount	9300
Education, &c. 100×11 =	1100
Neat	8200

∵ 8200—7400=800 to much.

Sup.		*Er.*
5000	×	750
9000		800
4500000		4000000

Then, 4500000+4000000=8500000, and 750+800=1550 ∵ 8500000÷1550=5483 £. 17 *s.* 5 *d.* $\frac{1}{31}$, the Answer required.

35. ARITHMETICAL PROGRESSION.

(2) Thus, $\overline{1+12}\times 6=78$ Strokes, Answer.

(3) First $2+188=190$ Sum of the Extremes.
And $94\div 2=47$ = half the Number of Terms.
Product 8930 Feet, a Mile=5280 Feet.
Then 5280)8930(1 Mile, 5 Feet, 21p $3\frac{1}{2}$ Feet, the Answer.

(4) First $2+200$ =202 Sum of the Extremes.
And $100\div 2=$ $\times$ 50 half the Numb. of Terms.
A Mile=yds. 1760)10100(5m. 5fur. 36p. 2yds. the Answer.

(5) First 9£. 19s. =199 Shillings.
Then $1+199$ =200 Sum of the Extremes.
And $100\div 2$ $\times$ 50 half the Number of Terms.
2,0)1000,0(500£. the Answer.

(7) First $60-6=54$ Diff. of the Extremes.
Then $19-\ 1=18$ Number of Terms, less one.
Then $54\div 18=\ 3$ the common Difference sought.
Then 3 added to 6 and every other Term respectively, gives each Day's Journey as follows, $6+9+12+15+18+21+24+27+30+33+36+39+42+45+48+51+54+57+60=627$ Miles from London.

(8) First $50-5=45$ Difference of the Extremes.
And $10-1=\ 9$ Number of Terms, less one.
Then $45\div 9=5$ Common Difference.
Then each Payment and the whole Debt will be as follows, viz. $5+10+15+20+25+30+35+40+45+50=275$ £. the whole Debt.

(10) First $60-6=54$ Diff. of the Extremes.
And $54\div 3=18$ ∵ $18+1=19$ Days, the Answer.

(12) First $10-1=9$ Number of Terms, less one.
And $9\times 2=18$. Then $23-18=5$, the Answer.

(14) First $100\div 2=50$ what each travelled.
And $50\div 5=10$. Also $5-1=4$. Then $4\times 2=8$. Likewise $8\div 2=4$ ∵ $10-4=6$ his first Day's Journey, and $6+2=8$ the 2d. 10 the 3d. 12 the 4th. and 14 the 5th. Sum of which =50.

Again.

Again, $4)50(12\frac{1}{2}$. Also $4-1=3$. Then $3\times3=9$. And $9\div2=4\frac{1}{2}$ ∵ $12\frac{1}{2}-4\frac{1}{2}=8$ his first Day's Journey, and $8+3=11$ his 2d. 14 his 3d. and 17 his 4th. Sum of which = 50 Leagues.

(16) First $100\times2=200$. Then $200-2=198$. And $198+1=199$, the last Term required.

(17) First $30\times2=60$. Also $60-2=58$.
Again $58+6=64$. And $64\times30=1920$. ∵ $1920\div2=960s.=48$ £. due, Answer.

(19) First $300\times2=600$. And $600\div9=66\frac{2}{3}$. Also $66\frac{2}{3}-4\times2(8)=58\frac{2}{3}$ ∵ $58\frac{2}{3}\div8=7\frac{1}{3}$ common Difference. Then the Miles he travelled each Day will be as follows, viz.

1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
4,	$11\frac{1}{3}$,	$18\frac{2}{3}$,	26,	$33\frac{1}{3}$,	$40\frac{2}{3}$,	48,	$55\frac{1}{3}$,	and $62\frac{2}{3}$,

which added together = 300, the whole Distance, Proof.

QUESTIONS.

(21) For A's Race thus.

First $4+40=44$, Sum of the first and last Terms. Then 44×5, half the Number of Terms $=220$ Yards. A's first Race per Prop. the first.

Then $901\times4=3604$ yds. A's first Term of the last Race.
Also $910\times4=3640$ A's last Term.
their Sum 7244 which $\times5=36220$ A's last Race.

Then to find his whole Ground, put 220 first Term. And 36226 the last Term, which (by Prop. 1.) is $36440\times5=182200$ Yards gone by A.

For B's, thus.

First $11\times4=44$ first Term. And $20\times4=80$ last Term. Then $44+80=124$ their Sum, which $\times5=620$ Yards, B's first Race.

∵ $911\times4=3644$ yds. B's first Term of the last Race.
$920\times4=3680$ B's last Race.
their Sum 7324 which $\times5=36620$ yds. B's last Race.

Then $620+36620=37240$ Sum of the first and last Terms.
which $\times5=$ 186200 Yards, gone by B.
Deduct 182200 ditto by A.
4000 Yards, common Difference.

Which

Which added continually to each of their shares, shew that

	Yds.		*Miles.*	*fur.*	*yds.*
A in all ran	182200	=	103	4	40
B	186200	=	105	6	80
C	190200	=	108	0	120
D	194200	=	110	2	160
E	198200	=	112	4	200
F	202200	=	114	7	20
G	206200	=	117	1	60
H	210200	=	119	3	100
I	214200	=	121	5	140
K	218200	=	123	7	180
Sums	2002000	=	1137	4	0

If 2002000 : 300£. :: 182200
182200

2002,000)54660,000(27£. 6*s.* 0½*d.* $\frac{590}{1001}$ A's part.

If 2002000 : 300£. :: 186200
186200

	£.	s.	d.		
2002,000)55860,000(	27	18	0½	$\frac{14}{1001}$	B's part.
	27	6	0½	590	
Common Difference		11	11¾	$\frac{425}{1001}$	

Which by continually adding to each we shall have

	£.	s.	d.				
First	27	6	0½	590	=	A's.	Part.
Then	27	18	0½	14	=	B's.	
	28	10	0¼	439	=	C's.	
	29	2	0	864	=	D's.	
	29	14	0	288	=	E's.	
	30	5	11¾	713	=	F's.	
	30	17	11¾	137	=	G's.	
	31	9	11½	562	=	H's.	
	32	1	11¼	987	=	I's.	
	32	13	11¼	411	=	K's.	
Proof	300	0	0				

From

From London to York suppose 180 Miles, which $\times 4 = 720$ Miles, twice a back

Their Sum Total 1137 m. 4 fur.
Distance of York 720

Short of the Undertaking $417\frac{1}{2}$ Miles.

(22) By the Nature of the Qustion, the Number of Calves that were calved at the End of these Years, will be as follows

B $\begin{cases} 3. & 4. & 5. & 6. & 7. & 8. & 9. & 10. & 11. & 12. & 13. & 14. & 15. & 16. \\ 1. & 1. & 1. & 2. & 3. & 4. & 6 & 9. & 13. & 19. & 28. & 41. & 60. & 88. \\ 17. & 18. & 19. & 20. & \text{Years.} \\ 120. & 189. & 277. & 406. & \text{respectively,} \end{cases}$ which are found by adding the last to the last but two.

Then of the whole Terms $1+1+1+2+3+4+6+9+13+19 \ldots +D+E+F+G$ be represented by S, when D, E, F, and G, denote the four last Terms, we shall then have $1+1+1+2+3+4+6+9+13+19+28 \ldots +D=S-E-F-G$, which being taken from the above, we have $1+1+1+1+2+3+4+6+9+13 \ldots +F=E+G+F$; and by adding G to both Sides of the Equation, we then get $1+1+1+1+2+3+4+6+9+13 \ldots +F-G=E+F+2G$; which will be the Man's Stock of Cows and Calves at the End of any Number of Years; which in this Case $E=189+F=77+2G=812$. Their Sum will be 1278, the Answer.

From the above Solution, it appears that the whole Stock of Cows and Calves at the End of any Number of Years, will be equal to the Number of Cows that would calve at the End of three Years after the given Time.

35. GEOMETRICAL PROGRESSION.

(2) First $\begin{cases} 0. & 1. & 2. & 3. & 4. & 5. & 6. \\ 1. & 2. & 4. & 8. & 16. & 32. & 64. \end{cases}$

Then $64 \times 64 = 4096 = 6+6 = 12$th Term.
And $4096 \times 4096 = 16777216 = 24$th Term.
Also $64 \times 32 = 2048 = 5+6 = 11$th Term.
$\because$ $16777216 \times 2048 = 34359738368$ Farthings, or 36th Term, which reduced to Pounds, will give 35791394£. 2s. 8d. the Answer.

0 1 2 3 4 5

(4) 500. 750. 1125 16875. 2531. 25. 3796. 875

5+4=9 Number of Terms less 1.

Then 3796, 875 × 2531, 25=9610839, 84375.

Which ÷500 the first Term =£. 19221. 13. 7. 125.

(6) First $\begin{cases} 0. & 1. & 2. & 3. & 4. & 5. & 6. \\ 1. & 2. & 4. & 8. & 16. & 32. & 64. \end{cases}$

Then 6+6+6=18 Number of Terms less 1.

64 × 64 × 64=262144 last Term.

262144—1÷2—1=262143, + 262144=524287 Farthings: or, 546 £. 2*s.* 7¾*d.* what 20 Horses came to. ∵ 20)546 2 7¾(27£. 6*s.* 3½*d.* $\frac{7}{20}$ per Head, the Answer.

(7) First $\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ 2. & 6. & 18. & 54. & 162. & 486 \end{cases}$

Then 486 × 486=236196=5+5=10th Term.

And 486 × 162= 78732=5+4=9th Term.

Product 18596183472 last Term.

—2 First.

Ratio 3—1=2)18596183470

9298091735 this added to the last Term gives

1,00)27894275207 the Sum of all the Terms at ½ per [100.

		£.	s.	d.
Value of the Pins 278942752 $\frac{7}{100}$ Qrs.=		290565	7	4
Lace comes to £. 8. 1. 8 × 5 × 4 (20)=		161	13	4
	Answer gained	290403	14	0

(8) 0 1 2 3 4 5

10. 100. 1000. 10000. 100000. 1000000.

Then 1000000 × 1000000=1000000000000

—10

Ratio 10—1=9)999999999990

Pints.

768,0)111111111111,0(144675924$\frac{455}{768}$,

A Bush-

A Bushel = pts. 64) 14467502(226056 $\frac{1}{8}$ Bushels at 4s.

226056 $\frac{1}{8}$ at 3s. 4d.

2

45211.4

=6=$\frac{1}{8}$ of 4s.

Answer £ 452114.6

(10) First 12×12=144 Square of the first Term.
And 12—10=2 Diff. of the first and second Term.
Then 2)144(72 Miles, the Answer.

37. PERMUTATION.

(1) First 1×2×3×4×5×6×7=5040 Days.
Then 365)5040(13 Years, 295 Days, the Answer.

(2) First 1×2×3×4×5×6×7×8×9×10×11×12 = 479001600 Rounds. Which × 3=1437004800 Seconds. And in a Year there are 35576,00)14370048,00(45 Years.

A Day =864,00)169128,00(19 Days.

An Hour =36,00)502,00(13 Hours.

A Minute =6,0)340,0(56 Men.

Remain 40 Seconds.

45 Years, 19 Days, 13 Hrs. 56 Min. 40 Secs. the Answer.

(3) First 1×2×3×4×5×6×7×8×9=362880 Days.
And 20 Guineas =5040 Pence.
Then, as 362880 Days : 5040 d. : : 365 D. : 5$\frac{215}{4536}$ d. per Year.

38. REDUCTION OF VULGAR FRACTIONS.

CASE I.

(2) 2832)12848(4 Then 16)$\frac{2832}{12848}$(=$\frac{177}{803}$, the Answer.

1520)2832(1

1312)1520(1

208)1312(

64)288(3

16)64(4

(3)

(3) 144)560(3 Then 16)$\frac{144}{560}$(=$\frac{9}{35}$, the Anſwer.

128)144(1

16)128(8

(4) 192)336(1 Then 48)$\frac{192}{336}$(=$\frac{4}{7}$, the Anſwer.

144)192(1

48)144(3

(5) 1476)1938(1 Then 6)$\frac{1476}{1938}$(=$\frac{246}{323}$, Anſwer.

462)1476(3

90)462(5

12)9(7

6)12(2

(7) 6)$\frac{42000}{54000}$(=$\frac{7}{9}$, the Anſwer.

(8) Thus 8)$\frac{24}{112}$(=$\frac{3}{14}$ 12)$\frac{96}{324}$(=$\frac{8}{27}$ 12)$\frac{60}{144}$($\frac{5}{12}$
And 12)$\frac{120}{360}$(=$\frac{1}{3}$
Anſwer $\frac{3}{14}$, $\frac{8}{27}$, $\frac{5}{12}$, and $\frac{1}{3}$, the Fractions required.

CASE II.

(9) Thus $\frac{2}{3}$ of $\frac{1}{2}$=$\frac{2}{6}$, which reduced by the laſt Caſe =$\frac{1}{3}$ the Anſwer.

(10) Thus $\frac{4}{9}$ of $\frac{5}{7}$ of $\frac{9}{10}$=$\frac{180}{630}$ reduced, =$\frac{2}{7}$, the Fraction required.

(11) Thus $\frac{2}{9}$ of $\frac{3}{4}$ of $\frac{4}{1}$=$\frac{24}{36}$ or $\frac{2}{3}$, the Fraction required.

(12) Thus $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{7}{8}$=$\frac{21}{168}$ or $\frac{1}{8}$, the Fraction required.

Note. If a Numerator of one Term in a Compound Fraction, be equal to a Denominator in another Term, cancel or reject both, and divide theſe Numerators and Denominators; which are diviſeable by each other, or by the ſame Number, which Quotients multiply into the remaining Numerators and Denominators, reduce the Compound Fraction to a ſingle one in the loweſt Terms.

As

As for Example suppose the last.
Thus $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{7}{8}=\frac{1}{8}$ the Fraction as before.
For the 3's and 7's divide off.

CASE III.

(16) Thus $\frac{12}{1}$, $\frac{27}{1}$, and $\frac{176}{1}$, the Fractions required.

(17) Thus $27\times12=324$. Then $\frac{324}{12}$, the Fraction required.

(18) Thus $7\times4+2=30$. Then $\frac{30}{7}$, the Fraction required.

(19) Thus $16\times2+1=33$. Then $\frac{33}{2}$, the Fraction required.

(20) Thus $142\times23+17=3283$. Then $\frac{3283}{23}$, the Answer.

(21) Thus $146\times37+21=5423$. Then $\frac{5423}{37}$, the Answer.

CASE IV.

(22) Thus $33\div2=16\frac{1}{2}$, the Answer.

(23) Thus $3283\div23=142\frac{17}{23}$, the Answer.

(24) Thus $5423\div37=146\frac{21}{37}$, the Answer.

CASE V.

(26) $\frac{1}{3}$. $\frac{7}{8}$. $\frac{5}{9}$ and $\frac{11}{12}$

First $1\times8\times9\times12=864$
Also $7\times3\times9\times12=2268$ New Numerators.
$5\times3\times8\times12=1440$
$11\times9\times8\times3=2376$
And $3\times8\times9\times12=2592$ Common Denominator.
Answer $\frac{864}{2592}$, $\frac{2268}{2592}$, $\frac{1440}{2592}$ and $\frac{2376}{2592}$.

(7) $\frac{3}{4}$. $\frac{11}{12}$ and $\frac{7}{10}$

First $3\times12\times10=360$
Also $11\times4\times10=440$ New Numerators.
$7\times12\times4=336$
And $4\times12\times10=480$ Common Denominator.
Answer $\frac{360}{480}$, $\frac{440}{480}$, and $\frac{336}{480}$.

(28) First $\frac{1}{6}$ of $\frac{7}{8}=\frac{7}{48}$. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{7}{48}$.

Then $1\times4\times5\times48=960$
Also $1\times3\times5\times48=720$ New Numerators.
$1\times3\times4\times48=576$
$7\times5\times4\times3=420$
And $3\times4\times5\times48=2880$ Common Denominat.
Answer $\frac{960}{2880}$, $\frac{720}{2880}$, $\frac{576}{2880}$, and $\frac{420}{2880}$.

$\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}$ and $\frac{1}{6}$.

(29) $\left.\begin{array}{ll} \text{First} & 1\times3\times4\times5\times6=360 \\ \text{Also} & 1\times2\times4\times5\times6=240 \\ & 1\times2\times3\times5\times6=180 \\ & 1\times2\times3\times4\times6=144 \\ & 1\times2\times3\times4\times5=120 \end{array}\right\}$ New Numerators.

And $2\times3\times4\times5\times6=720$ Common Denominator.

Answer $\frac{360}{720}$, $\frac{240}{720}$, $\frac{180}{720}$, $\frac{144}{720}$, and $\frac{120}{720}$.

(30) First $\frac{1}{6}$ of $\frac{3}{1}=\frac{3}{6}$ or $\frac{1}{2}$. $\frac{5}{6}$, $\frac{7}{10}$, $\frac{3}{4}$ and $\frac{1}{2}$.

$\left.\begin{array}{ll} \text{Then} & 5\times10\times4\times2=400 \\ \text{Also} & 7\times6\times4\times2=336 \\ & 3\times6\times10\times2=360 \\ & 1\times4\times10\times6=240 \end{array}\right\}$ New Numerators.

And $6\times10\times4\times2=480$ Common Denominator.

Answer $\frac{400}{480}$, $\frac{336}{480}$, $\frac{360}{480}$, and $\frac{240}{480}$.

CASE VI.

(33) Thus $\frac{3}{4\times21}=\frac{3}{84}$ or $\frac{1}{28}$, the Fraction required.

(34) Thus $\frac{1\times20\times12}{620}=\frac{24|0}{62|0}$ or $\frac{12}{31}$, the Fraction req.

(35) Thus $\frac{3}{4\times4\times12\times27}=\frac{3}{5184}$ or $\frac{1}{1728}$, the Fract. req.

(36) Thus $\frac{5}{7\times12\times20}=\frac{5}{1680}$ or $\frac{1}{336}$, the Fraction req.

(37) Thus $\frac{1\times21\times12\times4}{1344}=\frac{1008}{1344}$ or $\frac{3}{4}$, the Fract. req.

(38) Thus $\frac{5}{6\times20\times12}=\frac{5}{1440}$ or $\frac{1}{288}$, the Fraction req.

(39) Thus $\frac{3\times112}{448}=\frac{336}{448}$ or $\frac{3}{4}$, the Fraction required.

(40) Thus $\frac{3}{4\times16\times16\times112}=\frac{3}{114688}$, the Fract. req.

(41) Thus $\frac{5\times12\times20}{144}=\frac{1200}{144}$ or $\frac{25}{3}$, the Fraction req.

(42) Thus $\frac{7\times3\times8\times40}{9}=\frac{6720}{9}$ or $\frac{2240}{3}$, the Fract. req.

(43)

(43) Thus $\frac{7}{45}\frac{\times 4\times 4}{}=\frac{112}{45}$, the Fraction required.

(44) Thus $\frac{11}{12}\frac{}{\times 63}=\frac{11}{756}$, the Fraction required.

(45) Thus $\frac{7}{9}\frac{\times 48\times 8}{}=\frac{2688}{9}$ or $\frac{896}{3}$, the Fraction req.

(46) Thus $\frac{7}{142}\frac{\times 36}{}=\frac{252}{142}$ or $\frac{126}{71}$, the Fraction required.

(47) Thus $\frac{3}{5}\frac{}{\times 8\times 36}=\frac{3}{1440}$ or $\frac{1}{480}$, the Fraction req.

(48) Thus $\frac{7}{142}\frac{\times 7\times 24\times 60\times 60}{}=\frac{4233600}{142}$ or $\frac{2116800}{71}$, the Fraction required.

(49) Thus $\frac{10}{11}\frac{}{\times 60\times 24}=\frac{10}{15840}$ or $\frac{1}{1584}$, the Fract. req.

CASE VII.

(51) 367 Moidores.
×27

1126)9909(7*s.*

Remains 837*s.*
×12

1126)10044(7*d.*

Remains 972*d.*
×4

1126)3888(3 Qrs.

Answer 7*s.* $7\frac{3}{4}$.

(52) 5 Guineas.
×21

12)105

Answer 8*s.* 9*d.*

(53) 5*s.*
12

8)60

Answer $7\frac{1}{2}d.$

(54) £. *s.*
3 12
×4

5)14 8

Answer 2 17 $7\frac{1}{5}$.

(55) 5£.
20

7)100

Answer 14*s.* $3\frac{3}{7}d.$ $\frac{5}{7}$.

(56)

(56) *lb.*

411

×12

720)4932(6 oz.

Remains 612

×20

72,0)1224,0(17 dwts.

504

Answer 6 oz. 17 dwts.

(59) 7

×8

8)56

Answer 7 Furlongs.

Ell. Eng.

(60) 5

×5

8)25

Answer $3\frac{1}{8}$ Qrs.

(62) *Hhd. W.*

11

×63

252)693(2 Galls.

Remains 189

×4

252)756(3 Qts.

Answer 2 Galls. 3 qts.

(57) *cwt.*

7

×4

9)28

Answer 3 qrs. 3 lb. 1 oz. $12\frac{4}{9}$ dwts.

(58) *Tons.*

63

×20

124)1260(10 cwt.

Remains 20

×112

124)2240(18 lb.

1000

Remains 8

×16

124)128(1 oz.

Remains 4

Answer 10 C. 18 lb. $1\frac{1}{31}$ oz.

(61) *Acres.*

7

×4

8)28

Answer 3 Rds. 20 pol.

(63) 7 *Bar. B.*

×36

9)252

Answer 28 Galls.

(64) *Chal.*

25
×36

72)900(12 Bush.

Remains 36 Bush.
×4

72)144(2 Pecks.

Answer 12 Bush. 2 Pecks.

(65) *Months.*

7
×4

8)28

Answer Weeks 3. 4. 3. 12

(66) 5 *Days.*
×24

7)120

Answer Hours 17. 8. $34\frac{2}{7}$

CASE VIII.

(68) First $4\frac{1}{2}d.$ = 9 Halfpence; and $1s.$ = 24. Ans. $\frac{9}{24}$ or $\frac{3}{8}$.

(69) First 2£. $17s.$ $7\frac{1}{5}d.$ = 3456 Fifths.
And $3s.$ $12d.$ — = 4320 Ans. $\frac{3456}{4320}$ or $\frac{4}{5}$.

(70) First 8 . $2d.$ = 98 Pence.
And 1£. $1s.$ = 252 Answer $\frac{98}{252}$ or $\frac{7}{18}$.

(71) First 6 oz. 17 dwts. 21 grs. = 3309 grs.
And 1 lb. = 5760 Ans. $\frac{3309}{5760}$ or $\frac{1103}{1920}$.

(72) First 3 qrs. 3 lb. 1 oz. $12\frac{5}{9}$ = 200705 Ninths.
And 1 Cwt. = 358048 Answer $\frac{200705}{358048}$.

(73) First 10 Cwt. 18 lb. $1\frac{1}{31}$ oz. = 564480
And 1 Ton. = 1111040 or $\frac{63}{124}$ Ans.

(74) First 3 Qrs. $\frac{1}{8}$ = 25 Eighths.
And 1 Ell. Eng. = 40 or $\frac{5}{8}$, the Answer.

(75) First 2 Feet 6 inch = 30 Inches.
And 1 Yard = 36 or $\frac{5}{6}$, the Answer.

(76) First 4 Fur. 32 p. = 192 Poles.
And 1 Mile = 320 or $\frac{3}{5}$, the Answer.

(77) First 3 Roods. 2 p. = 122 Poles.
And 1 Acre = 160 or $\frac{61}{80}$, the Answer.

(78) Thus $\frac{42}{63}$ or $\frac{2}{3}$, the Fraction required.

(79) Thus $\frac{28}{36}$ or $\frac{7}{9}$, the Fraction required.

(80)

(80) First 4 Bush. 2 Pks $= \frac{58 \text{ Pecks.}}{144}$ or $\frac{29}{72}$, the Anſwer.
And a Chaldron $= 144$

(81) First 1 W. 1 d. 12 h. $= \frac{252 \text{ Hours.}}{672}$ or $\frac{3}{8}$, the Anſwer.
And 1 Month $= 672$

(82) First 243 D. 8 h. $= \frac{5840 \text{ Hours.}}{8760}$ or $\frac{2}{3}$, the Anſwer.
And 1 Year $= 8760$

39. ADDITION OF VULGAR FRACTIONS.

(2) Thus $\frac{3}{7}+\frac{1}{4}+\frac{5}{6}=\frac{72}{168}+\frac{42}{168}+\frac{140}{168}=\frac{254}{168}$ or $1\frac{43}{84}$ Anſwer.

(3) Thus $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}=\frac{360}{720}+\frac{480}{720}+\frac{540}{720}+\frac{576}{720}+\frac{600}{720}$ $=\frac{2556}{720}$ or $3\frac{11}{20}$, the Sum required.

(4) Firſt $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.
Then $\frac{8}{15}+\frac{13}{15}=\frac{21}{15}$ or $1\frac{2}{5}$, the Sum required.

(5) Firſt $\frac{3}{7}$ of $\frac{5}{6}=\frac{5}{14}$, and $\frac{5}{9}$ of $\frac{3}{4}=\frac{5}{12}$.
Then $\frac{5}{14}+\frac{5}{12}=\frac{60}{168}+\frac{70}{168}=\frac{130}{168}$ or $\frac{65}{84}$, the Sum req.

(6) Firſt $\frac{1}{5}$ of $\frac{5}{6}=\frac{1}{6}$, (ſee Note Caſe II.)
Then $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{72}{144}+\frac{48}{144}+\frac{36}{144}+\frac{24}{144}=\frac{180}{144}$ or $1\frac{1}{4}$, the Sum required.

(11) Firſt $\frac{1}{8}$ of 1*s.* $=\frac{1}{160}$ of 1£. per Caſe VI.
Then $\frac{201}{460}+\frac{1}{160}=\frac{32160}{73600}+\frac{460}{73600}=\frac{32620}{73600}$ or $\frac{1631}{3680}$£. which by Caſe VII. $=8$*s.* $10\frac{1}{4}$*d.* $\frac{11}{23}$.

(12) Firſt $\frac{3}{8}$ of a oz. $=\frac{3}{96}$ or $\frac{1}{32}$ of a lb.
Then $\frac{1}{4}+\frac{1}{32}=\frac{32}{128}+\frac{4}{128}=\frac{36}{128}=\frac{9}{32}$ lb. or 3 oz. 7 dwts. 12 grs. the Sum required.

(13) Firſt $\frac{1}{3}$ of a lb. $=\frac{1}{336}$ of a Cwt.
Then $\frac{5}{8}+\frac{1}{336}=\frac{1680}{2688}+\frac{8}{2688}=\frac{1688}{2688}=\frac{211}{336}$ Cwt. or 2 qrs. 14 lb. 5 oz. $5\frac{1}{3}$ drs. the Anſwer.

(14) Firſt $\frac{2}{5}$ of an Ell Eng. $=\frac{2}{4}$ of a Yard.
Then $\frac{3}{4}+\frac{2}{4}=\frac{5}{4}$ yd. or 1 yd. 1 qr. the Sum required.

(15) Firſt $\frac{2}{3}$ of a Yard. $=\frac{2}{5280}$ or $\frac{1}{2640}$ of a Mile.
Then $\frac{5}{6}+\frac{1}{2640}=\frac{13200}{15840}+\frac{6}{15840}=\frac{13206}{15840}$ Miles, or 6 Fur. 26 p 4 yds. 1 ft. the Sum required.

(16) Firſt $\frac{1}{2}$ of a Peck $=\frac{1}{288}$ of a Chaldron.
Then $\frac{3}{8}+\frac{1}{288}=\frac{864}{2304}+\frac{8}{2304}=\frac{872}{2304}=\frac{109}{288}$ or 13 Buſh. 2 pks. 1 gall. the Sum required.

(17) First $\frac{3}{4}$ of a Week $=\frac{3}{16}$ of a Month.
Then $\frac{3}{16}+\frac{5}{6}=\frac{18}{96}+\frac{80}{96}=\frac{98}{96}=\frac{49}{48}$ Mon. or 1 m. 14 hrs. the Sum required.

(18) First $\frac{3}{8}$ of an Hour $=\frac{3}{672}$ or $\frac{1}{224}$ of a Week.
Then $\frac{1}{224}+\frac{5}{8}=\frac{8}{1792}+\frac{1120}{1792}=\frac{1128}{1792}=\frac{141}{224}$ or 4 D. 9 h. 45 m. the Sum required.

(19) First $\frac{2}{5}$ of 12£. $=\frac{24}{5}$ or $4\frac{4}{5}$£. and $\frac{1}{5}$ of $\frac{9}{10}=\frac{9}{50}$.
Also $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{1}{20}=\frac{1}{40}$.
Then $\frac{4}{5}+\frac{3}{7}+\frac{9}{50}+\frac{1}{40}=\frac{56000}{70000}+\frac{30000}{70000}+\frac{12600}{70000}+\frac{1750}{70000}$ $=\frac{100350}{70000}$ or $1\frac{607}{1400}$.
Then $4+4+1\frac{607}{1400}=9\frac{607}{1400}$£. or 9£. 8*s.* 8*d.* $\frac{8}{35}$, the Sum required.

40. SUBTRACTION OF VULGAR FRACTIONS.

(2) Thus $\frac{11}{12}-\frac{2}{9}=\frac{99}{108}-\frac{24}{108}=\frac{75}{108}$, the Diff. required.
(3) Thus $\frac{19}{64}-\frac{2}{13}=\frac{247}{832}-\frac{128}{832}=\frac{119}{832}$, the Diff required.
(4) Thus $\frac{14}{17}-\frac{7}{11}=\frac{154}{187}-\frac{119}{187}=\frac{35}{187}$, the Diff. required.
(5) First $\frac{3}{4}$ of $\frac{7}{8}=\frac{21}{32}$.
Then $\frac{21}{32}-\frac{2}{5}=\frac{105}{160}-\frac{64}{160}=\frac{41}{160}$, the Diff. required.
(6) First $\frac{5}{6}$ of $\frac{7}{10}=\frac{7}{12}$, and $\frac{2}{3}$ of $\frac{1}{2}=\frac{1}{3}$.
Then $\frac{7}{12}-\frac{1}{3}=\frac{21}{36}-\frac{12}{36}=\frac{9}{36}$ or $\frac{1}{4}$, the Diff. required.
(7) Thus $10\frac{3}{4}-6\frac{5}{6}=10\frac{18}{24}-6\frac{20}{24}=3\frac{22}{24}$ or $3\frac{11}{12}$, the Diff.
(8) Thus $17\frac{11}{12}-16\frac{6}{7}=17\frac{77}{84}-16\frac{72}{84}=1\frac{5}{84}$, the Diff.
(9) First $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{3}{4}=\frac{35}{64}$.
Then $12-\frac{35}{64}=11\frac{29}{64}$, the Difference.
(10) First $\frac{3}{4}$ of a penny $=\frac{3}{48}$ or $\frac{1}{16}$ of 1*s.* per Case VI.
Then $\frac{2}{3}-\frac{1}{16}=\frac{32}{48}-\frac{3}{48}=\frac{29}{48}$*s.* or $7\frac{1}{4}$*d.* per Case VII. Anf.
(11) First $\frac{3}{4}$ of an oz. $=\frac{3}{7168}$ of a Cwt.
Then $\frac{1}{3}-\frac{3}{7168}=\frac{7168}{21504}-\frac{9}{21504}=\frac{7159}{21504}$C. or 1qr. 19lb. 4oz. $9\frac{1}{3}$drs. the Difference required.
(12) First $\frac{3}{8}$ of an Inch $=\frac{3}{288}$ or $\frac{1}{96}$ of a Yard.
Then $\frac{3}{4}-\frac{1}{96}=\frac{288}{384}-\frac{4}{384}=\frac{284}{384}=\frac{71}{96}$ yds. or 2 ft. $2\frac{5}{8}$ inc. the Difference required.
(13) First $\frac{3}{4}$ of a Peck $=\frac{3}{576}$ or $\frac{1}{192}$ of a Chaldron.
Then $\frac{3}{8}-\frac{1}{192}=\frac{576}{1536}-\frac{8}{1536}=\frac{568}{1536}=\frac{71}{192}$ Chal. or 13 bush. 1 pk. 2 galls. the Difference.
(14) First $\frac{3}{4}$ of an Hour $=\frac{3}{96}$ or $\frac{1}{32}$ of a Day.
Then $\frac{5}{6}-\frac{1}{32}=\frac{160}{192}-\frac{6}{192}=\frac{154}{192}=\frac{77}{96}$ Day; or 19 h. 15 min. the Difference.

MUL-

41. MULTIPLICATION OF VULGAR FRACTIONS.

(2) Thus $\frac{6}{11}\times\frac{7}{12}=\frac{7}{22}$ (see Note in Case II.) the Product required.

(3) Thus $12\frac{1}{2}\times 6=75$, the Product required.

(4) First $17\frac{3}{5}=\frac{88}{5}$.
Then $\frac{88}{5}\times\frac{7}{8}=\frac{77}{5}$ or $15\frac{2}{5}$, the Product required.

(5) First $2\frac{3}{4}=\frac{11}{4}$, and $\frac{3}{4}$ of $\frac{7}{8}=\frac{21}{32}$.
Then $\frac{11}{4}\times\frac{21}{32}=\frac{231}{128}$ or $1\frac{103}{128}$, the Product required.

(6) First $12\frac{3}{4}=\frac{51}{4}$, and $\frac{7}{8}$ of $\frac{12}{1}=\frac{84}{8}$ or $\frac{21}{2}$.
Then $\frac{51}{4}\times\frac{21}{2}=\frac{1071}{8}$ or $133\frac{7}{8}$, the Product required.

(7) First $\frac{3}{4}$ of $\frac{10}{11}=\frac{15}{22}$.
Then $\frac{15}{22}\times\frac{6}{1}=\frac{45}{11}$ or $4\frac{1}{11}$, the Product required.

(8) First $\frac{7}{8}$ of $\frac{3}{4}=\frac{21}{32}$, and $\frac{2}{5}$ of $\frac{5}{7}$ of $\frac{14}{1}=\frac{28}{7}$.
Then $\frac{21}{32}\times\frac{28}{7}=\frac{21}{8}$ or $2\frac{5}{8}$, the Product.

(9) First $3\frac{2}{3}=\frac{11}{3}$.
Then $\frac{11}{3}\times\frac{1}{7}=\frac{11}{21}$. Also $\frac{3}{5}$ of $\frac{3}{4}=\frac{9}{20}$.
Again $\frac{11}{21}\times\frac{9}{20}=\frac{33}{140}$, the Product required.

42. DIVISION OF VULGAR FRACTIONS.

(2) Thus $\frac{6}{7}\div\frac{3}{5}$ or $\frac{6}{7}\times\frac{5}{3}=\frac{30}{21}$, the Quotient required.

(3) First $\frac{2}{3}$ of $\frac{7}{8}=\frac{7}{12}$.
Then $\frac{2}{11}\div\frac{7}{12}$ or $\frac{2}{11}\times\frac{12}{7}=\frac{24}{77}$, the Quotient required.

(4) First $12\frac{1}{2}=\frac{25}{2}$, and $17\frac{2}{3}=\frac{53}{3}$.
Then $\frac{25}{2}\div\frac{53}{3}$ or $\frac{25}{2}\times\frac{3}{53}=\frac{75}{106}$, the Quotient required.

(5) First $12\frac{3}{4}=\frac{51}{4}$, and $3\frac{7}{8}=\frac{31}{8}$.
Then $\frac{51}{4}\div\frac{31}{8}$ or $\frac{51}{4}\times\frac{8}{31}=\frac{408}{124}$ or $3\frac{9}{31}$, the Quot. req.

(6) First $\frac{7}{8}$ of $\frac{3}{4}=\frac{21}{32}$, and $\frac{2}{5}$ of $\frac{12}{1}=\frac{24}{5}$.
Then $\frac{21}{32}\div\frac{24}{5}$ or $\frac{21}{32}\times\frac{5}{24}=\frac{105}{768}$, the Quotient required.

(7) First $7\frac{2}{7}=\frac{51}{7}$, then $\frac{51}{7}\div\frac{9}{1}$ or $\frac{51}{7}\times\frac{1}{9}=\frac{51}{63}$, the Quot.

(8) Thus $\frac{96}{1}\div\frac{146}{1}$ or $\frac{96}{1}\times\frac{1}{146}=\frac{96}{146}$ or $\frac{48}{73}$, the Quot. req.

(9) First $14\frac{3}{7}=\frac{101}{7}$, and $\frac{3}{4}$ of $\frac{12}{1}=\frac{9}{1}$.
Then $\frac{101}{7}\div\frac{9}{1}$ or $\frac{101}{7}\times\frac{1}{9}=\frac{101}{63}$, the Quotient required.

(10) First $142\frac{7}{12}=\frac{1711}{12}$, and $12\frac{3}{5}=\frac{63}{5}$.
Then $\frac{1711}{12}\div\frac{63}{5}$ or $\frac{1711}{12}\times\frac{5}{63}=\frac{8555}{756}$ or $11\frac{239}{756}$, the Quotient required.

(11) First $\frac{7}{8}$ of $\frac{6}{1}=\frac{21}{4}$, and $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{11}{12}=\frac{33}{56}$.
Then $\frac{21}{4}\div\frac{33}{56}$ or $\frac{21}{4}\times\frac{56}{33}=\frac{1176}{132}$ or $8\frac{10}{11}$, the Quot. req.

1. If the Divisor and Dividend have both the same Denominator, the Quotient may be found by dividing one Numerator by the other.

Thus $\frac{7}{11}\div\frac{4}{11}$; thus $7\div4=\frac{4}{7}$, the Quotient required.

2. If the Divisor and Dividend have each the same Numerator, divide one of the Denominators by the other, which will give the Quotient required.

Divide $\frac{8}{17}$ by $\frac{8}{11}$; thus $17\div11=\frac{11}{17}$, the Quotient.

3. If a Number can be found that will divide both the Numerators, or both the Denominators, (viz. those of the Divisor and Dividend) without a Remainder; use those Quotients instead of the given Numerators and Denominators, which will give the Result in its lowest Terms.

Divide $\frac{8}{15}$ by $\frac{9}{16}$; thus $\frac{8}{15}\div\frac{9}{16}=\frac{1}{5}\div\frac{3}{2}=\frac{3}{10}$, the Quot. req.
For 8 and 16 divide by 8, also 15 and 9 divide by 3.

43. THE RULE OF THREE DIRECT IN VULGAR FRACTIONS.

(2) If $2\frac{2}{5}$ yds. : $3\frac{3}{4}$ £. :: $4\frac{4}{5}$ yds. or $\frac{12}{5}$: $\frac{15}{4}$ £. :: $\frac{24}{5}$ yd.

$$\begin{array}{l} \text{Then } 24\times15\times5 \\ \text{And } 12\times4\times5 \end{array}\Bigg\}=\frac{1800}{240}\text{ £. or 7 £. 10}s.\text{ the Answer.}$$

(3) If $\frac{3}{4}$ lb. : $5\frac{1}{2}$ *s.* :: $42\frac{3}{8}$, or as $\frac{3}{4}$: $\frac{11}{2}$ *s.* : $\frac{339}{8}$ lb.

$$\begin{array}{l} \text{Then } 339\times11\times4 \\ \text{And } 3\times2\times8 \end{array}\Bigg\}=\frac{14916}{960}\text{ £. or 15 £. 10}s.\ 9d.\text{ Answ.}$$

(4) If $\frac{7}{9}$: $14\frac{2}{3}$ *s.* :: 8 Cwt. or $\frac{7}{9}$ *s.* : $\frac{44}{3}$ :: $\frac{8}{1}$.

$$\begin{array}{l} \text{Then } 44\times9\times8 \\ \text{And } 7\times3\times1 \end{array}\Bigg\}=\frac{3168}{21}s.\text{ or 7 £. 10}s.\ 10\tfrac{1}{4}d.\ \tfrac{1}{7}\text{ Answ.}$$

(5) If $10\frac{1}{2}$: 100 £. :: $2700\frac{1}{2}$, or as $\frac{21}{2}$: $\frac{100}{1}$ £. :: $\frac{2701}{2}$,

$$\begin{array}{l} \text{Then } 2701\times2\times100 \\ \text{And } 21\times1\times2 \end{array}\Bigg\}=\frac{540200}{42}\text{ £. or 283 £. 11}s.\ \tfrac{1}{2}d.\ \tfrac{2}{5},$$

the Answer.

(6) If $112\frac{5}{8}$ £. : 100 £. : : 1270 £. or as $\frac{901}{8}$ £. : $\frac{100}{1}$: : $\frac{1270}{1}$.

$$\begin{array}{l} \text{Then } 1270\times8\times100 \\ \text{And } 901\times1\times1 \end{array}\Bigg\}=\frac{1016000}{901}\text{ £. or 1127 £. 12}s.$$

$8\frac{1}{2}d.\ \frac{470}{901}$, the Answer.

(7) First $4\frac{3}{4}=\frac{19}{4}$, and $22\frac{3}{8}=\frac{179}{8}$, ∵ $\frac{179}{8}\times\frac{19}{4}=\frac{3401}{32}$ yds. in the $22\frac{3}{8}$ Pieces. Then,

If 1 : $8\frac{3}{4}$ *s.* ∵ $\frac{3401}{32}$, or as $\frac{1}{1}:\frac{35}{4}$ *s.* : $\frac{3401}{32}$ yds.

$$\left.\begin{array}{l}\text{Then } 3401\times35\\ \text{And } \quad 32\times4\end{array}\right\}=\frac{119035}{128}\text{£.}$$ or 46£. 9*s.* $11\frac{1}{2}$*d.* $\frac{1}{8}$ Answer.

(8) If 5*s.* $6\frac{1}{2}$*d.* : 3 qrs. :: 100£. 10*s.* 6*d.* or as $\frac{267}{960}$£. : $\frac{3}{5}$ Ell. :: $\frac{4021}{40}$£.

$$\left.\begin{array}{l}\text{Then } 4021\times3\times960\\ \text{And } \quad 267\times5\times40\end{array}\right\}=\frac{11580480}{53400}$$ or $\frac{289512}{1335}$ Ells. ∵ $\frac{289512}{1335}\div\frac{12}{1}=\frac{289512}{16020}$, or $18\frac{32}{445}$ Ells in each Piece, the Answer.

44. THE RULE OF THREE INVERSE.

(2) If $25\frac{1}{3}$£. : $6\frac{3}{4}$ mo. :: $10\frac{3}{4}$£. or as $\frac{76}{3}:\frac{27}{4}::\frac{43}{4}$.

$$\left.\begin{array}{l}\text{Then } 76\times27\times4\\ \text{And } \quad 43\times3\times4\end{array}\right\}=\frac{8208}{516}$$, or 15 mo. 3 w. $4\frac{51}{129}$ d. Ans.

(3) If 4 : $12\frac{3}{4}$ ho. :: 12 or as $\frac{4}{1}:\frac{51}{4}$ ho. :: $\frac{12}{1}$ per.

$$\left.\begin{array}{l}\text{Then } 51\times4\times1\\ \text{And } \quad 1\times4\times12\end{array}\right\}=\frac{204}{48}=\frac{17}{4}$$, or $4\frac{1}{4}$ Hours, Answer.

(4) If $12\frac{2}{3}$ oz. : 5*s.* :: $8\frac{3}{4}$ oz. or as $\frac{38}{3}:\frac{5}{1}$*s.* :: $\frac{35}{4}$.

$$\left.\begin{array}{l}\text{Then } 38\times5\times4\\ \text{And } \quad 35\times1\times3\end{array}\right\}=\frac{760}{105}$$, or 7*s.* $2\frac{3}{4}$*d.* $\frac{3}{7}$, the Answer.

(5) If $6\frac{2}{3}$ m. : $100\frac{2}{3}$£. :: $3\frac{5}{6}$ yrs. or as $\frac{5}{9}$ yr. : $\frac{302}{3}$£. :: $\frac{23}{6}$ yrs.

$$\left.\begin{array}{l}\text{Then } 302\times5\times6\\ \text{And } \quad 33\times9\times3\end{array}\right\}=\frac{9060}{621}$$, or 14£. 11*s.* $9\frac{1}{4}$*d.* $\frac{55}{469}$, Answ.

(6) If $5\frac{7}{12}$ *s.* : $26\frac{5}{8}$ yds. :: $8\frac{1}{2}$*s.* or as $\frac{67}{12}:\frac{213}{8}::\frac{17}{2}$.

$$\left.\begin{array}{l}\text{Then } 67\times213\times2\\ \text{And } \quad 12\times8\times17\end{array}\right\}=\frac{28542}{1632}$$, or 17 yds. $\frac{133}{272}$ qrs. Answ.

45. THE DOUBLE RULE OF THREE.

(2) First 14£. 6*s.* 8*d.* $=14\frac{1}{3}$ or $\frac{43}{3}$£.

Then 12 . 16 . $\frac{43}{3}$	Now $12\times16=192$ Divisor per Sect. 15.
18 . 34.	And $\frac{43}{3}\times34\times18=\frac{26316}{3}$ Dividend.

∵ $\frac{26316}{3}\div\frac{192}{1}=\frac{26316}{576}$, or 45£. 13*s.* 9*d.* the Answer.

(3) First 16£. 13s. 4d. × $16\frac{2}{3}$£. or $\frac{50}{3}$, and 6£. 17s. 6d. = $6\frac{7}{8}$ or $\frac{55}{8}$.

Cwt. m. £.

Then 40 . 30 . $\frac{50}{3}$ | Now $\frac{50}{3} \times \frac{80}{1} = \frac{4000}{3}$ Divisor.
— . 80 . $\frac{55}{8}$ | Also $\frac{40}{1} \times \frac{30}{1} \times \frac{55}{8} = \frac{66000}{8}$ Dividend.

∴ $\frac{66000}{8} \div \frac{4000}{3} = \frac{99}{16}$ or 6 Cwt. 21 lb. the Answer.

(4) First 26£. 19s. 4d. = $26\frac{29}{30}$ or $\frac{809}{30}$, and 14£. 15s. = $14\frac{3}{4}$ or $\frac{59}{4}$.

Per. m. £.

Then 12 . 3 . $\frac{809}{30}$ | Now $\frac{809}{30} \times \frac{36}{1} = \frac{29124}{30}$ Divisor.
36 . — $\frac{59}{4}$ | Also $\frac{59}{4} \times \frac{12}{1} \times \frac{3}{1} = \frac{2124}{4}$ Dividend.

∴ $\frac{2124}{4} \div \frac{29124}{30} = \frac{63720}{116496}$ m. or $16\frac{833}{14562}$ Days, Answer.

M. d. w.

(5) 30 . 11 . 1 | Now $\frac{11}{5} \times \frac{1}{1} = \frac{11}{5}$ Divisor.
— . $\frac{11}{5}$. 4 | Also $\frac{30}{1} \times \frac{11}{1} \times \frac{4}{1} = \frac{1320}{1}$ Dividend.

∴ $\frac{1320}{1} \div \frac{11}{5} = \frac{6600}{11}$ or 600 Men, the Answer.

(6) First $2\frac{1}{2} = \frac{5}{2}$, and $3 - \frac{1}{10} = 2\frac{9}{10}$ or $\frac{29}{10}$, also $\frac{1}{8}$ of $\frac{3}{5}$ of $\frac{21}{1} = \frac{63}{40}$ s.

T. m. £.

Then $\frac{5}{2}$. $\frac{29}{10}$. $\frac{63}{40}$ | Now $\frac{29}{10} \times \frac{5}{2} = \frac{29}{2}$ Divisor.
$\frac{1}{20}$. 1 . — | Also $\frac{63}{40} \times \frac{1}{20} \times \frac{1}{1} = \frac{63}{80}$ Dividend.

∴ $\frac{63}{80} \div \frac{29}{2} = \frac{126}{2320}$ s. or $\frac{378}{725}$ qr. the Answer.

45. QUESTIONS IN VULGAR FRACTIONS.

(1) Thus $99\frac{9}{9}$ the Answer. For $\frac{9}{9} = 1$, and $99 + 1 = 100$.

(2) First $\frac{3}{16}$ of $\frac{18}{7}$ of $\frac{141}{213} = \frac{7614}{14910}$ or $\frac{1269}{2485}$.
∴ $\frac{1}{1} - \frac{1269}{2485} = \frac{1216}{2485}$ Answ. For $\frac{1269}{2485} + \frac{1216}{2485} = 1$, Proof.

(3) First $\frac{1}{25}$ of $\frac{7}{8} = \frac{7}{200}$, and $\frac{1}{16}$ of $\frac{47}{19} = \frac{47}{304}$.
∴ $3\frac{7}{200} - \frac{47}{304} = 3\frac{2128}{60800} - \frac{9400}{60800} = 2\frac{53528}{60800}$ or $2\frac{6591}{7600}$ the Answer.

For $2\frac{53528}{60800} - \frac{2128}{60800} = 3\frac{51400}{60800}$ which added to $\frac{9400}{60800} = 3$, the Proof.

(4) First $\frac{1}{11}$ of $\frac{12}{1} = \frac{12}{11}$, and $\frac{1}{19}$ of $\frac{27}{1} = \frac{27}{19}$.
Then $\frac{12}{11} + \frac{27}{19} = \frac{228}{209} + \frac{297}{209} = \frac{525}{209} = 2\frac{107}{209}$.
Again $7\frac{1}{2} = \frac{15}{2}$, then $\frac{1}{3}$ of $\frac{15}{2} = \frac{5}{2}$; also $1\frac{1}{4} = \frac{5}{4}$, and $\frac{20}{30}$ of $\frac{5}{4} = \frac{20}{24}$; then $\frac{5}{2} - \frac{20}{24} = \frac{31}{24}$ or $1\frac{7}{24}$, which added to $8 = 9\frac{7}{24}$.
∴ $9\frac{7}{24} - 2\frac{107}{209} = 9\frac{1463}{5016} - 2\frac{2568}{5016} = 6\frac{3911}{5016}$ the Answer.

(5) First $\frac{1}{3}$ of $\frac{7}{8}$ of $\frac{11}{4} = \frac{231}{128}$.
∴ $\frac{1}{1} \div \frac{231}{128} = \frac{128}{231}$ the Number, therefore $\frac{128}{231} \times \frac{128}{231} \times \frac{128}{231} = \frac{2097152}{12326391}$, the Cube of that Number.

(6) First $\frac{16}{3}$ of $\frac{3}{16}=\frac{48}{48}=1$, and $9\frac{28}{37}=\frac{361}{37}$.
$\because\ \frac{361}{37}\times\frac{1}{1}=\frac{361}{37}$, the Number.
Then $\frac{361}{37}\times\frac{361}{37}=\frac{130321}{1369}$ or $95\frac{266}{1369}$, the Anſwer.

(7) First $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}=\frac{3}{10}$; alſo $\frac{1}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}=\frac{22}{273}$.
Then, as $\frac{3}{10}:\frac{22}{273}::\frac{12000}{1}:\frac{880000}{237}=3223$ £. 8*s.* $80\frac{1}{4}$*d.* $\frac{45}{91}$, Value of the Ship.
$\because\ 12000+3223$ £. 8*s.* $10\frac{1}{4}$*d.* $\frac{45}{91}=15223$ £. 8*s.* $10\frac{1}{4}$*d.* $\frac{45}{91}$, the Anſwer.

(8) First $\frac{3}{4}$ of $\frac{3}{5}=\frac{9}{20}$ ſold. Then, as $\frac{9}{20}:\frac{1710}{1}::\frac{20}{20}:\frac{34200}{9}=3800$ £. the Anſwer.

(9) His whole Eſtate $\frac{83}{83}-\frac{34}{83}=\frac{49}{83}$. Then $\frac{34}{83}$ of $\frac{49}{83}=\frac{1666}{6889}$ youngeſt Son's Part; and $\frac{34}{83}=\frac{2822}{6889}$ eldeſt.
$\because\ \frac{2822}{6889}-\frac{1666}{6889}=\frac{1150}{6889}=57$ £. 3*s.* 4*d.* or $\frac{1543}{6}$.
Alſo $\frac{2822}{6889}+\frac{1666}{6889}=\frac{4488}{6889}$ both the Sons Part.
So that $\frac{6889}{6889}-\frac{4488}{6889}=\frac{2401}{6889}$, Widow's Part.
Then by rejecting the Common Denominator we ſhall have the following Proportion

	£.	*s.*	*d.*
As $1156:\frac{1543}{6}::2401:\frac{3704743}{6936}=$ Widow had the Uſe of.	534	2	$8\ \frac{1616}{3468}$
As $1156:\frac{1543}{6}::2822:\frac{2177170}{3468}=$ eldeſt Son had.	627	15	$9\frac{1}{4}\ \frac{2852}{3468}$
As $1156:\frac{1543}{6}::1666:\frac{1285219}{3468}=$	370	19	$5\frac{1}{4}\ \frac{2244}{2468}$
His whole Eſtate	1622	10	$10\frac{3}{4}\ \frac{3824}{3468}$

(10) First $\frac{19}{30}-\frac{11}{39}=\frac{741}{1170}-\frac{339}{1170}=\frac{411}{1170}$.
Then, as $\frac{411}{1170}:\frac{1081}{2}$ £. $::\frac{1170}{1170}:\frac{1264770}{822}=1538$ £. 12*s.* $11\frac{3}{4}$*d.* $\frac{257}{411}$, the Anſwer.

(11) First 4000 his whole Stock.
Then $\frac{5}{8}$ of $\frac{2}{3}$ of $4000=1666\frac{2}{3}$ Mad Tom took.

$2333\frac{1}{3}=\frac{7000}{3}$ left.
$\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{7000}{3}=583\frac{1}{3}$ Raving Ned took.

1750 left.
$\frac{7}{10}$ of $\frac{17}{20}$ of $1750=1041\frac{1}{4}$ Poſitive Jack took.

$708\frac{3}{4}=708\frac{48}{64}$ left.
$\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{2835}{4}=132\frac{57}{64}$ Dolly had.

Anſwer $575\frac{55}{64}$ left.

(12) As $\frac{5}{12} : 2200 :: \frac{12}{12} : \frac{26400}{5} = 5280$£. Brother's Fortune.
And $5280 \times 3\frac{1}{8} = 16500$; Also $1\frac{1}{2} = \frac{3}{2}$.
$\because 16500 \div \frac{3}{2} = \frac{33000}{3} = 11000$£. Father's Part.

(13)

	£.	s.
First 17 Hhds. at 34£. each $= 34 \times 17 =$	578	0
As $\frac{4}{7} : \frac{578}{1} :: \frac{7}{7} : \frac{4046}{4} =$	1011	10
13 Guns at 18£. 10s. each = 18£. 10s. $\times 13 =$	240	10
Value of what was cast overboard	1830	0

Then $\frac{3}{7}$ of $\frac{9}{13} = \frac{27}{91} = 1830$£. and $\frac{27}{91} + \frac{64}{91} = \frac{91}{91}$.
$\because$ as $\frac{27}{91} : \frac{1830}{1} :: \frac{64}{91} : \frac{117120}{27} = 4337$£. 15s. $6\frac{2}{3}$d. the Value which came into Port.

(14) First $16131\frac{7}{10} = \frac{161317}{10}$.
Then $\frac{7}{8}$ of $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{161317}{10} = \frac{3387657}{800} = 4234$£. 11s. $5\frac{1}{10}$d. A had at first.
And $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{3387657}{800} = \frac{10162971}{5000} = 2032$£. 11s. $10\frac{1}{2}$d. $\frac{54}{525}$ sold B.
$\because \frac{161317}{10} - \frac{3387657}{800} = \frac{129053600}{8000} - \frac{3387657|0}{800|0} = \frac{9417603}{800}$.
Also $\frac{9}{11}$ of $\frac{9517703}{800} = \frac{85659327}{8800} = 9734$£. $3\frac{1}{4}$d. $\frac{47}{55}$ Cousin P paid.

(15) First $\frac{1}{24} = \frac{12}{408}$ X, $\frac{1}{24} = \frac{17}{408}$.
Then $\frac{12}{408} + \frac{17}{408} = \frac{29}{408}$ performed in one Day by X, and Z.
And $\frac{1}{12} = \frac{34}{408}$ performed in one Day by all three working together.
$\because \frac{34}{408} - \frac{29}{408} = \frac{5}{408}$ done in one Day by Y.
Therefore as $\frac{5}{408}$: 1 Day :: 1 Work : $\frac{408}{5} = 81\frac{3}{5}$ Days, the Answer.

(16) First $\frac{16}{16} - \frac{3}{16} = \frac{13}{16}$ left at 6 Months End.
Then $\frac{2}{3}$ of $\frac{13}{16} = \frac{26}{48}$; and $\frac{13}{16} - \frac{26}{48} = \frac{624}{768} - \frac{416}{768} = \frac{208}{768} =$ 348 left.
$\because$ as $\frac{208}{768} : \frac{348}{1} :: \frac{768}{768} : \frac{267264}{208} = 1284$£. 18s. $5\frac{1}{2}$d. $\frac{2}{13}$ Ans.

(17) As $\frac{3}{5} : \frac{19312}{1} :: \frac{5}{5} : \frac{96560}{3} = 32186$£. 13s. 4d. two Years Rent; which $\div$ by 2 = 16093£. 6s. 8d. the yearly income required.

(18) First 880 Guineas = 924£.
Then as $\frac{3}{20} : \frac{924}{1} :: \frac{20}{20} : \frac{18480}{3} = 6160$£.
$\because 2200 + 6160 = 8366 = \frac{4}{5}$ of his whole Fortune.
Now as $\frac{4}{5} : \frac{8360}{1} :: \frac{5}{5} : \frac{41880}{4} = 10450$£. the Answer.

(19) Here as the Son was to have twice as much as the Mother, and the Mother twice as much as the Daughter, so we will suppose the Estate to be divided as follows, viz. $4+2+1=7$ the whole Estate; then as she had both a Son and a Daughter, the Mother must have but $\frac{2}{7}$ of the Estate; whereas had it been only a Daughter, she would have had $\frac{2}{3}$.

Then $\frac{2}{3}-\frac{2}{7}=\frac{14}{21}-\frac{6}{21}=\frac{8}{21}=2000$£.

$\therefore$ as $\frac{8}{21} : \frac{2000}{1} :: \frac{1}{3} : \frac{42000}{24}=1750$£. the Answer.

(20) First Cock runs off 3 Galls. $=\frac{3}{103}$ of the Cistern in a Minute; and $1\frac{1}{4}$ min. $=\frac{5}{4}$.

Then, as $\frac{5}{4} : \frac{3}{103} :: 1 : \frac{12}{515}$ runs off by the second in a Minute.

And $\frac{3}{103}+\frac{12}{515}=\frac{15}{515}+\frac{12}{515}=\frac{27}{515}$ runs off by both in a Minute.

$\therefore$ as $\frac{27}{515} : 1 :: 1 : \frac{515}{27}=19$ min. $4\frac{4}{9}$ sec. the Answer.

(21) First $\frac{1}{4}+\frac{1}{3}+\frac{1}{6}=\frac{18}{72}+\frac{24}{72}+\frac{12}{72}=\frac{54}{72}=\frac{27}{36}$.

Then as $\frac{27}{36} : \frac{45}{1} :: \frac{36}{36} : \frac{1620}{27}=60$ Crowns, the Answer.

(22) Here for one twelfth, read 50.

First $\frac{1}{2}=\frac{6}{12}$ Apples; $\frac{1}{4}=\frac{3}{12}$ Pears; $\frac{1}{6}=\frac{2}{12}$ Plums.

Then $\frac{6}{12}+\frac{3}{12}+\frac{2}{12}=\frac{11}{12}$; also $\frac{12}{12}-\frac{11}{12}=\frac{1}{12}=50$ Cherries.

$\therefore$ $50\times 6=300$ Apples.
$50\times 3=150$ Pears.
$50\times 2=100$ Plums.
50 Cherries.

Answer in all 600 Trees.

(23) First $1=\frac{4}{4}$, and $\frac{1}{2}=\frac{2}{4}$.

Then $\frac{4}{4}+\frac{4}{4}+\frac{2}{4}+\frac{1}{4}=\frac{11}{4}=99$ by the Question.

$\therefore$ As $\frac{11}{4} : 99 :: 1 : \frac{396}{11}=36$ Scholars.

(24) First $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}=\frac{15}{30}+\frac{10}{30}+\frac{6}{30}=\frac{31}{30}$.

Then $\frac{31}{30}+\frac{20}{30}=\frac{61}{30}$; and $74-\frac{4}{5}=73\frac{1}{5}=\frac{366}{5}=\frac{2196}{30}$.

$\therefore$ $\frac{61}{30} : \frac{2196}{30} :: 1 : \frac{2196}{61}=36$ Years, the Answer.

(25) $\left.\begin{array}{l}\text{First } \frac{3}{8} \text{ of } \frac{2}{3}=\frac{1}{4} \text{ B's.} \\ \text{And } \frac{3}{10} \text{ of } \frac{2}{3}=\frac{1}{5} \text{ C's,}\end{array}\right\}$ First Acquisition, their Sum $=\frac{9}{20}$.

Then $\frac{2}{3}-\frac{9}{20}=\frac{13}{60}=\frac{91}{420}$ left.

$\therefore$ $\frac{1}{8}$ of $\frac{13}{60}=\frac{13}{420}=$ E's first Acquisition.

Also $\frac{91}{420}-\frac{13}{420}=\frac{78}{420}$ D's, thus ended the first Heat.

Again

$$\left.\begin{array}{l}\text{Again } \frac{1}{2} \text{ of } \frac{1}{4} = \frac{1}{8} \text{ B's} \\ \text{Retained } \frac{1}{5} \text{ C's} \\ \text{And } \frac{13}{70} + \frac{1}{16} = \frac{139}{560} \text{ D's} \\ \text{Also } \frac{13}{420} + \frac{1}{16} = \frac{157}{1680} \text{ E's}\end{array}\right\} \text{Part at the End of the second Scuffle.}$$

Proceeding $\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{20}$ A's.

$\frac{1}{3}$ of $\frac{1}{5} + \frac{1}{8} = \frac{233}{120}$ B's.

$\frac{2}{7}$ of $\frac{1}{5} + \frac{139}{560} = \frac{171}{560}$ D's.

Then $\frac{1}{20} + \frac{1}{15} + \frac{2}{35} = \frac{73}{420}$.

$$\left.\begin{array}{l}\frac{1}{5} - \frac{73}{420} = \frac{11}{420}, \text{ and } \frac{1}{2} \text{ of } \frac{11}{420} = \frac{11}{840} \text{ C's} \\ \frac{157}{168} + \frac{11}{840} = \frac{179}{1680} \text{ E's}\end{array}\right\} \text{Part of the third Smuſs.}$$

Further $\frac{1}{20} + \frac{1}{15} = \frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{60} = \frac{7}{80}$, loſt by A and B.

Then $\frac{5}{16}$ of $\frac{7}{80} + \frac{1}{4}$ of $\frac{1}{20} = \frac{51}{1280}$ A's.

$$\left.\begin{array}{l}\text{Alſo } \frac{5}{16} \text{ of } \frac{7}{80} + \frac{1}{4} \text{ of } \frac{1}{25} \text{ of } \frac{1}{8} = \frac{649}{3840} \text{ B's} \\ \frac{1}{8} \text{ of } \frac{7}{80} + \frac{11}{840} = \frac{323}{13440} \text{ E's}\end{array}\right\} \text{Part after the laſt Smuſs.}$$

$$\left.\begin{array}{l}\frac{1}{8} \text{ of } \frac{7}{80} + \frac{171}{840} = \frac{1417}{4480} \text{ D's} \\ \frac{1}{8} \text{ of } \frac{7}{80} + \frac{179}{1680} = \frac{1579}{13440} \text{ E's}\end{array}\right\} \text{Part after the laſt Smuſs.}$$

$$\left.\begin{array}{l}\text{Then } \frac{51}{1280} + \frac{1}{15} = \frac{2863}{26880} \text{ A's} \\ \text{And } \frac{649}{3840} + \frac{1}{15} = \frac{6335}{26880} \text{ B's} \\ \text{Alſo } \frac{323}{13440} + \frac{1}{15} = \frac{2438}{26880} \text{ C's} \\ \frac{1417}{4480} + \frac{1}{15} = \frac{10294}{26880} \text{ D's} \\ \frac{1579}{13440} + \frac{1}{15} = \frac{4950}{26880} \text{ E's}\end{array}\right\} \text{Share carried off at the laſt.}$$

So that if the Number of Sugar-Plumbs were 26880.

$$\left.\begin{array}{lr}\text{A got} & 2863 \\ \text{B} & 6335 \\ \text{C} & 2438 \\ \text{D} & 10294 \\ \text{E} & 4950\end{array}\right\} = 26880, \text{ the Sum.}$$

47. ADDITION OF DECIMALS.

(1)		(2)		(3)	
	,0476		,427		,274
	21,476		64,075		,076
	,0067		27,6421		,64762
	,64		10,8		,0706
	17,6		,0074		,47
	,20764		104,046842		,007
					968,42
Sum	39,97794		206,998342		969,96522

To

1. To add Decimals, wherein there are single Repetends.

RULE.

Make every Line end at the same Place, filling up the Vacancies by the repeating Digits, and annexing a Cypher or Cyphers to the finite Terms; then add as before; only increase the Sum of the Right-hand Row, with as many Units as it contains Nines, and the Figure in the Sum under that place will be a Repetend.

EXAMPLES.

(1)	(2)	(3)
47,674	11,4	14,276421
4,0264	6,14274	7,4
32,6	91,8	21,646
6,14	37,671	9,27
27,0646	146,476741	31,1474

In each of the above Examples there are single recurring Figures, which before they are added must be made to end together, and then they will stand as follows.

(1)	(2)	(3)
47,67444	11,444444	14,276421
4,02642	6,142740	7,444444
32,66666	91,788888	21,646666
6,14444	37,671111	9,277777
27,06466	146,476741	31,147444
117,57664	293,522925	83,792754

2. To add Decimals having Compound Repetends.

RULE.

Make the Repetends similar and conterminous; then add as before, only increase the Right-hand Figure by as many Units as are carried from the Column of Figures, wherein all the Repetends begin together: lastly, dash off for a Repetend as many Places as were so in the Numbers added together.

EXAMPLES.

(1)	(2)
14,1472	,246
,7684246	3,67
7,064	27,0427694
26,0062	9,9431

Now

Now the two last Examples being made similar and conterminous, will become as follows,

(1)		(2)
14,1472472		,2462462
,7684246		3,6767676
7,0640640		27,0427694
26,0062062		9,9431431
47,9859421		40,909264

18. SUBTRACTION of DECIMALS.

		(1)	(2)	(3)
	From	176,	647,	74,6407
	Take	10,764	,00746	69,5
	Diff.	165,236	646,99254	5,1407

To subtract Decimals that have a single Repetend.

RULE.

Make both end together as in Addition; and if the Repetend of the Number to be subtracted, be greater than the Repetend of the Number it is to be taken from, then the Right-hand Figure of the Remainder must be less by Unity, than it would be; or instead of borrowing 10, as in whole Numbers or Infinites, borrow in this Place 9, the rest as usual; and the Right-hand place or Figure will be a Repetend.

EXAMPLES.

	(1)	(2)	(3)
From	41,74	24,1466	16,126
Take	21,94648	19,9	4,1942764

These Examples being made to end together as before directed in Addition, will stand as follows,

	(1)	(2)	(3)
From	41,74444	24,14466	16,1262626
Take	21,94648	19,99999	4,1942764
Diff.	19,79795	4,14466	1,9319862

49. MULTIPLICATION of DECIMALS.

	(1)	(2)	(3)
	,17506	8,04704	27,42
	×,76	×,2575	×3,56
Prod.	,1331304	1,1869384	97,6152

(4)	(5)	(6)	(7)
5745	,4	,047	,000476
×,0675	×,2	×,046	×,00078
Prod. ,03877875	,08	,002042	,00000037122

(8) ,47 × by 0008=,000376, the Product.

CONTRACTIONS.

(9) 2,74 × by 10=27,4. (10) ,2746 × 100=27,46.
(11) 1,076 × 1000=1076. (12) ,42768 × 10000=4276,8.

(14) 3,47678
× 287672 Mul. inverted.

69536
24338
2086
243
28
1
96,232

(15) 47,689464
× 49671,62

95378928
28613678
476895
333826
28613
4292
191
1248,36423

(1) If the Right-hand Figure of the Multiplicand be a Circulate,

RULE.

Multiply the Multiplicand as before, by every Figure in the Multiplier; observing to increase the Right-hand of each resulting Line, by as many Units as there are Nines in the Product of the first Figure in that Line, and the Right-hand Figure of each Line will be a Circulate; and before you add them together, make them all end at the same Place as shewn in Addition.

EXAMPLES.

(1) 147,64
×,7
Prod. 103,351

(2) 42,644
×,276
255853
2986955
8528444
117,71253

(2) When the Right-hand Figure of a Multiplier be a Circulate.

RULE.

Multiply by, as by a finite Digit, ſetting the Product one Place extraordinary towards the Left-hand, then divide the Product by 9, continuing the Quotient if needful till it arrives at a Circulate; then beginning at the Place under the Right-hand Figure of the Multiplicand, cut off for Decimal Parts.

EXAMPLES.

(3)

$$\begin{array}{r} 46{,}276\dot{2} \\ 8 \\ \hline 9)370{,}2096 \\ \hline \text{Prod. } 41{,}1344 \\ \hline \end{array}$$

(4)

$$\begin{array}{r} 261{,}27\dot{6} \\ {,}4\dot{7} \\ \hline 9)18{,}28932 \\ \hline 2{,}03214\dot{6} \\ 104{,}5104 \\ \hline \text{Prod. } 106{,}54254\dot{6} \\ \hline \end{array}$$

(3) When the Multiplicand and Multiplier are each a ſingle Circulate.

RULE.

The firſt Line (or that produced by multiplying the Circulate in the Multiplier) muſt be managed as in Note 2. only the Right-hand Figure muſt be increaſed by as many Units as there are Nines in the Product of the firſt Figure of that Line; the Products of the reſt muſt be managed as directed in Note 1.

(6)

$$\begin{array}{r} 141{,}1\dot{4} \\ 8{,}4\dot{7} \\ \hline 9)98801 \\ \hline 109667 \\ 564577 \\ 11291555 \\ \hline 1196{,}58001 \\ \hline \end{array}$$

(5)

$$\begin{array}{r} 24{,}607\dot{2} \\ {,}49\dot{6} \\ \hline 9)1476432 \\ \hline 164048 \\ 22146500 \\ 98428888 \\ \hline 12{,}183943\dot{7} \\ \hline \end{array}$$

50. DIVISION OF DECIMALS.

	Divis. Divid. Quot.		Divis. Divid. Quot.
(1)	6,5)1735,5(267	(2)	,0084),8332(99,19047+
(3)	,0574)49,3066(859	(4)	,47)17,46(37,14893+
(5)	7,476)186,900(25	(6)	,7875)14,4100(18,2984+
(7)	,04)104,00(2600	(8)	,008)6,000(750
(9)	34)1229,42112(36,15944	(10)	4,7)754,4578(160,5293
(11)	604,25)246,1476(,40736+	(12)	119)7,268401(,061079
(13)	647)6,7258(,010395+	(14)	678),0008136(,0000012
(15)	12),0072(,006	(16)	2,46),016728(,0068

CONTRACTIONS.

(17)	1,0)24,6(2,46	(18)	1,00),4076(,00476
(19)	1000)487,67(,48767	(20)	1,000)474,6(,4746

(22) 2137,2)913,08(,426
85488
5820
4274
1436
1282
154

(23) 240,649)6109,2674(25,3866
481298
129628
120325
9303
7220
2083
1925
158
144
14
14

(24) 52,7438)165,9923(3,14
1582314
77609
52744
24865
21097
3768

(1) If the Dividend be a Repetend.

RULE.

If it be a single Repetend, bring down the circulating Figure, until the Quotient either repeats, or is as exact as required; but if the Repetend in the Dividend be a compound one, then bring down the circulating Figures in the same Order they stand in; and when you have got through them all, bring down the first Figure in the Repetend over again, and so proceed until your Quotient either repeats, or be as exact as necessary.

EXAMPLES.

6,84)14,44(2,11176+
764
804
1204
5204
4164
60

46,849)1694,647(36,170412
289177
80836
339874
193176
57804
109557
15859

(2) If the Divisor be a single Repetend.

RULE.

Place the Dividend under itself, but removed one Place towards the Right-hand, which subtract, and the Remainder will be a new Dividend; which divide by the Divisor in the same Manner as if it was a terminate Number.

Divide 42,86 by 8.
428
,8)3859
,48225

Divide 6,426 by 68.
642
,68)5,784(8,505882+

(3) If the Divisor and Dividend consist of terminate Numbers joined to the Repetend.

RULE.

Place the Divisor and Dividend under each other, but removed one Place towards the Right; then subtract the lower Line from the upper one, the Remainder will be a new Divisor and Dividend, which proceed with as before directed.

EXAMPLES.

```
(1)    7414,864)81869,8694(
        741486    8186,9869
       ________  __________
       763,378   73682,8825(9 6024+
                 __________

(2)    64,842)9,46946(
        6,484  94694
       ______  ______
       58,358  8,52252(14603+
               _______
```

(4) If a compound Repetend is found in your Divisor, or in both your Divisor and Dividend.

RULE.

Proceed as in the last Case with your Divisor and Dividend, only remove them each so many Places towards the Right-hand, as they are Places in the Repetend of the Divisor; but if the Divisor is a compound Repetend without any terminate Figures, divide by it as a terminate Number; first subtracting the Dividend from itself, as above directed.

EXAMPLES.

```
(1) Divide 147,47683 by 8,467
           8,467)147,47683268
           8       14742683
           _____ ___________
           8,459)147,27940585(17,41085+
           _____ ___________
```

Divide

(3) Divide 4176,4266268 by 37,2694.

37,2694) 4176,4266268
37   4176426

37,2657) 4176,0089842(112,06393+

(4) Divide 47,69647 by 476.
4749

,476)47,64873(100,1+

51. REDUCTION OF DECIMALS.

CASE I.

(2) 2)1,0(,5 (3) 4)3,0(,75 8)1,0(,125, and 16)100(,625
(4) 24)5,0(,2083 (5) First $\frac{5}{12}$ of $\frac{3}{4}=\frac{5}{16}$; then 16)5,000(,3125
(6) 7)4,0(,57142+ (7) 3)2,0(,6
(8) First $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{7}{8}=\frac{21}{64}$; then 64)21,0(,328125
(9) First $\frac{1}{4}$ of $\frac{5}{12}=\frac{5}{48}$; then 48)5,00000(,10416.

CASE II.

First 6*s.* 9*d.*$=\frac{27}{80}$£. per Case VIII. Sect. 38.

(2) Thus 6*s.* 9*d.*=27 Three-pences, and 1£.=80.
Then 80)27,0(,3375, the Decimal required.

(3) Thus 21)9,00(,42857+, the Decimal required.

(4) First 14*s.* 6½*d.*=349 Half-pence, and a Moidore = 648 ditto.
Then 648)349,0(,53858+, the Decimal required.

(5) First 18*s.* 4½*d.*=441 Half-pence, and 1£.=480 ditto.
Then 480)441,0(,91875, the Decimal required.

(6) First $\frac{1}{4}$ of a Penny $=\frac{1}{320}$£. per Case VI. Sect. 38.
Then 320)1,000(,003125, the Decimal required.

(7) First 1lb.=240 dwts. 240)11,00(,04583, the Dec. req.

(8) First 1lb.=256 drs. then 256)10,00(,03906+, the Dec.

(9) First $\frac{3}{4}$qrs. 14lb.=98lb. then 112)98,0(,8741, the Dec.

(10) A Yd.=36 Inches, then 36)6,0(,16, the Decimal req.

(11) First a League =24 fur. then 24)6,0(,25, the Dec.

(12) First 18 gall. 2 qts.=74 qts. and a hhd.=252 ditto.
Then 252)74,0(,29365+, the Decimal required.

(13) First 3 qts. 1 pt.=7 pts, and a bar.=256 pts. Then 256)7,00(,02734+, the Decimal required.

(14) First an Acre =160 Perches, then 160)8,00(,05, the Decimal required.

(15) First 4 bush. 2 pks.=18 pks and a chald.=144 pks. Then 144)18,0(,125, the Decimal required.

(16) Thus 60)12,0(,2 the Decimal required.

(17) First 12 Days =288 hrs. and 365 d. 6 h.=8766 hrs. Then 8766)288,00(,03285+, the Decimal required.

CASE III.

(2) *£.*

```
    ,3375
      ×20
   ------
s. 6,7500
      ×12
   ------
d. 9,00
```

Answer 6*s.* 9*d.*

(3) *Guinea.*

```
    ,45
     ×7×3=21
   ----
    3,15
       3
   ----
s.  9,45
      12
   ----
d.  5,40
       4
   ----
    1,6
```

Ans. 9*s.* 5*d.* ¼,6.

(4) *Moidore.*

```
    ,72708
       ×9×3=27
   -------
    6,54372
         ×3
   -------
s. 19,63116
         12
   -------
d.  7,57392
          4
   -------
    2,29568
```

Ans. 19*s.* 7*d.* ½,29+

(5) *Lb.*

```
 ,00243
     12
 ------
 ,02916
     20
 ------
 ,58320
   4×6=24
 ------
 2,3328
      6
 ------
13,99680
```

Answer 14 Grs.

(6) *Ton.*

```
     ,3375
        20
    ------
cwt. 6,7500
         4
    ------
qrs. 3,00
```

Ans. 6 Cwt. 3 Qrs.

(7) *Lb.*

```
     ,0306
        16
     -----
      6336
        16
     -----
drs. 10,1376
```

Ans. 10 drs. 1376.

(8)

Cwt.	*Yard.*	*League.*
(8) ,875	(9) ,16669	(10) ,259
4	3	3
Qrs. 3,500	,50007	,777
28	×12	8
lb. 14,0	6,00084	Fur. 6,216
Answer 3 Qrs. 14 lb.	Answ. 6 Inches +	40
		8,640
		Answer 6 Fur. 8,64 Poles.

Hhd.	*Bar.*	*Acre.*
(11) ,29365	(12) ,875	(13) ,05
×63	×32	4
Galls. 18,5 nearly.	28,000	,20
4	Answ. 28 Galls.	40
Qts. 2,0		8,00
Answer 18 Galls. 2 Qts.		Answ. 6 Fur. 8,64 Poles.

Chaldron.	*Day.*
(14) ,125	(15) ,4765
×36	×24
Bush. 4,500	Hrs. 11,4360
4	60
Pks. 2,0	Min. 26,1600
Answer 4 Bush. 2 Pks.	60
	Sec. 9,60
	60
	36,00

Answer 11 Hrs. 26 Min. 9 Sec. 36 M.

53. EXTRACTION OF THE SQUARE ROOT.

(2)

```
        60516(246 the Root required.
        4
     44)205
        176=44×4
    486)2916
        2916=486×6
```

Now 246×2,46=60516, the Proof.

(3)

```
        765( or 87,5099, or rather 87,31+ the Root.
    167)1258
   1745)8900
 175009)1750000
1750189)17491900
   Rem. 1740199
```

(4)

```
         39342864(6272,389+ the Root.
     122)334
    1247)9028
   12542)29964
  125443)488000
 1254468)11167100
12544769)113135600
    Rem. 232679
```

(5) 8209667940,5290(90607,21792 + the Root.

1806)10966

181207)1307940

1812142)3949152

18121441)32486890

181214427)1436544900

1812144349)16804391100

18121443582)49509195900

Rem. 13266308736

(6) ,000729(,027 the Root.
4

47)329

(7) 2)1,41421 + the Rt.
1

24)100

281)400

2824)11900

28282)60400

282841)383600

Rem. 100759

(8) 2,27109570(1,50701 +
1

25)127

3007)21095

301401(467000

Rem. 165599

(9) 36,00000625(6,0000005 + the Root.

120000005)625000000

Rem. 249999975c0

VUL-

54. VULGAR FRACTIONS.

(12) Thus $\sqrt{\frac{25}{81}}=\frac{5}{9}$, the Root required.

(13) Thus $\sqrt{\frac{225}{324}}=\frac{15}{18}$, or $\frac{5}{6}$, the Root.

(14) First $10\frac{39}{49}=\frac{529}{49}$. Then $\sqrt{\frac{529}{49}}=\frac{23}{7}$, or $3\frac{2}{7}$, the Root.

(15) First $\frac{4608}{6272}=\frac{36}{49}$. Then $\sqrt{\frac{36}{49}}=\frac{6}{7}$, the Root.

(16) First $\frac{726}{864}=\frac{121}{144}$. Then $\sqrt{\frac{121}{144}}=\frac{11}{12}$, the Root.

(17) First $27\frac{9}{16}=\frac{441}{16}$. Then $\sqrt{\frac{441}{16}}=\frac{21}{4}$, or $5\frac{1}{4}$ the Root.

(18) First $8\frac{3}{7}=8,428571428$. Then $\sqrt{8,428571428}=$ $2,9032+$, the Root.

(19) First $\frac{497}{676}=,7352071005$. Then $\sqrt{,7352071005}=$ $,85744+$, the Root.

(20) First $76\frac{14}{17}=76,8235941176$. Then $\sqrt{76,8235941176}$ $=8,7649+$, the Root.

55. THE USE OF THE SQUARE ROOT.

(2) First $124\times67=8308$. Then $\sqrt{8308}=91,148225+$ the mean Proportion nearly.

For $67 : 91.148225 :: 91.148225 : 124$ nearly.

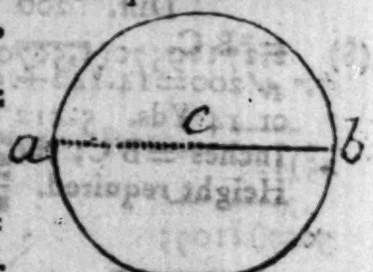
(4) First 9 A. 2R. 15 P. = 1535 Perches. And $5.5\times5.5=$ 30.25 Yards in 1 Perch.

$\because$ $1535\times30.25=46433.75$ Yds. the superficial Content.

Then $\sqrt{46433.75}=215.484918$ Yds.$+$ $=215$ Yds. 1 Ft. 5,467 Inc. the Side required.

(5) Thus $\sqrt[2]{160}=12.649+$, the Side required.

(7) First $4840\times2=9680$ Yards, Content of the two Acres.



Then as $355 : 452 :: 9680 :$ 12324 Yds. $=^2ab$.

$\because$ $\sqrt[2]{12324}=111=ab$.

Therefore $111\div2=55\frac{1}{2}$ Yds. $=$ *cb*, the Length of the Cord req.

(8) First $140,5\times140,5=19740.25$ $=^2AC$.

And $55,5\times55,5 = 3080.25$ $=^2AB$.

$16660=$ ^2BC.

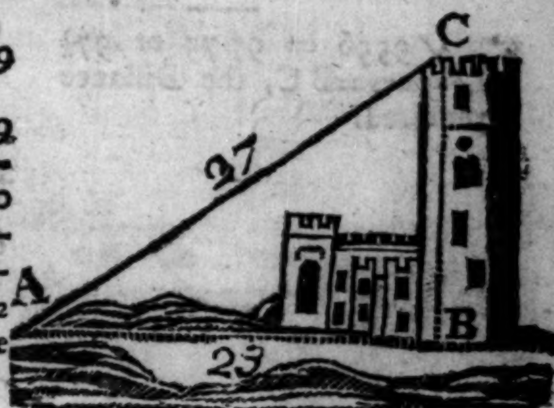
$\because$ $\sqrt{16660}=129,07$ Yds. $=BC$, the Height required.

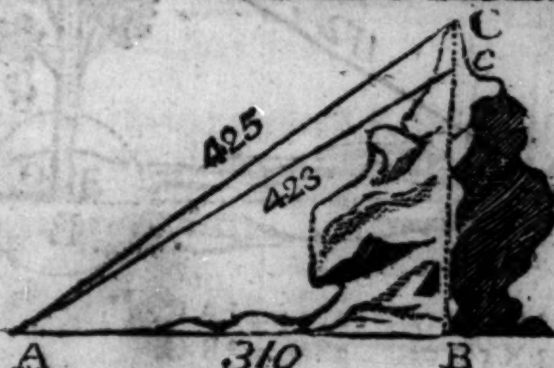
(9) First $40 \times 40 = 1600 = {}^2CE$, or ${}^2DE$.
And $33 \times 33 = 1089 = {}^2AC$.

$\because \quad \sqrt{511} = 22.6 = AE$.

Again $21 \times 21 = 441 = {}^2BD$.
Then $1600 - 441 = 1159 = {}^2EB$.
$\because \sqrt{1159} = 34{,}04 = EB$.
Therefore $22.6 + 34.04 = 56.64 +$ Ft. $= AE + EB = AB$, the Answer.

(10) First $27 \times 27 = 729 = {}^2AC$.
And $23 \times 23 = 529 = {}^2AB$.
Diff. $200 = {}^2BC$.
$\because \sqrt{200} = 14.142 +$ or 14 Yds. 5.112 Inches $= BC$, the Height required.





(11) First $425 \times 425 = 180625 = {}^2AC$.
And $310 \times 310 = 96100 = {}^2AB$.
Diff. $84525 = {}^2BC$.

∴ $\sqrt{84525}=290.73183=BC$, Height of the Light-house and Rock.

Again $423\times423=178929={}^{2}Ac$.
$-\ 96100={}^{2}AB$.

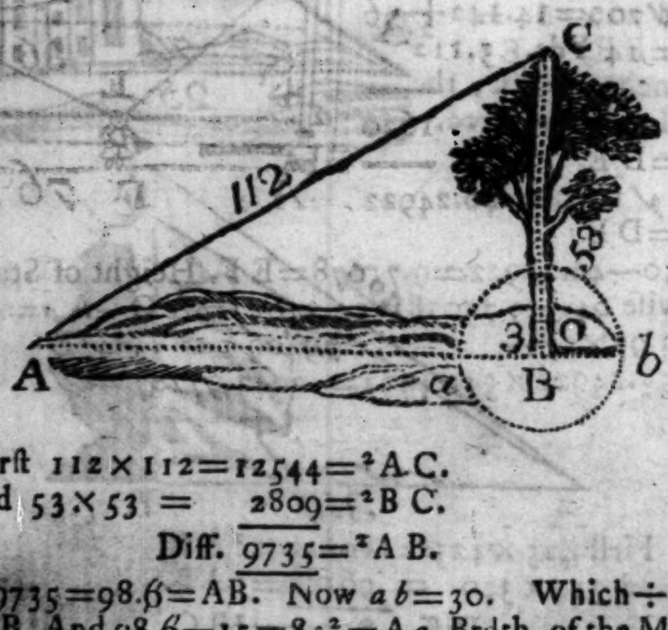
Diff. $82829={}^{2}Bc$.

∴ $\sqrt{82829}=287,80027=Bc$, Height of the Rock.
Therefore $290,73183-287,80027=2.9356$ Fathoms.
Or 17,6+ Ft.$=Cc$, the Height of the Light-house.

(12) First $50\times50=2500={}^{2}AB$.
And $84\times84=7056={}^{2}AC$.

Sum $9556={}^{2}BC$.

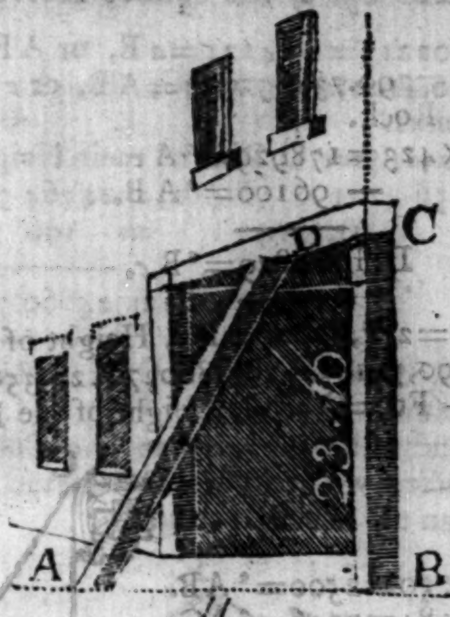
∴ $\sqrt{9556}=97.75$ or $.97\frac{3}{4}$ Leagues$=BC$, the Distance required.



(13) First $112\times112=12544={}^{2}AC$.
And $53\times53=\ \ 2809={}^{2}BC$.

Diff. $9735={}^{2}AB$.

∴ $\sqrt{9735}=98.6=AB$. Now $ab=30$. Which$\div2=15$ $=aB$. And $98.6-15=83\frac{2}{3}=Aa$. Brdth. of the Mt. req.



(14) First $11 \times 11 = 121 = {}^2AB$.
And $23.83 \times 23.83 = 568.027 = {}^2BC$.

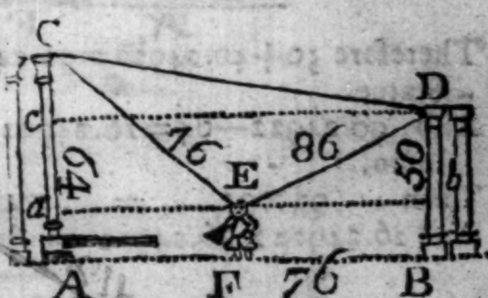
Sum $689.027 = AC$.

∴ $\sqrt{689.027} = 26.249$, or 26 Ft. 29 Inches $= AC$, the Length required.

(15) First $86 \times 86 = 7396 = {}^2ED$.
And $76 \times 76 = 5776 = FB$, or Eb.

1620 $= DE$.

∴ $\sqrt{1620} = 40.24922 = DE$.



Also $50 - 40.24922 = 9.75078 = EF$. Height of Statue.
Likewise $64. - 9.75078 = 54,24922 = AC - Aa = aC$.
Again $97 \times 97 - \quad = 9409 \quad = {}^2EC$.
And $54.24922 \times 54.24922 = 2942.97787 = aC$.

Diff. $6466.02213 = aE$, or AF.

$\because \sqrt{6466.022213} = 80.4121 = a$E, or AF.

Now $76 + 80.4121 = 156.4121 =$ AB, or cD, Distance of the Columns.

And $64 - 50 = 14 = c$C Diff. of their Heights.

Therefore $156.4121 \times 156.4121 = 24464.745 = {}^2$CD.

$$14 \times 14 \quad = \quad 196 \quad = {}^2\text{C}c.$$

$$\text{Sum } \underline{24660\ 745} = {}^2\text{CD}.$$

$\because \sqrt{24668.745} = 157.06 + =$ CD, Distance of the Top of the Columns.

But if the Statue be higher than the Columns, then by working as before we shall find the Statue to be 40.24922 Feet higher than the lower Column.

Therefore $50 + 40.24922 = 90.24922 =$ EF, Height of the Statue.

Also $90.24922 - 64 = 26.24922$ higher than highest Column,

Then $\quad 97 \times 97 \quad = \quad 9409 \quad = {}^2$CE.

And $26.24922 \times 26.24922 = 689.02135$

$$\text{Diff. } \underline{8719.97845}$$

$\because \sqrt{8719.97845} = 93.3808$ AF.

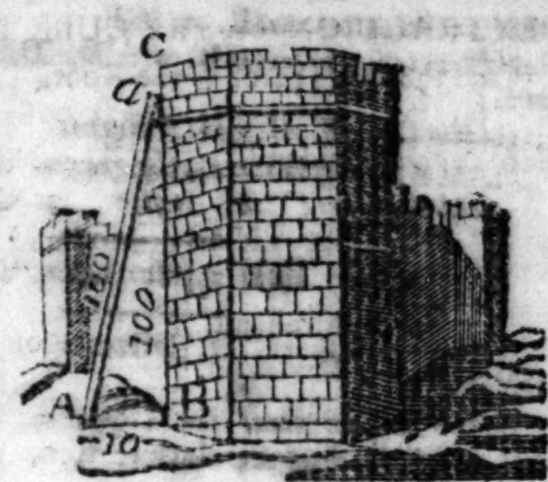
So that $76 + 93.3808 = 169.3808 =$ AF + FB, Distance of the Columns.

Again $169.3808 \times 169.3808 = 28689.85540864 = {}^2c$D.

And $\quad 14 \times 14 \quad = \quad 196 \quad = {}^2c$C.

$$\text{Sum } \underline{28885.85540864} = {}^2\text{CD}.$$

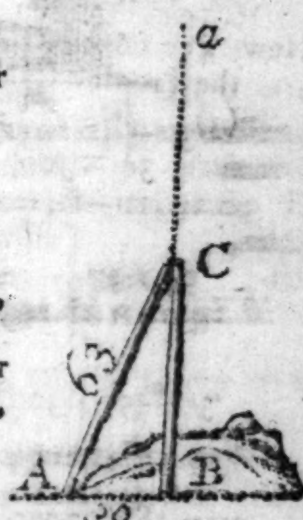
$\because \sqrt{28885.85540864} = 169.95839$ Ft. = CD. Distance required. (16)



(16) First $100 \times 100 = 10000 = {}^2Aa$.
And $10 \times 10 = 100 = {}^2AB$.

Diff. $9900 = {}^2Ba$.

Then $\sqrt{9900} = 99.49874 = Ba$.
$\because 100 - 99.49874 = ,50125 = aC$, which is 6 Inches nearly. Q. E. F.



(17) First $63 \times 63 = 3969 = {}^2AC$, or C D.
And $30 \times 30 = 900 = {}^2AB$.

Diff. $3069 = {}^2BC$.

Then $\sqrt{3069} = 55.3985$ Yds. $= BC$, Piece standing.
$\therefore 63 + 55.3985 = 118.3985$ Yds. or 118 Yds. 1 Foot $2\frac{1}{4}$ Inches $= Ba$, Height of the Pole.

(18 Thus $\sqrt{23716} = 154$ Men, the Answer.

56. THE EXTRACTION OF THE CUBE ROOT.

(2) 92398647(452.08+ the Root.
64
3rr=4800)28398 Dividend.
24000=3rre
2000=3ree — Here r=40, and e=5.
125=eee
27125 Subtrahend.
3rr=607500)1273647 Dividend.
1215000=3rre
5400=3ree — Here r=450, and e=2.
8=eee
1220408
3rr=6129120000)53239000000 Dividend.
49032960000=3rre
8678400=3ree — Here r=45200, and e=8.
512=eee
49041638912 Subtrahend.
4197361088 Rem.

Now 452.08×452.08×452.08+4197361088=92398647, the Proof.

(3) 2716243264(1395.24+ the Root.
3rr=300)1716 Dividend. — Here r=10, and e=3.
1197 Subtrahend.
3rr=50700)519243 Dividend. — Here r=130, and e=9.
488610 Subtrahend.
3rr=5796300)30624264 Dividend. — Here r=1390, and e=5.
29085875 Subtrahend.
3rr=583807500)1538389000 Dividend. — Here r=13950, and e=2.
1167782408 Subtrahend.
3rr=58397491200)370606592000 Dividend. — Here r=13950, and e=2.
233596661824 Subtrahend.
37009936976 Remains.

(4)

(4) 91)4,497+ the Root.

4800)27000 Dividend.

21184 Subtrahend. Here $r=40$, and $e=4$.

580800)5816000 Dividend.

5334849 Subtrahend. Here $r=440$, $e=9$.

60480300)481151000 Dividend.

424622473 Subtrahend. Here $r = 4490$, and $e=7$.

57128527 Remains.

(5) 67527834239(4072.18+ the Root.

$3rr=480000$)3527834 Dividend.

3419143 Subtrahend. Here $r=40$, and $e=7$.

$3rr=49694700$)108691239 Dividend.

99438248 Subtrahend. Here $r = 4070$, and $e=2$.

$3rr=4974355200$)9252991000 Divid.

4974477361 Subtr. Here $r=40270$ and $e=1$.

$3rr=497459952300$)4278513639000 Divid.

3979757803232 Subtr. Here $r = 7210$, and $e=8$.

298755835768 Remains.

 (6)

(6) 4764,750)16,827+ the Root.

$3rr=300$)3764 Dividend.
3096 Subtrahend.
} Here $r=10$, and $e=6$.

$3rr=76800$)668750 Dividend.
645632 Subtrahend.
} Here $r=160$, and $e=8$.

$3rr=8467200$)23118000 Dividend.
16954568 Subtrahend.
} Here $r=1680$, and $e=2$.

$3rr=848737200$)6163432000 Dividend.
5943633293 Subtrahend.
} Here $r=16820$, and $e=7$.

219798717

(7) 36155.027576(33 0665

$3rr=2700$)9155
8937

$3rr=32670000$)218027576 Dividend.
196376616 Subtrahend.
} Here $r=3300$, and $e=6$.

$3rr=3278890800$)21650960000 Dividend.
19676915496 Subtrahend.
} Here $r=33060$, and $e=6$.

$3rr=328008806800$)1974044504000 Divid.
1640065333625 Subtr.
} Here $r=330660$, & $e=5$.

333979170375 Remains.

(8)

(8) 67667.921875(40.75 the Root.

$3rr=480000$)3667.921 Dividend. } Here $r=408$, and $e=7$.
3419143 Subtrahend.

$3rr=49694700$)248778875 Dividend. } Here $r=4070$, and $e=5$.
248778875 Subtrahend.

(9) 219365329(603.1+ the Root.

$3rr=1080000$)3365329 Dividend. } Here $r=600$, and $e=3$.
3256227 Subtrahend.

$3rr=109082700$)109102000 Divid. } Here $r=6030$, and $e=1$.
109100791 Subtr.

1209 Remains.

(10) 3105926.917(145.9+ the Root.

$3rr=300$)2105 Dividend. } Here $r=10$, and $e=4$.
1744 Subtrahend.

$3rr=58800$)361926 Dividend. } Here $r=140$, and $e=5$.
304625 Subtrahend.

$3rr=6307500$)57301917 Dividend. } Here $r=1450$, and $e=9$.
57120579 Subtrahend.

181338 Remains.

(11) ,000421875(,075 the Root.

3rr=14700)78875 Dividend.
78875 Subtrahend.
} Here r=70, and e=5.

(12) 28022810.390625(303.7472+ the Root.

3rr=270000)1022810 Dividend.
818127 Subtrahend.
} Here r = 300, and e=3.

3rr=27542700)204683390 Dividend.
193244653 Subtrahend.
} Here r = 3030, and e=7.

3rr=2767010700)11438737625 Divid.
11082607350 Subtr.
} Here r = 30370, and e=4.

3rr=276773962800)356130275000 Divid.
276774874021 Subtr.
} Here r = 303740, & e=7.

3rr=27677578524300)79355400979050 Div.
55355193497528[4] Su.
} Here r= 3037470, and e=2.

24000207481472 Remains.

57. To Extract the Cube Root of Vulgar Fractions.

(13) First $\frac{324}{1500}=\frac{27}{125}$, then $\sqrt[3]{\frac{27}{125}}=\frac{3}{5}$, the Root req.

(14) First $\frac{352}{1188}=\frac{8}{27}$, then $\sqrt{\frac{8}{27}}$, $=\frac{2}{3}$, the Root.

(15) First $5\times125+104=729$, then $\sqrt[3]{\frac{729}{125}}=\frac{9}{5}$, or $1\frac{4}{5}$, Root.

(16) First $405\times125+104=50653$, then $\sqrt{\frac{50653}{125}}=\frac{37}{5}$ or $7\frac{3}{5}$, the Root.

(17)

(17) First $5\frac{3}{5}=5{,}6$, then $\sqrt[3]{5{,}600}=1{,}775+$ the Root.

(18) First $7\frac{5}{7}=7{,}714285\frac{5}{7}$, then $\sqrt[3]{7{,}7142857}=1{,}775$, the Root required.

58. THE USE OF THE CUBE ROOT.

(1) First $\sqrt[3]{21952}=28$, Side of the Cube.
Then $28\times28=784$, the Content required.

(2) $\left.\begin{array}{l}\text{First } 125\times125\times125\\ \text{And } \ \ 25\times\ 25\times\ 25\\ \text{Also } \ \ 15\times\ 15\times\ 15\end{array}\right\}\times3=\left\{\begin{array}{r}5859375\\46875\\10125\end{array}\right.$

$\therefore \sqrt[3]{5859375}=180{,}05$ Keel.
And $\sqrt[3]{46875}=36{,}05$ Midship-beam.
Also $\sqrt[3]{10125}=21{,}6$ Depth in the Hold.

(3) $\left.\begin{array}{l}\text{First } 125\times125\times125\\ \text{And } \ \ 25\times\ 25\times\ 25\\ \text{Also } \ \ 15\times\ 15\times\ 15\end{array}\right\}\div2=\left\{\begin{array}{r}976562{,}5\\7812{,}5\\1687{,}5\end{array}\right.$

Which Numbers extracted will give 99.202; 19.84 and 11.906 the Dimensions required.

(4) First $75\times75\times75=421875$, and $100\times100\times100=1000000$.

Then, as 421875 C. K. : 300 Tons, :: 1000000 C. K. : 711.111+ Tons, or 711 Tons, 2 cwt. 24,64 lb. the Burthen required.

(5) First $4\times4\times4=64$, Cube of the Diameter.
Then, as 18 lb : 64 inc. :: 114 lb. : 405,3 inc. Cube of the Diameter.
$\because \sqrt[3]{405{,}3}=7{,}4+$, the Diameter required.

(6) First $11{,}5\times11{,}5\times11{,}5=1520{,}875$, and $20{,}83\times20{,}83\times20{,}83=9037{,}905787$.
Then, as 1520,875 : 1000 lb. :: 9037,905787 : 5942,5697 lb. +, the Weight required.

(7) First $189\div7=27$, whose Cube Root is 3.
Then $3\times7=21$, the lesser Mean, and $21\times3=63$, the greater Mean.
For, as 7 : 21 :: 63 : 189, the Proof.

(8)

(8) First $256 \div 4 = 64$, then $\sqrt[3]{64} = 4$, which $\times$ by $4 = 16$, the lesser Mean, and $16 \times 4 = 64$ the greater.

For, as $4 : 16 :: 64 : 256$, the Proof.

59. To EXTRACT THE FIRST SURSOLID ROOT.

RULE.

1. Having pointed the given Resolvend into Periods of five Figures, seek such a Sursolid Number in the Table, or otherwise, as comes nearest to the first Period of the Resolvend, whether greater or less; and call the respective Root, either more than just, or less than just, as it falls out; annexing so many Cyphers to it, as there are remaining Periods of whole Numbers in the Resolvend.

2. Find the Difference between the Resolvend and the Sursolid Number, so taken, by subtracting the lesser from the greater.

3. Find the Cube of the foresaid Sursolid Root, with its annexed Cyphers, which also may be done by the Table, and multiply that Cube into five, the Index of the Sursolid, and divide the Difference between the Resolvend and the Sursolid Number by that Product; by which it will be depressed to a Square, and when pointed into Periods of two Figures each, call it the new Resolvend.

4. Make the first Root without Cyphers a Divisor, enquiring how often it may be found in the first Period of the new Resolvend; with this Consideration, if the Root, now a Divisor, be less than just, annex twice the Quotient Figure to it; but if more than just, subtract twice the Quotient Figure from a Cypher, either annexed or supposed to be annexed, to that Divisor or Root, multiplying it so increased or diminished, with the said Quotient Figure; setting down the Units Place of the Product under the pointed Figure of that Period, subtracting it as in Division.

TABLE.

1	2	3	4	5	6	7	8	9
1	32	243	1024	3125	7776	16807	32768	59049

EXAM-

EXAMPLES.

(1) Extract the Surſolid Root of 12309502009375, the Reſolvend pointed.

The neareſt Surſolid Number to 1230, the firſt Period of the Reſolvend is 1024, whoſe Root is 4 (by the Table) leſs than juſt.

Therefore 12309502009375
—1024

2069502009375 their Difference.

Now, as there are two Periods remaining in the Reſolvend, place two o's to 4, the Root of the firſt Period.

Next the Cube of 400=64000000.

And 64000000 × 5=320000000 the Diviſor.

Then 320000000)2069502009375(6497, &c.

First Root =400
+2×10 = 20

1ſt. Diviſor =420)6467(15 laſt Root.
+20×2×5= 30 42 400 Firſt.

450 2267 415 Root (true) required.
2250

Rem. 17 to be rejected.

Now 415×415×415×415×415=12309502009375, the Proof.

(2) Extract the Surſolid Root of 232783459873.

Now the neareſt Surſolid Number to 232 is 243 (per Table) whoſe Root is 3, being more than juſt.

Therefore 243000000000
—232783459873

Rem. 10216540127 for a Dividend.

300 Cubed =27000000, which × 5=135000000 Diviſor.

Then 135000000)10216540127(756,7810 new Reſolvend.

First

First Root = 300
—2 × 2 = —4
300 first Root.
1 Divifor = 296)756,7810(2,566 laft Ditto, too much.
—4—2 × ,5 = —5,0 592
2-97,434 the Root only too little by 2 in the lowest Figure.
2 Divifor = 291,0)164,78(
—1—2 × 06 = —1,12 145,50
3 Divifor = 289,88)19,2810(
&c. &c.

60. To EXTRACT THE ROOT OF THE SECOND SURSOLID, OR SEVENTH POWER.

RULE.

Having pointed the Refolvend into Periods of feven Figures, feek out fuch a Number by the Table, as comes neareft to the firft Period of the Refolvend, whether greater or leffer, calling its Root more than juft, or lefs than juft, annexing a proper Number of Cyphers.

2. Find the Difference between the Refolvend and that Number of the feventh Power, by fubtracting the leffer from the greater.

3. Find the Surfolid or fifth Power of that Root, with its annexed Cyphers, by the Table; and multiply that Surfolid Number into feven, the Index of the Refolvend.

4. Make that Product a Divifor, by which the forefaid Difference muft be divided; fo that it may be depreffed to a Square, and pointed as fuch.

5. Make the firft Root without a Cypher, a Divifor working with it and the new Refolvend, as in Surfolid, only here you muft increafe or diminifh the Divifor with three quotient Figures.

TABLE.

1	2	3	4	5	6	7	8	9
1	128	2187	16384	78125	279936	823543	2097152	4782969

EXAM-

EXAMPLES.

What is the second Surſolid Root of

382986553955078125 Reſolvend pointed.
—2187 the neareſt of the ſeventh Power.

164286553955078125 their Difference.

The firſt Root is 300, being leſs than juſt, and the fifth Power of 300=2430000000000, which being multiplied by 7=17010000000000 for a Diviſor, by which the aforeſaid Difference muſt be divided, which contracted muſt ſtand thus 1701) 16428655(9658,23 &c.

First Root = 300
+3×20 = +60

1 Diviſor = 360)9658(300 25
60+3×05 = +75 72 325 the true Root.

2 Diviſor = 435)2458
2175

Rem. 283 to be rejected.

70. The RULE of THREE in DECIMALS.

(2) Firſt 2½ lb.=2,5. And 1£. 5s.=1,25. Alſo 14¾=14,75.

Then, as 2,5 lb. : 1,25£. :: 14,75 lb. : 8,85£.=8£. 17s. the Anſwer.

(3) Firſt 1 lb.=,008928 58 C. 11¾d.=,0489583£.

And 4hhds. each, 4C. 2 qrs. 14 lb.=4,625×4=18,5C.

Then, as, 00892858 : ,0489583£. :: 18,5C. : 101,4417£. =101£. 8s. 10d. the Anſwer.

(4) Firſt 4 Cheſts each, : 2C. 3qrs. 14 lb.=2,875 C.×4 =11,5C. And 906£. 10s.=906,5£.

Then, as 11,5 : 906,5 :: 00892857 : 7038=14s. ¾d. .6 the Anſwer.

(5) Firſt 4 Tuns, 201½ galls.=63×4+201,5=453,5 gal. And 240£. 16s. 6d=240,825£. Alſo 24½=24,5.

Then 453,5−24,5=429 galls. the Remainder.

As 429 galls. : 240,825£ :: 1 gall. : ,561 34£.=11s. 2½d. 8+the Anſwer.

(6)

(6) First $4\frac{74}{123}=4,16626$, which $\div$ 12 $=$,3471883 ft. this added to 10 ft. $=$ 10,3471883 ft. $=$ to 6,5 Cubits.

Then, as 6,5 cub. : 10,3471883 :: 1 cub. : 1,591875 + = 1 ft. 7.1025 Inches, the Answer.

(7) First 78,4 $\div$ 8 = 9,8, the second Number.

Then, as 5 : 9,8 :: 8 : 15,68 the fourth Number.

$\because$ 15,68 — 9,8 = 5,88, the Answer.

(8) First $13\frac{1}{3}=13,3$. And 13,3 × 282 = 3760 cubic Inches, the Content of the Cask.

Then 3760 × ,52835 = 1986,596 oz. Weight of the Oil.

Which $\div$ 16 = 124,16225 lb. Also $7\frac{1}{2}$ = 7,5 lb. Weight of 1 Gall.

Therefore, as 7,5 lb. : 1 gall. :: 124,16225 lb. : 16,555 = 16 gall. 4.4 pts. the Answer.

(9) First 4,63 — 1,5688 = 3,0612 oz. Difference.

And a solid Foot and a half = 1728 + 864 = 2592 solid Inc.

Then as 1 inc. : 3,0612 oz :: 2592 inc. : 7934,6304 oz. = 4,4278 cwt. or 4 cwt. 1 qr. 19 lb. 14 oz. 9,88 drs. Answ.

(10) Now, as 5 : 8 :: 75 : 120, the greater Number.

Then 75 + 120 = 195, their Sum.

And 120 — 75 = 45, their Difference.

Also 195 × 45 = 8775, Prod. of their Sum and Diff.

120 × 120 = 14400, Square of the gr. and 75 × 75 = 5625 of the lesser.

14400 — 5625 = 8775, Diff. of those Squares.

14400 + 5625 = 20025, Sum of ditto.

120 $\div$ 75 = 1,6, Quot. of the gr. divided by the lesser.

75 $\div$ 120 = ,625, Quot. of the lesser div. by the greater.

1,6 × 1,6 = 2,56 Square of the greater Quote.

,625 × ,625 = ,390625, Sq. of the lesser Quote.

Answ. 2,950625, Sum of those Squares.

(11) Now, as 8 : 7 :: 224 : 196, the lesser Number.

Then 224 + 196 = 420, their Sum.

And 420 × 420 = 176400, Square of their Sum.

Also 240 — 196 = 28, Diff.

28 × 28 = 784, Square of that Diff.

224 $\div$ 196 = 1,142856, Quot. of greater $\div$ lesser.

196 $\div$ 224 = ,875, Quot. of the lesser $\div$ greater.

$\left.\begin{array}{l} 1,142856 \times 1,142856 = 1,30612 + \\ ,875 \times ,875 = ,765625 \end{array}\right\}$ Square of those Quotes.

420 × 28 + 28 = 11788, Prod. of their Sum and Difference, and the Diff. added.

Lastly

Laſtly, $11788 \times 11788 = 138956944$, Square of the Sum of their Diff. added to the Prod. of their Sum and Diff.

			Miles.	
(12) He that rows towards	}	London goes	{ 7,5	} per Ho.
And he that rows from	}		2,5 }	
		Sum	10	

∵ $34 \div 10 = 3,4$ Hours, before they will meet.

Then, as 1 h. : 2,5 m. :: 3,4 h. : 8,5, or $8\frac{1}{2}$ Miles from London.

And $34 - 8\frac{1}{2} = 25\frac{1}{2}$ Miles from Chertſey, the Anſwer.

(13) Firſt $13,5 - 7,5 = 6$ Inches Diff.

Then, as 4 : 9 :: 6 : 13,5 lb. the Anſwer.

(14) Firſt 18200£. at 3 per cent. = 546£. (per Sect. XIX.) ſpent on the Funeral and Monument.

Then $18200 - 546 = 17654$£. left.

Again 17654£. at 9£. per cent. = 1583,86 gave to his Couſins.

Then $17654 - 1588,86 = 16065,14$£. what he had then left.

Again $\frac{2}{7}$ of $16065,14 = 4590,04$£. paid for the Seat.

Then $16065,14 - 4590,04 = 11475,1$ left.

Again $11475,1 \div 8 = 1434,3875$ paid for Horſes.

Then $11475,1 - 1434,3875 = 10040,7125$ left.

Alſo $10040,7125 - 550$£. $= 9490,7125$ left, after he had ſpent 550£. on his Miſtreſs.

Now, as 12 : 2000 :: 19 : 3166,6£. ſpent in riotous living.

∵ $9490,7125 - 3166,6 = 6324,04583$£. or 6324£. 11*d*. the Anſwer.

(15) Firſt $6 \times 6 = 36$. And $3 \times 3 = 9$ Square of their Diſtances.

Then, Recip. as 36 : 1 :: 9 : 4, ſo that A's place is four times as hot as B's.

(16) Firſt $81000000 \times 81000000 = 6561000000000000$.

Then, Recip. as

1 : 6561000000000000 :: 2 : 3280500000000000.

∵ $\sqrt[2]{3280500000000000} = 57275649$ Miles, the Anſwer.

(17) Firſt $81 \times 81 = 6561$. And $424 \times 424 = 179776$, Squares of their Diſtances; the Cyphers being omitted.

Y Then

Then, Recip. as 179776 : 1 :: 6561 : 27,4; so that the Sun's influence on the Earth to that on the Planet Jupiter, is as 27,4, to 1.

(18) First $32 \times 32 = 1024$. And $777 \times 777 = 603729$, the Squares of the Distances, Cyphers omitted.

Then, Recip. as 603729 Sat. : 1 :: 1024 : 589,584 Merc. nearly.

∴ The solar Influence on Mercury to that of Saturn, is as 589,584 to 1, nearly.

(19) First $115 \times 115 = 13225$ Square.

As 13225 : 1 :: 1 : 13225 Degrees hotter.

(20) The square Roots of the Distances being as the Times, viz. as the $\sqrt{1} : \sqrt{2}$ :: is the Time of falling through the whole required Height.

Now the $\sqrt[2]{1} = 1$, and $\sqrt[2]{2} = 1,4142$, from which take 1, Remains ,4142.

∴ as, 4142 : 1,4142 :: 1 : 3,414 Secs. the Descent; the Square of which is 11,6574.

Then A^2^1 : 16,083 :: 11,6574 : 187,48 ft. the Tower's Height.

(21) First $7970 \times 7970 \times 7970 = 506261573000$.

And $2170 \times 2170 \times 2170 = 10218313000$.

Then, as 10218313000 : 506261573000 :: 1 : 49,5445, Times bigger than the Moon.

∴ as 32,5 : 100 :: 49,5445 : 40,117, Q. E. F.

That is, the Earth contains 40,117 times more Matter than the Moon.

(22) First $81000000 + 240000 = 81240000$, Sun from a full Moon.

And $81000000 - 240000 = 80760000$, Sun from a new Moon.

Now $\left\{\begin{array}{l} 8124 \times 8124 = 65999376 \\ 8076 \times 8076 = 65221776 \end{array}\right\}$ Squares of their Distances, Cphs. omitted.

Recip. 65221776 : 1 :: 65999376 : ,9882.

So that the Proportion of Light and Heat a new Moon has to that of a full one, is

As 1 to ,9882, or as 458329 to 452929 in whole Numbers.

(23) First $11 \times 11 = 121$, and the Square of 1 is 1.

Then, as 1 : 16,083 :: 121 : 1946,083, Q. E. F.

Or

$$\text{Or } 16{,}083 \times \left\{\begin{array}{l} 1 = 16{,}083 \\ 3 = 48{,}25 \\ 5 = 80{,}416 \\ 7 = 112{,}583 \\ 9 = 144{,}75 \\ 11 = 176{,}916 \\ 13 = 219{,}083 \\ 15 = 241{,}25 \\ 17 = 273{,}416 \\ 19 = 305{,}583 \\ 21 = 337{,}75 \end{array}\right\} \text{in the} \left\{\begin{array}{l} \text{1st.} \\ \text{2d.} \\ \text{3d.} \\ \text{4th.} \\ \text{5th.} \\ \text{6th.} \\ \text{7th.} \\ \text{8th.} \\ \text{9th.} \\ \text{10th.} \\ \text{11th.} \end{array}\right\} \text{Secs. of Time.}$$

Answ. 1946,083 as before.

(24) First 10×10 = 100, and 6×6=36, Square of their Descents.

$$\text{Then, as } 1 : 16{,}083 :: \left\{\begin{array}{l} 100 : 1608{,}3 \\ 36 : 579. \end{array}\right\} \text{their Depths.}$$

Answer 1029,3 Difference.

(25) First 19,5×19,5=380,25 sq. of the Descent.
Then As 1 : 16,083 :: 380,25 : 6115,6875 Feet.
Which ÷6=1019 faths. 1 ft. 8¼ inc. Q. E. F.

(26) Thus, as ,16,083 : 1^2 :: 400 : 24,8702.
∴ $\sqrt[2]{24{,}8702} = 4{,}987+$, or 5 secs. nearly, the Answer.

(27) The Semi-diameter of the Earth = 3980 Miles, or 21014400 ft.
Then, as 16,083 : 1^2 :: 21014400 : 1306594,82.
∴ $\sqrt{1306594{,}82} = 1135{,}554$ sec. = 18 min. 55″ 33‴. Q. E. F.

62. THE DOUBLE RULE OF THREE IN DECIMALS.

(1) First, as 1,1£. : 12 Per. :: 1,6£. : 17,45 Persons.
Then, Recip. 25,2£. : 17,45 :: 18,9£. : 23 Persons, nearly, the Answer.

(2) As 19 lb. : 4,25 lb. :: 13 lb. : 2,908 lb.
Also 2,908 : 6 :: 10 : 20,6334 lb. worth 63 lb. at 5*d.* per lb. =63×5=315*d.* or 1,3125£.
Then, as 20,6334 : 1,3125£. :: 112 lb. : 7,12437£. = 7£. 2*s.* 5*d.*¾ ,9 (nearly) the Answer.

$$\begin{array}{llll|l} & £. & P. & yr. & I. \\ (3) & 100 & . & 1 & 3,5 \\ & & — & 1,25 & 38,5 \end{array} \quad \text{Then } \frac{10 \times 1 \times 38,5}{3,5 \times 1,25} = \frac{3850}{4,375}£. = 880£. \text{ the Anſwer.}$$

(4) Firſt from Decemb. 11th. to May 10th. = 150 Days.
And from Sept. 3d. to Chriſtmas day = 113.
Alſo 91 Guin. = 95,55£. and 100 Mar. = 66,6£.
Then, as 95,55£. : 150 Days, :: 66,6£. : 215 Days.
∵ 215 — 113 = 102 Days.
Recip. as 66,6 : 102 :: 40£. : 169,83 Days, or rather 170 Days, the Anſwer.

(5) *Days.* *w.*

$$\text{As} \left\{\begin{array}{c} 12 \\ 14 \\ 15 \\ 18 \end{array}\right\} \textit{Work Day} : 1 :: 1 : \left\{\begin{array}{l} \frac{1}{12} = ,0833333 = A.B.C. \\ \frac{1}{14} = ,0714285 = B.C.D. \\ \frac{1}{15} = ,0666666 = A.C.D. \\ \frac{1}{18} = ,0555555 = A.B.D. \end{array}\right\} \text{Will do part of the whole work.}$$

All working three Days will do ,276984 Part of the Work.

Then, as ,276984 : 3 :: 1 : 10,8309505 Days, all working

14 = B. C. D.
— 10,83085 A. B. C. D.

As 3,16905 : 10,8395 :: 14 : 47,848 Days by A.

15 = A. C. D.
— 10,83095

As 4,16905 : 10,8395 :: 15 : 38,969 Days by B.

18 = A. B. D.
— 10,83095

As 7,16905 : 10,8395 :: 18 : 27,194 Days by C.

12 = A. B. C.
— 10,83095

As 1,16905 : 10,8395 :: 12 : 111,1766 Days by D.

alone.

(6)

(6) First 34″=,009444 ho. and 5 Rods,=,015625 Mile.
Then, as 1 ho. : 12 Miles. :: ,009444 ho. : ,113328 m.
∵ ,113328 + ,015625 = ,128953 m. or 680,9 ft. the Hare had started.
Now 20−12=8, Dog gained in running 20.
Again, as 8 : 20 :: ,12895 : 2,57906 fur.=$1702\frac{1}{4}$ ft. run by the Greyhound.
∵ 8 : 1 ho. :: 128953 m. : ,016119 ho.=58″ ,0284, run by the Greyhound.

63. VIBRATION OF PENDULUMS.

(8) Recip. as 39,2 : 3600 : $\left\{\begin{matrix} 12 : 11760. \\ 6 : 23520. \end{matrix}\right.$

∵ $\sqrt[2]{11760}$=108,444, and $\sqrt[2]{23520}$=153,362.
Then 153,362×60 = 9201,72
And 108,444×60 = 6506,64

Answer 2695,08 the Difference.

(9) A Pendulum which vibrates 60 Times in a Minute, will vibrate 60×60=3600 Times, in an Hour; the Square of which =12960000, and the Square of 30=900; also of 100=10000. Then Recip.

12960000 : 39,2 inc. :: $\left\{\begin{matrix} 900 : 564480 \\ 10000 : 50803,2 \end{matrix}\right.$

12)513676,8 Inches.

Answer, Diff. 42806,4 Feet.

(10) First a Pendulum which vibrates Thirds, will vibrate 60 Times in a Second.
∵ As 1^2 : 39,2 :: 60^2 (3600) : ,0108 inc. the Length of that which vibrates Thirds.
Secondly, a Pendulum which vibrates Seconds, will make 60 Vibrations in a Minute.
∵ As 60^2 (3600) : 39,2 :: ^{21} : 11760 inc.=$2\frac{5}{12}$ mls. the Length of a Pendulum which vibrates once in a Minute.
Thirdly, a Pendulum which vibrates Seconds, will make (60×60)=3600 Vibrations in an Hour.

$\because$ As 3600^2 (12960000) : 39,2 :: ^{2}1 : 508032000 inc. $=8018\frac{9}{11}$ mls. the Length of the Pendulum, which vibrates once in an Hour.

Lastly, a Pendulum which vibrates Seconds will make $(60\times60\times24)=86400$ Vibrations in a Day, or 24 Hrs.

$\because$ As 86400^2 (7464960000) : 39,2 :: ^{2}1 : 292626432000 inc. $=4618188\frac{1533}{1760}$ mls. the Length of a Pendulum which vibrates once in a Day.

(11) First $60\times60=3600$.

Then, as 39,2 inc. : 3600 sec. :: 18 inc. : 7840 sec.

$\because$ $\sqrt[2]{7840}=88,54378$, Vibrations in one Minute.

Then 60)88,54378(1,475729 Vibrations in a Second.

Also 1,475729)8,000000(5,421 sec. in eight Vibrations.

Now $5,421\times5,421=29,387$.

Then, as ^{2}1 : 16,083 ft. :: 29,387 : 472,640916 ft. Sound was returning.

Again, as 1150 ft. : 1 sec. :: 472,640916 : ,41099″, Time Sound was returning.

$\because$ 5,421 — ,411 = 5,01 Seconds, Time of the Body's Descent.

And $5,01\times5,01=25,1$. Also $25,1\times16,083=403,69$ ft. the Depth of the Well.

64. FELLOWSHIP.

(59) First 229£. 13s. 4d. = 229,6; $5\frac{5}{9}=5,5$; $4\frac{3}{7}=4,428571$; $4\frac{1}{9}=4,1$; and $3\frac{2}{3}=3,6$.

Then Nell takes up 3,6, as often as Anne takes up 4,1; then as often as Moll takes up 5,5, Anne takes up 4,428571; and Nell must take

$$\frac{3{,}6\times4{,}428571}{4{,}1}=3{,}949809,$$

For, as 4,1 : 3,6 :: 4,428571 : 3,949809.

$$\left.\begin{array}{rl} \text{Hence Moll takes} & 5{,}555555 \\ \text{Anne} & 4{,}428571 \\ \text{And Nell} & 3{,}949809 \end{array}\right\}\text{Ratio of their Shares.}$$

Sum 13,933935

$$\because \text{As } 13{,}933935 : 229{,}6\text{£.} :: \left\{\begin{array}{lrrr} & \text{£.} & s. & d. \\ 5{,}555555 : 91{,}569 = & 91 & 11 & 4\frac{3}{4}\text{ M.} \\ 4{,}428571 : 72{,}995 = & 72 & 19 & 10\frac{3}{4}\text{ A.} \\ 3{,}949809 : 65{,}102 = & 65 & 2 & \frac{1}{2}\text{ N.} \end{array}\right.$$

65. SIM-

65. SIMPLE INTEREST.

Theorem I. $Ptr=I$. the Intereſt.

(2) Here is given $P=260,875$, $t=5,5$, and $r=,045$.

Then per Theo. $260,875 \times 5,5 \times ,045 = 64,5665625 =$ 64£. 11*s.* $3\frac{3}{4}$*d.* ,9, the Intereſt required.

(3) Here is given $P=85$, $t=4,5$, and $r=,05$.

Then per Theo. $85 \times 4,5 \times ,05 = 19,125$£. $= 19,2\dot{6}$£. the Anſwer.

(4) Here $P=1000$, $t=4,\dot{6}$, and $r=,055$.

Then by Theo. $1000 \times 4,\dot{6} \times ,055 = 256,\dot{6} =$ 256£. 13*s.* 4*d.* the Intereſt.

(5) Firſt, from May 12, 1764, to Nov. 24, 1769, is 5 yrs. 196 ds. or 5,536986 yrs. $=t$, $r=,0375$, and $P=500$.

Then per Theo. $5,53696 \times ,0375 \times 500 = 103,8184875 =$ 103£. 16*s.* $4\frac{1}{4}$*d.* ,748, the Anſwer.

When the Intereſt is for Days.

(7) Here $r=,00012328767$, $t=220$, and $P=370,5$.

Then per Rule $,00012328767 \times 370,5 \times 220 = 10,049+$ $=10$£. $11\frac{3}{4}$*d.* the Intereſt required.

(8) Firſt, from July 1, to Feb. 24, following, is 238 Days, $=t$, $P=600$, and $r=,00016438356$.

Then $00016438356 \times 238 \times 600 = 23,47397223628 = 23$£. 9*s.* $5\frac{3}{4}$*d.* the Intereſt.

Theorem II. $Ptr+P=A$, the Amount.

(9) Here $P=284,5$, $t=7$, and $r=,035$.

Then per Theo. $284,5 \times 7 \times ,035 + 284,5 = 354,2025 =$ 354£. 4*s.* $\frac{1}{2}$*d.* ,4, the Amount required.

(10) Here $P=672,25$, $t=5,5$, and $r=,045$.

Then per Theorem, $672,25 \times 5,5 \times ,045 + 672,25 =$ $838,631875 = 838$£. 12*s.* $7\frac{1}{2}$*d.* ,6, the Anſwer.

(11) Here $P=500$, $t=6,328767$, and $r=,0475$.

Then per Theorem VI. $328767 \times ,0475 \times 500 + 500 =$ $650,30821625 = 650$£. 6*s.* $1\frac{3}{4}$*d.* 8+, the Amount.

Theorem III. $\frac{I}{tr}=P$, the Principal.

(12) Here $I=69,675$, $t=3$, and $r=,05$.

Then per Theo. $\frac{69,675}{3 \times ,05} = \frac{69,675}{,15} = 464,5 =$ 464£. 10*s.* the Principal.

(13)

(13) Here $I=64,35$, $t=5,5$, and $r=,045$.

Then per Theo. $\frac{64,35}{5,5\times,045}=\frac{64,3500}{,2475}=260$ £. the Answ.

(14) Here $I=67,790625$, $t=4$, and $r=,04$.

Then per Theo. $\frac{67,790625}{4\times,04}=\frac{67,790625}{,16}=423$ £. 13 *s.* 9¾*d.* ,75+, the Principal required.

Theorem IV. $\frac{a}{tr+1}=P$, the Principal.

(15) Here $a=354,22083$, $t=7$, and $r=,035$.

Then per Theo. $\frac{354,22083}{,035\times7+1}=\frac{354,22083}{1,245}=284,5147=$ 284 £. 10*s.* 3½*d.* ,112, the Answer.

(16) Here $A=500,460416$, $t=6,416$, and $r=,05$.

Then per Theo. $\frac{500,460416}{6,416\times,05+1}=\frac{500,460416}{1,32083}=$ 378,89826+ = 378 £. 17*s.* 11½*d.* ,2296, the Answer.

(17) Here $A=100$, $t=7,6027397$, and $r=,0475$.

Then per Theo. $\frac{100}{7,6027397\times,0475+1}=\frac{100}{1,36113}=$ 73,4675+ = 73 £. 9*s.* 4*d.* ,2, the Principal.

Theorem V. $\frac{I}{pr}=t$, the Time.

(18) Here $I=69,675$, $P=464,5$, and $r=,05$.

Then per Theo. $\frac{69,675}{464,5\times,05}=\frac{69,675}{23,225}=3$ yrs. the Time required.

(19) Here $I=64,35$, $P=260$, and $r=,045$.

Then per Theo. $\frac{64,35}{260\times,045}=\frac{64,35}{11,7}=5\frac{1}{2}$ yrs. the Answer.

(20) Here $I=130,47916$, $P=500$, and $r=,065$.

Then per Theo. $\frac{130,47916}{,065\times500}=\frac{130,47516}{32,5}=4,0144+=4$ Years, 5,25 Days, the Answer.

(21)

Theorem VI. $\frac{a-P}{Pr}=t$, the Time.

(21) Here $a=354,22083$, $P=284,5$, and $r=,0325$.

Then per Theo. $\frac{354,22083-284,5}{284,5\times,0325}=\frac{70,72083}{9,24625}=7,65405$ $+=7$ Years, 238,7 Days, the Anſwer.

(22) Here $a=847,875$, $P=672,25$, and $r=,0475$.

Then per Theo. $\frac{847,875-672,25}{672,25\times,0475}=\frac{175,625000}{31,931875}=5\frac{1}{2}$ yrs. the Anſwer.

(23) Here $a=500,464583$, $P=378,9$, and $r=,05$.

Then per Theo. $\frac{500,464583-378,9}{378,9\times,05}=\frac{121,564583}{18,945}=$ 6,41467=6 Years, 5 Months, (nearly) the Anſwer.

Theorem VII. $\frac{I}{pt}=r$, the Rate.

(24) Here $I=69,675$, $P=464,5$, and $t=3$.

Then per Theo. $\frac{69,675}{464,5\times3}=\frac{69,675}{1393,5}=,05$, or 5 per Ct. the Rate required.

(25) Here $I=64,35$, $P=260$, and $t=5,5$.

Then per Theo. $\frac{64,35}{260\times5,5}=\frac{64,35}{1430}=,045$, or $4\frac{1}{2}$ per Cent. the Anſwer.

(26) Here $I=235,46$, $P=560,635416$, and $t=7$.

Then per Theo. $\frac{235,46}{560,635416\times7}=\frac{235,46666666}{3924,447916}=,06$; or 6 per Cent the Anſwer.

Theorem VIII. $\frac{a-P}{Pt}=r$ the Rate.

(27) Here $a=354,22083$, $P=284,5$, and $t=7$.

Then per Theo. $\frac{354,22083-284,5}{284,5\times7}=\frac{69,72083}{1991,5}=,035$, or $3\frac{1}{2}$ per per Cent. the Anſwer.

(29) Here $a=500,4635416$, $P=378,9$, and $t=6$.

Then per Theo. $\frac{500,4635416-378,9}{378,9\times 6}=\frac{121,5635416}{2273,4}=$ $,053472+$, or $5£.\ 6s.\ 11\frac{1}{4}d.\ ,312$, per Cent. the Answ.

(30) Here $a=847,875$, $P=672,25$, and $t=5,5$.

Then per Theo. $\frac{847,875-672,25}{672,25\times 5,5}=\frac{175,62500}{3697,375}=,0475$, or $4\frac{3}{4}$ per Cent. the Answer.

66. Of ANNUITIES, PENSIONS, &c. in ARREARS, at SIMPLE INTEREST.

Theorem IX. $\overline{\frac{ttu-tu}{2}\times r}:+tu=A$, the Amount.

(2) Here $u=250$, $t=6$, and $r=,03$.

Then per Theo. $\overline{\frac{250\times 6\times 6-250\times 6}{2}\times ,03}:+250\times 6=$ $\overline{\frac{9000-1500}{2}\times ,03}:+1500=\overline{3750\times ,03}+1500=112,5$, $+1500=1612,5$, or $1612£.\ 10s.$ the Answer.

(3) Here $u=125$, $t=12$, and $r=,015$, per Note.

Then per Theo. $\overline{\frac{125\times 12\times 12-125\times 12}{2}\times ,015}:+125\times$ $12=\overline{\frac{18000-1500}{2}\times ,015}:+1500=\overline{8250\times ,015}+$ $1500=123,75+1500=1623,75=1623£.\ 15s.$ the Ans.

(4) Here $u=62,5$, $t=24$, and $r=,0075$, per Note.

Then per Theo. $\overline{\frac{62,5\times 24\times 24-62,5\times 24}{2}\times ,0075}:+$ $62,5\times 24=\overline{\frac{36000-1500}{2}\times ,0075}:+1500=$ $\overline{17250\times ,0075}+1500=129,375+1500=1629,375=$ $1629£.\ 7s.\ 6d.$ the Answer.

Theorem X. $\frac{2a}{ttr-tr+2t}=U$, the Annuity.

(5)

(5) Here $a=627,2$, $t=7$, and $r=,04$.

Then per Theorem, $\frac{627,2\times 2}{7\times 7\times ,04-7\times ,04+7\times 2}=\frac{1254,4}{1,96-,28+14}=\frac{1254,4}{15,68}=80£.$ the Answer.

(6) Here $a=1612,5$, $t=6$, and $r=,03$.

Then per Theo. $\frac{1612,5\times 2}{6\times 6\times ,03-6\times ,03+6\times 2}=\frac{3225.0}{12,9}=$ $250£.$ the Answer.

Theo. for Half Yearly Payment, $\frac{4a}{ttr-tr+2t}=t.$

(7) Here $a=1623,75$, $t=12$, and $r=,015$.

Then per Theo. $\frac{1623,72\times 4}{12\times 12\times ,015-12\times ,015+12\times 2}=$ $\frac{6495}{1,98+24}=\frac{6495,00}{25,98}=250£.$ the Answer.

Theo. for Quarterly Payment, $\frac{8a}{ttr-tr+2t}=t.$

(8) Here $a=1629,375$, $t=24$, and $r=,0075$.

Then per Theo. $\frac{1629,375\times 8}{24\times 24\times ,0075-24\times ,0075+24\times 2}=$ $\frac{13035}{4,14+48}=\frac{13035,00}{52,14}=250£.$ the Answer.

Theo. XI. $\frac{2a-2ut}{utt-ut}=r$, the Rate.

(9) Here $a=627,2$, $u=80$, and $t=7$.

Then per Theo. $\frac{627,2\times 2-80\times 2\times 7}{80\times 7\times 7-80\times 7}=\frac{1254,4-1120}{3920-560}=$ $\frac{134,40}{3360}=,04$, or 4 per Cent. the Answer.

(10) Here $a=1612,5$, $u=250$, and $t=6$.

Then per Theo. $\frac{1612,5\times 2-250\times 2\times 6}{250\times 6\times 6-250\times 6}=\frac{3225-3000}{9000-1500}=$ $\frac{225,00}{7500}=,03$, or 3 per Cent. the Answer.

Theo.

Theo. for Half Yearly Payments, $\frac{4a-4ut}{utt-ut}=r$.

(11) Here $a=1623,75$, $u=125$, and $t=12$, per Note.

Then per Theo. $\frac{1623,75\times4-125\times4\times12}{125\times12\times12-125\times12}=\frac{6495-6000}{18000-1500}$ $=\frac{495,00}{16500}=,03$, or 3 per Cent. the Answer.

Theo. for Quarterly Payments, $\frac{8a-8ut}{utt-ut}=r$.

(12) Here $a=1629,375$, $u=62,5$, and $t=24$.

Then per Theo. $\frac{1629,375\times8-62,5\times8\times24}{62,5\times24\times24-62,5\times24}=$ $\frac{13035-12000}{36000-1500}=\frac{1035,00}{34500}=,03$, or 3 per Cent. Answ.

Theorem XII. First $\frac{2}{r}-1=X$. Then $\sqrt{\frac{2}{ur}+\frac{xx}{4}}:-\frac{x}{2}=T$.

(13) Here $a=627,2$, $u=80$, and $r=,04$.

Then per Theo. I. $\frac{2}{,04}-1=50-1=49=X$.

Then $\sqrt{\frac{627,2\times2}{80\times,04}+\frac{49\times49}{4}}:-\frac{49}{2}=\sqrt{\frac{1254,4}{3,2}+\frac{2401}{4}}$ $-24,5=\sqrt{392+600,25}-24,5=\sqrt{992,25}-24,5=$ $31,5-24,5=7$ Years, the Time required.

(14) Here $a=1612,5$, $u=250$, and $r=,03$.

Then per Theorem I. $\frac{2}{,03}-1=65,\dot{6}=X$.

Then $\sqrt{\frac{1612,5\times2}{250\times,03}+\frac{65,\dot{6}\times65,\dot{6}}{4}}:-\frac{65,\dot{6}}{2}=$ $\sqrt{\frac{3225}{7,5}+\frac{4307,73}{4}}:-32,8=\sqrt{1506,93}-32,8=38,8$ $-32,8=6$ Years, the Answer.

(15) Here $a=1623,75$, $u=125$, and $r=,015$.

Then per Theorem I. $\frac{2}{,015}-1=132,\dot{3}=X$.

Then

Then $\sqrt{\frac{1623{,}75\times 2}{125\times{,}015}+\frac{132{,}3\times 132{,}3}{4}}:-\frac{132{,}3}{2}=$ $\sqrt{\frac{3247{,}5}{1{,}875}+\frac{17507{,}68}{4}}:-66{,}1=\sqrt{1732+4376{,}92}$ $:-66{,}1=\sqrt{6108{,}92}-66{,}1=78.\quad 1-66{,}1=12$ Half Years or 6 Years, the Time required.

(16) Here $a=1629{,}375$, $u=62{,}5$, and $r={,}0075$.

Then per Theorem I. $\frac{2}{{,}007}-1=265{,}6=X$.

Then $\sqrt{\frac{1629{,}375\times 2}{62{,}5\times{,}0075}+\frac{265{,}6\times 65{,}6}{4}}:-\frac{265{,}6}{2}=$ $\sqrt{\frac{3258{,}75}{{,}46875}+\frac{70761{,}06}{4}}:-132{,}8=\sqrt{6952+17690{,}26}$ $-132{,}8=\sqrt{24642{,}26}-132{,}8=156{,}9-132{,}8=24$ quarterly Payments, or 6 Years, the Answer.

75. PRESENT WORTH OF ANNUITIES, &c.

Theorem XIII. $\frac{ttr-tr+2t}{2tr+2}:\times u=P$. The present Worth.

(18) Here $t=6$, $r={,}03$, and $u=250$.

Then per Theo. $\frac{6\times 6\times{,}03-6\times{,}03+6\times 2}{2\times 6\times{,}03+2}:\times 250=$ $\frac{1{,}08-{,}18+12}{{,}36\times 2}\times 250=\frac{12{,}9}{2{,}36}\times 250=5{,}4661017\times$ $250=1366{,}52675$, or 1366£. 10*s*. 6*d*. ,42, the present Worth required.

(19) Here $t=12$, $r={,}015$, and $u=125$.

Then per Theo. $\frac{12\times 12\times{,}015-12\times{,}015+12\times 2}{12\times 2\times{,}015+2}:\times 125$ $=\frac{2{,}16-{,}18+24}{{,}36+2}:\times 125=\frac{1{,}98+24}{2{,}36}:\times 125=$ $\frac{25{,}98}{2{,}36}\times 125=11{,}008474\times 125=1376{,}05925=1376$£. 1*s*. 2*d*. ,22, the present Worth.

(20) Here is given $t=24$, $v=62,5$, and $r=,0075$.

Then per Theo $\frac{24\times 24\times ,0075-24\times ,0075+24\times 2}{24\times 2\times ,0075+2}:\times$

$62,5=\frac{4,32-,18+48}{2,36}:\times 62,5=\frac{4,14+48}{2,36}:\times 62,5$

$=\frac{52,14}{2,36}:\times 62,5=22,09322\times 62,5=1380,82625=$

1380£. 16s. 6d. ,3, the Answer.

Theorem, XIV. $\frac{tr+1}{ttr-tr+2t}:\times 2p=$ U, the Annuity.

(21) Here is given $t=7$, $r=,04$, and $p=490$. To find U.

Then per Theo. $\frac{7\times ,04+1}{7\times 7\times ,04-7\times ,04+7\times 2}:\times 490\times 2$

$=\frac{1,28}{1,96-,28+14}:\times 980=\frac{1,28}{15,68}:\times 980=,0816326$

$\times 980=80$£. the yearly Rent.

(22) Here is given $t=6$, $r=,03$, and $p=1366,525$. To find U.

Then per Theo $\frac{6\times ,03+1}{6\times 6\times ,03-6\times ,03+6\times 2}:\times 1366,525\times 2$

$=\frac{1,18}{12,9}\times 2733,05=,0914728\times 2733,05=250$£. the Annuity required.

(23) Here is given $t=12$, $r=,015$, and $p=1376,25$. To find U.

Then per Theo. $\frac{12\times ,015+1}{12\times 12\times ,015-12\times ,015+12\times 2}:\times$

$1376,25\times 4=\frac{1,18}{25,98}\times 5505=,0454195\times 5505=$

250£. $8\frac{1}{2}$d. the Annuity required.

(24) Here is given $t=24$, $r=,0075$, and $p=1380,875$, To find U.

Then per Theo. $\frac{24\times ,0075+1}{24\times 24\times ,0075\times 12\times ,0075+24\times 2}\times$

$1380,875\times 8=\frac{1,18}{52,14}\times 11047=,022631\times 11047=$

250£. the Answer. Theo-

Theorem XV. $\frac{\overline{ut-p}\times 2}{2pt+ut-utt}=R$, the Rate.

(25) Here is given $u=80$, $t=7$, and $p=490$. To find R,

Then per Theorem, $\frac{\overline{80\times 7-490}\times 2}{490\times 2\times 7+80\times 7-80\times 7\times 7}=$

$\frac{70\times 2}{7420-3920}=\frac{140}{3500}=,04$, or 4 per Cent. Answ.

(26) Here is given, $u=250$, $t=6$, and $p=1366,525$. To find R.

Then per Theo. $\frac{\overline{250\times 6-1366,525}\times 2}{1366,525\times 2\times 6+250\times 6-250\times 6\times 6}$

$=\frac{133,475\times 2}{17898,3-900}=\frac{266,95}{8898,3}=,03=3$ per Cent. the Answ.

(27) Here is given $u=125$, $t=12$, and $p=1376,25$. To find R.

Then per Theo. $\frac{\overline{125\times 12-1376,25}\times 2}{1376,25\times 2\times 12+125\times 12-125\times 12\times 12}$

$=\frac{\overline{1500-1376,25}\times 2}{33030+1500-18000}=\frac{123,75\times 2}{34520-18000}=\frac{2475}{16530}$ $=,015$ half the Rate, which $\times 2=,03$ or 3, per Cent. the Answer.

(28) Here is given, $u=62,5$, $t=24$, and $p=1380,875$. Then per Theorem,

$$\frac{\overline{62,5\times 24-1380,875}\times 2}{1380,875\times 2\times 24+62,5\times 24-62,5\times 24\times 24}=$$

$=\frac{238,25}{31782}=,0075$, which $\times 4=,03$, or 3 per Cent. the Answer.

Theorem XVI. $\frac{2}{r}-\frac{2p}{u}-1=X$. Then $\sqrt{\frac{2p}{ur}+\frac{xx}{4}}-\frac{x}{2}=t$. the Time.

(29) Here is given $u=80$, $p=490$, and $r=,04$. To find t.

Theo. first $\frac{2}{04}-\frac{490\times 2}{80}-1=50-13,25=36,75=X$.

Then $\sqrt{\frac{490\times 2}{80\times ,04}+\frac{36,75\times 36,75}{4}}:-\frac{36,75}{2}=$ $\sqrt{\frac{980}{3,2}+\frac{1350,5625}{4}}:-18,375=\sqrt{306,25+307,640625}$ $-18,375=\sqrt{643,890625}-18,375=25,375-18,375$ $=7$ Years, the Time required.

(30) Here is given $u=250$, $p=1366,525$, and $r=,03$.

Then per Theo. first $\frac{2}{,03}-\frac{1366,525\times 2}{250}-1=66,\dot{6}-$ $11,9322=54,73446=X$.

Then $\sqrt{\frac{1366,525\times 2}{250\times ,03}+\frac{54,73446\times 54,73446}{4}}-$ $\frac{54,73446}{2}=\sqrt{\frac{2733,05}{7,5}+\frac{2995,86}{4}}-27,3=\sqrt{1113,36}-$ $27,3=33,3-27,3=6$ Years, the Answer.

(31) Here is given $u=125$, $p=1376,25$, and $r=,015$.

Then per Theo. first $\frac{2}{,015}-\frac{1376,25\times 2}{125}-1=133,\dot{3}-$ $23,02=110,31\dot{3}=X$.

Then $\sqrt{\frac{1376,25\times 2}{125\times ,015}+\frac{110,31\dot{3}\times 110,31\dot{3}}{4}}-\frac{110,31\dot{3}}{2}=$ $\sqrt{4510,2486}-55,15=67,15-55,15=12$ half yearly Payments, or 6 Years, the Time required.

(32) Here is given $u=62,5$, $p=1380,875$, and $r=,0075$.

Then per Theo. first $\frac{2}{,0075}-\frac{1380,875\times 2}{62,5}-1=266,\dot{6}$ $-45,188=221,4786=X$.

Then $\sqrt{\frac{1380,875\times 2}{62,5\times ,0075}+\frac{221,4786\times 221,4786}{4}}-$ $\frac{221,4786}{2}=\sqrt{5891,733+12263,1962}-110,7393=$ $\sqrt{18154,92}-110,7=134,7-110,7=24$ quarterly Payments, or 6 Years, the Time required.

ANNU-

76. ANNUITIES, &c. TAKEN IN REVERSION.

Theo. XVII. $\dfrac{ttr - tr + 2t}{2tr + 2} : \times u = p$, which change to a.

Then $\dfrac{a}{tr+1} = p$, the present Worth.

(34) Here is given, first $r = ,045$, $t = 7,5$, and $u = 8$.

Then per Theo. I. $\dfrac{7,5 \times 7,5 \times ,045 - 7,5 \times ,045 + 7,5 \times 2}{7,5 \times 2 \times ,045 + 2}$

$: \times 80 = \dfrac{2,53125 - ,3375 + 15}{} \times 80 = \dfrac{17,19375}{2,675} \times 80$

$= 64,2756 \times 80 = 514,2048 = p$.

Now per Theo. II. $p = a$, and r as before $= 5$.

Then $\dfrac{514,2048}{5 \times 045 + 1} = \dfrac{514,2046}{1,225} = 419,95759 = 419$ £. 19*s*. $1\frac{3}{4}$*d*. the Answer.

(35) Here is given $u = 40$, $t = 10$, and $r = 05$.

Then per Theo. I. $\dfrac{10 \times 10 \times ,05 - 10 \times ,05 + 10 \times 2}{10 \times 2 \times ,05 + 2} \times 40$

$\dfrac{4,5 + 20}{1 + 2} \times 40 = \dfrac{24,5}{3} \times 40 = 80,1\dot{6} \times 40 = 326,\dot{6} = p$.

Now per Theo. II. $a = 326,\dot{6}$, and $t = 7$.

Then $\dfrac{326,6}{7 \times ,05 + 1} = \dfrac{326,6}{1,35} = 241,9755 = 241$ £. 19*s*. 6*d*. ,12, the present Worth.

Theorem XVIII. $ptr + p = A$ the Amount. Change A, and call it p, and t here will = the Time of its Continuance.

Then $\dfrac{tr+1}{ttr - tr + 2t} : \times 2p = U$, the Annuity, &c.

(36) Here is given $p = 1220,11458\dot{3}$, $t = 4$, and $r = ,03$.

Then per Theorem I. $1220,111458\dot{3} \times 4 \times ,03 + 1220,111458\dot{3} = 146,413375 + 1220,1114583 = 1366,52483 = A$.

Now A changed, it becomes p, and $t=6$.

Then per Theo. II. $\dfrac{6\times,03+1}{6\times6\times,03-6\times,03+6\times2}\times$

$1366,52483\times2=\dfrac{1,18}{12,9}\times2733,049666=2723,049666$

$\times,0914728=250$£. the Annuity.

(37) Here is given $p=419,755208$, $t=5$, and $r=,045$.

Then per Theorem I. $419,755208\times5\times,045+419,755208$ $=94,4449218+419,755208=514,2001298=A$.

Now A changed becomes p, and t here $=7,5$.

Then per Theo. II. $\dfrac{7,5\times,045+1}{7,5\times7,5\times,045-7,5\times,045+7,5\times2}$

$\times514,2001298\times2=\dfrac{1,3375}{17,19375}\times1028,4002596=$

$,077789\times1028,4002596=80$£. the Answer.

(8) Here is given $p=241,975308$, $t=7$, and $r=,05$.

Then per Theorem I. $241,975308\times7\times,05+241,975308$ $=326,6666658=A$.

Now A changed becomes $p=326,6666658$, and $t=10$.

Then per Theo. II. $\dfrac{10\times,05+1}{10\times10\times,05-10\times,05+10\times2}$

$\times\ 326,6666658\ \times\ 2\ =\dfrac{1,5}{24,5}\times653,3333316=$

$653,3333316\times,06122448=80$£. nearly the Answer.

77. REBATE OR DISCOUNT.

Theorem XIX. $\dfrac{s}{tr+1}=P$, the present Worth.

(40) Here is given $s=1000$, $t=5$mo.$=,41\dot{6}$yr. and $r=,045$.

Then per Theo. $\dfrac{1000}{,41\dot{6}\times,045+1}=\dfrac{1000}{1,01875}=981,595=$

981£. 11s. $10\frac{1}{4}$d. the present Worth required.

(11) Here is given $s=9342$, $r=,04$, and $t=10$ mo. $=,8\dot{3}$yr.

Then per Theo. $\dfrac{9342}{,8\dot{3}\times,04+1}=\dfrac{9342}{1,03}=9038,9$ the present Worth.

$9342-9038,9=303,1=303$£. 2s. the Discount.

Theo-

Theorem XX. $Ptr+p=S$, the Sum due.

(41) Here is given $p=144,578125$, $t=,75$, and $r=,05$. Then per Theo. $144,578125\times,75\times,05+144,578125=5,421679+144,578125=149,9998$, or rather 150£. the Answer.

(42) Here is given $p=981,52083$, $t=,41\dot{6}$, and $r=,045$. Then per Theorem, $981,52083\times,41\dot{6}\times,045+981,52=18,403518+9981,52083=999,9243485=999$£. 18*s.* 5½*d.* the Sum due.

(43) Here is given $p=9111,18514\dot{6}$, $t=,8\dot{3}$, and $r=,04$. Then per Theo. $9111,185416\times,8\dot{3}\times,04+9111,185416=303,70618+9111,185146=9414,891326=9414$£. 17*s.* 9¼*d.* ,6, the Debt.

Theorem XXI. $\frac{s-p}{pr}=T$, the Time.

(44) Here is given $s=150$, $p=144,578125$, and $r=05$. Then per Theo. $\frac{150-144,578125}{144,578125\times,05}=\frac{5,42187500}{7,22890625}=$,75 yr. or 9 Months, the Time required.

(45) Here is given $s=1000$, $p=981,5208\dot{3}$, and $r=,045$. Then per Theo. $\frac{1000-981,5208\dot{3}}{981,5208\dot{3}\times,045}=\frac{18,47916}{44,16843749}=$,41$\dot{6}$, or 5 Months, the Answer.

(46) Here is given $s=9342$, $p=9111,185416$, and $r=,04$. Then per Theo. $\frac{9342-9111,185416}{9111,185416\times,04}=\frac{230,814584}{364,44741\dot{6}}=$,63332=7 mo. or 18 Days, the Answer.

Theorem XXII. $\frac{s-p}{pt}=R$, the Rate.

(47) Here is given $s=150$, $p=144,578125$, and $t=,75$. Then per Theo. $\frac{150-144,578125}{144,578125\times,75}=\frac{5,421875}{108,43359375}=$,055 or per Cent. the Rate required.

(48) Here is given $s=1000$, $p=981,5208\dot{3}$, and $t=,41\dot{6}$. Then per Theo. $\frac{1000-981,5208\dot{3}}{981,5208\dot{3}\times,41\dot{6}}=\frac{18,4791\dot{6}}{40,89570138}=$,045=4½ per Cent. the Answer.

(49)

(49) Here is given $s=9342$, $p=9111,185416$, and $t=,83$:
Then per Theo. $\frac{9342-9111,185416}{9111,185416\times,83}=\frac{230,814584}{769,26989582}=$ $,03=3$ per Cent. the Anſwer.

78. EQUATION OF PAYMENTS.

Theorem XXIII. $\frac{s}{tr+1}=P$, the preſent Worth.

(51) Here is given $r=,035$, the firſt $t=,5$, the ſecond $,6$, and the third $=,83$.
Then per Theo. the preſent Worth will be as follows, viz.

$$\begin{array}{lr}
 & £. \\
\text{Paid down} & 400 \\
1.\ \frac{500}{,5\times035+1}=\frac{500}{1,075}= & 491,40049 \\
2.\ \frac{250}{,6\times035+1}=\frac{250}{1,0213}= & 244,78605 \\
3.\ \frac{250}{,83\times035+1}=\frac{250}{1,025}= & 243,902439 \\
\text{the Sum} = & 1380,089179=p. \\
 & 1400\quad =s. \\
s-p=\text{Diff.} = & 19,910821=d.
\end{array}$$

Now per Theo. II. $\frac{d}{pr}=\frac{19,910821}{1380,8179\times,035}=\frac{19,910821}{48,303121265}$ $=,41013=5$ Months, nearly the true equate Time required.

QUESTIONS FOR EXERCISE.

(52) As $12,3£.:47$ Days $::949,5£.:18,45$ Days, the Anſwer.

(53) Here is given $a=542,4$, $p=384$, and $t=8,25$.
Then per Theo. VIII. $\frac{a-p}{pt}=\frac{542,4-384}{384\times8,25}=\frac{158,4}{3168}=$ $,05$ or 5 per Cent. the Anſwer.

79. COM-

79. COMPOUND INTEREST.

Theorem I. $pr^t = A$, the Amount.

(2) Here is given $p=246,5$, $t=7$, and $r=1,05$.
Then per Theorem,

$\overline{246,5 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05}$ $=246,5 \times 1,4071004 = 346,8502486 = 346$£. 17*s.* the Amount required.

(3) Here is given $p=500$, $t=30$, and $r=1,045$, which raised to the 30th Power, will give $3,7453181 = r^t$. (see Table I.)

Then $3,7453181 \times 500 = 1772,65905 = 1772$£. 13*s*, 2*d*. ,172, the Amount for 30 Years.

Again, suppose the Amount was required only for 30 d.

Then $r=1,0001206$, which raised to the 30th Power, $=$ 1,0036243 (see Table II.) the Amount of 1£. for 30 Days.

∵ $1,0036243 \times 500 = 501,81215 = 501$£. 16*s.* 3*d.* ,156, the Amount.

(4) Here is given $p=523$, $t=5$ yrs. 194 d. and $r=1,05$.
Now the Amount of 1£. for 5 yrs. is $1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 = 1,2762816$. (see Table I.)

Then, per Table II. the Amount of 1£. for 100 d. $=$ 1,0134569; for 90 $=$ 1,0121031; and for 4 $=$ 1,0005348.

∵ $1,2762816 \times 1,0134569 \times 1,0121031 \times 1,0005348 =$ 1,310976072, the Amount of 1£. for 5 yrs. 194 d.

Therefore $1,310976072 \times 523 = 685,640485656 = 685$£. 12*s.* 9½*d.* ,8, the Answer.

Theorem II. $\frac{a}{r^t} = P$.

(5) Here is given $a=243,10126$, $t=4$, and $r=1,05$.

Then per Theo. $\frac{243,10126}{1,05 \times 1,05 \times 1,05 \times 1,05} = \frac{243,10126}{1,2155063}$ $=200$£. the Answer.

(6) Here is given $a=346,85$, $t=7$, and $r=1,05$.
Then per Theorem,

$$\frac{346,85}{1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05} = \frac{346,85}{1,4071004}$$

$=246,5 = 246$£. 10*s.* the Answer.

(7)

(7) Here is given $a=1872,65905$, $t=30$, and $r=1,045$. Which raised to 30th Power $=3,7453181$. (see Table I.)

Then per Theo. $\frac{1872,65905}{3,7453181}=500$ £. the Answer.

(8) Here is given $a=685,64$, $t=5$ yrs. 194d. and $r=1,05$, which raised to the Power of $t=1,310976$.

Then per Theo. $\frac{685,64}{1,310976}=523$ £. the Answer.

Theorem III. $\frac{a}{p}=R^t$.

(9) Here is given $a=243,10126$, $p=200$, and $r=1,05$.

Then per Theo. $\frac{243,10126}{200}=1,2155063=R^t$.

Now 1st. 1,05)1,2155063(1,157625
2d. 1,05)1,157625(=1,1025
3d. 1,05)1,1025(=1,05
4th. 1,05)1,05(=1, and nothing remains.

Therefore the Number of the Divisions are 4, which are 4 Years, the Answer.

(10) Here is given $a=346,85$, $p=246,5$, and $r=1,05$.

Then per Theo. $\frac{346,85}{246,5}=1,4701004=R^t$.

Which $\div 1,05$ as in the last Example, till nothing remains, and the Number of Divisions will be 7, or 7 Yrs. the Answer.

(11) Here is given $a=1872,65905$, $p=500$, and $r=1,045$.

Then per Theo. $\frac{1872,65905}{500}=3,7453181=R^t$.

Which Number look for in Table I. under $4\frac{1}{2}$ per Cent. and you will find it to be even with 30 Yrs. the Answ.

(12) Here is given $a=685,64$, $p=523$, and $r=1,05$.

Then per Theo. $\frac{685,64}{523}=1,310976=R^t$. the nearest Number (less) to which under 5 per Cent. is even with 5 yrs. viz. 1,2762816.

Then $1,310976 \div 1,2762816 = 1,027184025$ the nearest Number to which, in Table II. is even with 100 Days, viz. 1,0134569.

Therefore $1,027184025 \div 1,0134569 = 1,01264437$ the nearest Number (less) to which in Table II. is even with 90 Days, viz. 1,0121031.

So

So $1{,}0126437 \div 1{,}0121031 = 1{,}0005348$, which is even with 4 Days, in Table II.

Therefore the Anſwer is 5 Years, 194 Days.

Theorem IV. $\frac{a}{p} = R_t$.

(13) Here is given $a = 243{,}10126$, $p = 200$, and $t = 4$.

Then per Theo. $\frac{243{,}10126}{200} = 1{,}2155013$, which is the fourth Power of r.

$\because \sqrt[2]{1{,}2155013} = 1{,}025$, and $\sqrt[2]{1{,}025} = 1{,}05$ or 5 per Cent. the Rate required.

(14) Here is given $a = 346{,}85$, $p = 246{,}5$, and $t = 7$.

Then per Theo. $\frac{346{,}85}{246{,}5} = 1{,}4071004 = r$, raiſed to the 7th Power.

Then per Rule in Page 238 $r = \sqrt[7]{1{,}4071004} = 1{,}05$ or 5 per Cent. the Anſwer.

(15) Here is given $a = 1872{,}65905$, $p = 500$, and $t = 30$.

Then per Theo. $\frac{1872{,}65905}{500} = 3{,}7453181 = r^t$, which Number look for in Table I. and even with 30 yrs. you will find it to be under $4\frac{1}{2}$ per Cent. the Anſwer.

(16) Here is given $a = 685{,}64$, $p = 523$, and $t = 5$ yrs. 194 d.

Then per Theo $\frac{685{,}64}{523} = 1{,}310976 = r^t$. Now the neareſt Number to which, even with 5 yrs. is under 5 per Cent. the Anſwer.

80. ANNUITIES, PENSIONS IN ARREARS.

Theorem V. $\frac{Ur^t - u}{r - 1} = A$, the Amount.

(2) Here is given $u = 100$, $t = 6$, and $r = 1{,}05$.

Then per Theorem

$$\frac{100 \times 1{,}05 \times 1{,}05 \times 1{,}05 \times 1{,}05 \times 1{,}05 \times 1{,}05 - 100}{1{,}05 - 1} =$$

$$\frac{100 \times 1{,}3400956 - 100}{{,}05} = \frac{134{,}00956 - 100}{{,}05} = \frac{34{,}00956}{{,}05}$$

$= 680{,}192 = 680$ £. 3 *s.* $9\frac{1}{4}$ *d.* the Amount.

(3)

(3) Here is given $u=70$, $t=7$, and $r=1,03$; which raised to the 7th Power $=1,2298733$. (see Table I.)

Then per Theo. $\frac{70\times 1,2298733-70}{1,03-1}=\frac{86,091131-70}{,03}=$ $\frac{16,091131}{,03}=536,37103=563£.\ 7s.\ 5d.$ the Answer.

(4) Here is given $u=30$, $t=30$, and $r=1,035$; which raised to the 30th Power $=2,8067937$ per Table I.

Then per Theo. $\frac{30\times 2,8067937-30}{1,035-1}=\frac{84,20381-30}{,035}=$ $\frac{54,20381}{,035}=1548,680257=1548£.\ 13s.\ 7\frac{1}{4}d.$ the Amount required.

Theorem VI. $\frac{ar-a}{r^t-1}=U$, the Annuity, &c.

(5) Here is given $a=297,25248$, $t=4$, and $r=1,04$.

Then per Theo. $\frac{297,25248\times 1,04-297,25248}{1,04\times 1,04\times 1,04\times 1,04-1}=$ $\frac{309,1425792-297,25248}{1,1698586-1}=\frac{11,890}{,1698586}=70£.$ the Annuity.

(6) Here is given $a=680,1912$, $t=6$, and $r=1,05$; which raised to the 6th Power $=1,3400956$.

Then per Theo. $\frac{680,1912\times 1,05-680,1912}{1,3400956-1}=\frac{34,00956}{,3400956}$ $=100£.$ the Answer.

(7) Here is given $a=536,37103$ (per Quest. 3.) $t=7$, and $r=1,03$; which raised to the seventh Power $=1,2298733$.

Then per Theo. $\frac{536,37103\times 1,03-536,37103}{1,2298733-1}=$ $\frac{552,46219-536,37103}{,2298733}=\frac{16,0911632}{,2298733}=70£.$ the Answer.

(8) Here $a=1548,680257$, $t=30$, and $r=1,035$; which raised to the 30th Power $=2,8067937$ per Table I.

Then

Then per Theo. $\frac{1548{,}680257 \times 1{,}035 - 1548{,}680257}{2{,}8067937 - 1} =$

$\frac{1602{,}884066 - 1548{,}680257}{1{,}8067937} = \frac{54{,}203819}{1{,}8067937} = 30\text{£}.$ the Anſwer.

Theorem VII. $\frac{\overline{ar+u}-a}{u} = R'$.

(9) Here is given $a = 297{,}25248$, $u = 70$, and $r = 1{,}04$.

Then per Theo. $\frac{297{,}25248 \times 1{,}04 + 70 - 297{,}25248}{70} =$

$\frac{309{,}1425792 - 297{,}25248}{70} = \frac{81{,}8900992}{70} = 1{,}16985856$ $= R'$, which proceed with as in Exam. IX. and the Number of Diviſions will be 4, that is, 4 yrs. the Time required.

(10) Here $a = 680{,}1912$, $u = 100$, and $r = 1{,}05$.

Then per Theo. $\frac{680{,}1912 \times 1{,}05 + 100 - 680{,}1912}{100} =$

$\frac{814{,}20076 - 680{,}1912}{100} = \frac{340{,}0956}{100} = 3{,}400956 = R'$, which proceed with as directed in the laſt Example, will give 7 yrs. the Anſwer.

(11) Here $a = 536{,}37103$, $u = 70$, and $r = 1{,}03$.

Then per Theo. $\frac{536{,}37103 \times 1{,}03 + 70 - 536{,}37103}{70} =$

$\frac{622{,}46219 - 536{,}37103}{70} = \frac{86{,}09113}{70} = 1{,}2298733 =$ R', which proceed with as before directed, will give 7 yrs. the Anſwer.

(12) Here $a = 1548{,}680257$, $u = 30$, and $r = 1{,}035$.

Then per Theo. $\frac{1548{,}68025 \times 1{,}035 + 30 - 1548{,}680257}{30}$

$= \frac{1632{,}884066 - 1548{,}680257}{30} = \frac{84{,}203809}{30} =$ $2{,}8067936 = R'$, which Number look for under $3\frac{1}{2}$ per Cent. and you will find it to be even with 30 yrs. the Anſwer. (ſee Table I.)

Theorem VIII. $\frac{ar}{u} - r^t = \frac{a-u}{u} = R^t$.

(13) Here is given $a=297,25248$, $u=70$, and $t=4$.
Then per Theorem, see the Appendix: by the Table, thus $297,25248 \div 70 = 4,246464 = R^t$. which in Table III. even with 4 yrs. is under 4 per Cent. the Answer.

(14) Here $a=680,1912$, $u=100$, and $t=6$.
Then per Table $680,1912 \div 100 = 6,801912 = r^t$. even with 6 yrs. you will find it to be under 5 per Cent.

(15) Here $a=536,37103$, $u=70$, and $t=7$.
Then $536,37103 \div 70 = 7,662729 = R^t$. then in Table III. even with 7 yrs. you will find it to be under 3 per Cent. the Answer.

(16) Here $a=1548,680257$, $u=30$, and $t=30$.
Then $1548,680257 \div 30 = 51,6226752 = R^t$. which even with 30 yrs. you will find it to be under $3\frac{1}{2}$ per Cent. the Answer.

81. PRESENT WORTH OF ANNUITIES.

Theorem. IX. $u - \frac{u}{r^t} : \div r - 1 = P$. the present Worth.

(2) Here is given $u=60$, $t=6$, and $r=1,04$. Which being involved to the 6th Power $= 1,265319$, per Table I.

Then per Theo. $60 - \frac{60}{1,265319} : \div 1,04 - 1 = \frac{60 - 47,41096}{,04} = \frac{12,58904}{,04} = 314,726 = 314$ £. 14 *s.* $6\frac{1}{4}$ *d.* the Answer.

(3) Here $u=1000$, $t=21$, and $r=1,045$. Which being involved to the 21st Power $=2,5202411$, per Table I.

Then per Theorem, $1000 - \frac{1000}{2,5202411} : \div 1,045 - 1 = \frac{1000 - 396,787434}{,045} = \frac{603,2125657}{,045} = 13404,72386 = 13404$ £. 14 *s.* $5\frac{3}{4}$ *d.* the Answer.

Theo-

Theorem X. $\frac{\overline{prt} \times r - prt}{r^t - 1}$ = U, the Annuity.

(4) Here is given $p = 323{,}1608$, $t = 8$, and $r = 1{,}05$; which being involved to the 8th Power $= 1{,}4774554$, per Table I.

Then per Theorem,

$$\frac{\overline{323{,}1608 \times 1{,}4774554 \times 1{,}05} - 323{,}1608 \times 1{,}4774554}{1{,}4774554 - 1}$$

$= \frac{501{,}3389 - 477{,}46567}{,4774554} = 50$£. the Annuity.

(5) Here $p = 314{,}726$, $t = 6$, and $r = 1{,}04$; which being involved to the 6th Power $= 1{,}265319$.

Then per Theorem,

$$\frac{\overline{314{,}726 \times 1{,}265319 \times 1{,}04} - 314{,}726 \times 1{,}265319}{1{,}265319 - 1} =$$

$\frac{414{,}157939 - 398{,}228785}{,265319} = \frac{15{,}92815}{,265319} = 60$£. the Annuity required.

(6) Here $p = 13404{,}72386$, $t = 21$, and $r = 1{,}045$; which being involved to the 21st Power $= 2{,}5202411$, per Table I.

Then per Theorem,

$$\frac{\overline{13404{,}72386 \times 2{,}5202411 \times 1{,}045} - \overline{13404{,}7236 \times 2{,}5202411}}{2{,}5202411 - 1} = \frac{353{,}0337712 - 337{,}83136}{1{,}5202411} = \frac{1520{,}2411}{1{,}5202411}$$

$= 1000$£. the Annuity.

Theo. XI. $\frac{u}{p + u - pr} = R^t$.

(7) Here is given $u = 50$, $p = 323{,}1608$, and $r = 1{,}05$.

Then per Theorem, $\frac{50}{323{,}1608 + 50 - 323{,}1608 \times 1{,}05}$

$= \frac{50}{33{,}84196} = 1{,}4774554 = R^t$. which proceed with as in Example III. and the Number of thoſe Diviſions will be 8, that is, 8 yrs. the Anſwer.

(8) Here $u=60$, $p=314{,}726$, and $r=1{,}04$.

Then per Theorem, $\dfrac{60}{314{,}726+60-314{,}726\times1{,}04}=$

$=\dfrac{60}{374{,}726-327{,}315}=\dfrac{60}{47{,}411}=1{,}265319=Rt.$ which proceed with, as in the last Example, and the Number of Divisions will be 6, which is 6 Years, the Answer.

(9) Here $u=1000$, $p=13404{,}72386$, and $r=1{,}045$.

Then per Theorem,

$$\frac{1000}{13404{,}72386+1000-13404{,}72386\times1{,}045}=$$

$$\frac{1000}{13404{,}72386-14007{,}936433}=\frac{1000}{396{,}787426}=$$

$2{,}520241\,1=Rt.$ which in Table I. under $4\frac{1}{2}$ per Cent. is even with 21 Years, the Answer.

Theorem XII. $\dfrac{u}{p}=\dfrac{u}{p}Rt.+Rt.-Rt.+1.$

To perform the three following Examples by the Theorem, see the Appendix.

(10) Here is given $u=50$, $p=323{,}1608$, and $t=8$. Then by the fifth Table, thus

First $323{,}1608\div50=6{,}463216$. Then even with 8 Years you will find it under 5 per Cent. the Answer.

(11) Here $u=60$, $p=314{,}726$, and $t=6$.

Then $314{,}726\div60=5{,}24543$, which Number look for in Table V. even with the given Time, and the nearest Number is under 4 per Cent. the Answer.

(12) Here $u=1000$, $p=13404{,}72386$.

Then $13404{,}72386\div1000=13{,}40472386$, which Number even with the given Time, you will find under $4\frac{1}{2}$ per Cent. the Answer.

82. ANNUITIES, LEASES, &c. TAKEN IN REVERSION.

Theorem I. $u-\dfrac{u}{r}:\div r-1=p$, which change to A.

Then $\dfrac{A}{Rt.}=p.$

(2) Here is given $u=1000$, $t=20$, and $r=1,05$. Which involved to the 20th Power $=2,6532977$ per Table I.

Then per Theo. I. $\overline{1000-\frac{1000}{2,6532977}}:\div\overline{1,05-1}=$ $\overline{1000-376,88948}\div,05=623,11052\div,05=1246,22104=p$.

Now per Rule II. $a=124622104$, $t=5$, and r as before; which being involved to the 5th Power $=1,276816$.

Then per Theo. II. $\frac{1246,22104}{1,2762816}=9767,60175=9767$ £. 12*s*. ,035.

Theorem, first $pr^t=A$, which change to P.

Then $\frac{pr^t\times r-pr^t}{r^t-1}=U$, the Annuity.

(3) Here $p=368,30419$, $t=2$, and $r=1,05$; which being involved to the 2d Power $=1,1025$ per Table I.

Then per Theo. I. $368,30419\times1,1025=406,05536=A$.

Now per Rule II. $p=406,05536$, $t=6$, and r as before; which being involved to the 6th Power $=1,3400956$.

Then per Theorem II.

$$\frac{\overline{406,05536\times1,3400956\times1,05}-\overline{406,05536\times1,3400956}}{1,3400956-1}$$

$=\frac{571,360652-544,153002}{,3400956}=\frac{272,0765}{,3400956}=80$ £. the Annuity required.

(4) Here $p=9767,60175$, $t=5$, and $r=1,05$; which being involved to the 5th Power, $=1,2762816$.

Then per Theorem I. $9767,60175\times1,2762816=12466,2103896=A$.

Now per Rule II. $p=12466,2103896$, $t=20$, and r as before; which being involved to the 20th Power $=2,6532977$ per Table I.

Then per Theorem II.

$$\frac{\overline{12466,2103896\times2,6532977\times1,05}-\overline{12466,2103896\times2,6532977}}{2,6532977-1}=\frac{34730,30572220-33076,56735457}{1,6532977}$$

$=\frac{1,65382836}{1,6532977}=1000$ £. the Annuity.

83. REBATE OR DISCOUNT.

Theorem XIII. $\frac{s}{r^t}=$P, the present Worth.

(2) Here $s=743,2375$, $t=6$, and $r=1,04$; which being involved to the 6th Power $=1,265319$.

Then per Theo. $743,2375\div1,265319=587,391005=$ 587£. 7s. 9¼d. the Answer.

Theorem XIV. $pr^t=$S, the Sum owed.

(3) Here $p=123,405208$, $t=4$, and $r=1,05$; which being involved to the 4th Power $=1,2155063=r^t$.

Then per Theorem, $123,405208\times1,2155063=150$ £. the Debt.

(4) Here $p=587,3391005$, $t=6$, and $r=1,04$; which being involved to the 6th Power $=1,265319$, per Table I.

Then per Theorem, $587,391005\times1,265319=743,237=$ 743£. 4s. 9d. the Answer.

Theorem XV. $\frac{s}{p}=r^t$.

(5) Here $p=123,405208$, $s=150$, and $r=1,05$.

Then per Theorem, $150\div123,405208=1,2155063=r^t$; which being continually divided by 1,05 till nothing remains, and the Number of those Divisions will be 4, that is, 4 Years, the Answer.

(6) Here $p=587,3391005$, $s=743,235$, and $r=1,04$.

Then per Theorem, $743,235\div587,3391005=1,265319$; which proceed with as directed in the last Example, and the Number of Divisions will give 6 Years, the Answer.

Theorem XVI. $\frac{s}{p}=$R^t.

(7) Here $s=150$, $p=123,405208$, and $t=4$.

Then per Theo. $\frac{150}{123,405208}=1,2155063=\overset{4}{r}$.

$\because \sqrt[2]{1,2155063}=1,1025$, and $\sqrt[2]{1,1025}=1,05$ or 5 per Cent. the Answer.

(8) Here $s=743{,}235$, $p=587{,}3391005$, and $t=6$.

Then per Theo. $743{,}235 \div 587{,}3391005 = 1{,}265319 = \overset{6}{r}$.

Then $\sqrt[2]{1{,}265319} = 1{,}124864$.

And $\sqrt[3]{1{,}124864} = 1{,}04$ or 4 per Cent. the Answer. (See the Guide, page 208.)

84. PURCHASING FREEHOLD, OR REAL ESTATES.

Theo. XVII. $\frac{u}{r-1} = \text{P}$, the present Worth.

(2) Here $u=25$, and $r=1{,}045$.

Then per Theo. $\frac{25}{1{,}045-1} = \frac{25}{{,}045} = 555{,}5 = 555$£. 11*s*. $1\frac{1}{4}$*d*. ,28, the Answer.

Theo. XVIII. $\frac{\text{P}+u}{p} = \text{R}$, the Rate per Cent.

(3) Here $p=10000$, and $u=500$.

Then per Theo. $\frac{10000+500}{10000} = \frac{10500}{10000} = 1{,}05$, or 5 per Cent. the Answer.

(4) Here $p=555{,}5$, and $u=25$.

Then per Theo. $\frac{555{,}5+25}{555{,}5} = \frac{580{,}5}{555{,}5} = 1{,}045$ or $4\frac{1}{2}$ per Cent. the Answer.

Theo. XIX. $\text{P} \times \overline{r-1} = \text{U}$, the Annuity.

(5) Here $p=10000$, and $r=1{,}05$.

Then per Theo. $10000 \times \overline{1{,}05-1} = 10000 \times {,}05 = {,}500$£. the Answer.

(6) Here $p=555{,}5$, and $r=1{,}045$.

Then per Theo. $555{,}5 \times \overline{1{,}045-1} = 555{,}5 \times {,}045 = 25$£. the Answer.

85. PUR-

85. PURCHASING FREEHOLD ESTATES IN REVERSION.

Theorem XX. first $\frac{u}{r-1}=P$, which change to A.

Then $\frac{a}{R^t}=P$, the present Worth.

(1) Here $u=500$, and $r=1,05$.

Then per Theo. I. $\frac{500}{1,05-1}=\frac{500}{,05}=10000=P$, which change to a.

Then per Theo. II. $a=10000$, $t=4$, and r as before; which involved to the 4th Power $=1,2155063$.

Then per Theo. II. $\frac{10000}{1,2155063}=8227,024=8227£.\ 5\frac{1}{4}d.$ the Answer.

Theorem XXI. first $pr^t=A$, which change to P.

Then $\frac{pr^tr-pr}{r}=U$, the Annuity.

(2) Here $p=8227,024$, $t=4$, and $r=1,05$; which being involved to the 4th Power $=1,2155063$.

Then per Theo. I. $,8227,024\times1,2155063=10000,=A$.

Now (per Rule II.) $p=10000$, and r as before.

Then per Theo. II. $\frac{10000\times1,05\times1,05-10000\times1,05}{1,05}$

$=\frac{11025-10500}{1,05}=\frac{525,00}{1,05}=500£.$ the Answer.

86. QUESTIONS FOR EXERCISE.

(1) First $486,5£.-94=392,5£.$ the Annuity.

Now the present Worth of 1£. for 7, 14, and 21 Yrs. at 5 per Cent. is 5,7863737, 9,8986409, and 12,8211527. (per Table V.)

Then

$$\left.\begin{matrix}5,7863737\\9,8986409\\12,8211527\end{matrix}\right\}\times392,5=\left\{\begin{matrix}2271,1507\\3885,2165\\5032,30243\end{matrix}\right.\begin{matrix}=2271\ 3\ 0\\=3885\ 4\ 4\\=5032\ 6\ 0\end{matrix}\left.\vphantom{\begin{matrix}a\\b\\c\end{matrix}}\right\}\text{P. W. for}\left\{\begin{matrix}7\\14\\21\end{matrix}\right.$$

(column heads: £. s. d. — Yrs.)

(2) First $15+5=20$ yrs. and 186£. 7s 6d. $=186,375£.$

Then the Worth of 1£. for 20 yrs.	=	12,4622103
And ditto for 15 yrs.	=	10,3796580
	Diff.	2,0825523

∴ $2,4719385 \times 186,375 = 460,70753 = 460$ £. 14*s.* $1\frac{1}{2}$*d.* the Fine required.

(3) First $7+14+10=31$ Years.
Now the present Worth of 1£. at 5 per Cent. for 31, 21, and 7 yrs. is 5,7863734, 12,8211527, and 15,5928104.

Then

$$\left.\begin{matrix}15,5928104\\12,8211527\\5,7863734\end{matrix}\right\}\times 50 = \left\{\begin{matrix}779,64052\\641,057635\\289,31867\end{matrix}\right\}\text{Present Worth of 50£. for}\left\{\begin{matrix}31\\21\\7\end{matrix}\right.\ \textit{Years.}$$

	£.	*s.*	*d.*		
∴ 289,31867	= 289	6	$4\frac{1}{2}$	Val. of	A's.
Also 641,057635 — 289,31867	= 351	14	$9\frac{1}{2}$	Val. of	B's.
And 779,64052 — 641,057635	= 138	11	$7\frac{3}{4}$	Val. of	C's.

(4) By Table V. the present Worth of 300£. per Annum for 7 Years at 4 per Cent. is $6,002547 \times 300 = 1800,61641$.

Then	£.	*s.*	*d.*			
$1800,61641+150=$	1950	12	4	Value of	A's	Offer.
$6,002547+250+400=$	1900	10	$3\frac{1}{4}$	Value of	B's	Offer.
$6,002547\times 200+650=$	1850	8	$2\frac{1}{2}$	Value of	C's	Offer.
	1800	0	0	Value of	D's	Offer.

Hence it appears that A's offer is better by above $\left\{\begin{matrix}50\\100\\150\end{matrix}\right\}$ than $\left\{\begin{matrix}\text{B's.}\\\text{C's.}\\\text{D's.}\end{matrix}\right.$

(5) First $54,85-7,35=47,5$£. the Annuity.
Now as the Payment is Half-yearly, we shall have $u = 475 \div 2 = 23,75$, $t=6$, and $r=1,05$; (see the Guide p. 223) which raised to the 6th Power $=5,0756921$, per Table V.
∴ $5,0756291 \times 23,75 = 120,54768735 = 120$£. 10*s.* $11\frac{1}{2}$*d.* the Answer.

(6) First $153-50=103$£. for 12 yrs. at 5 per Cent.
Then per Table VI. 1£. will purchase for 12 yrs. at 5 per Cent. an Annuity of ,1128254 per Annum.
Then $,1128254 \times 103 = 11,6210162 = 11$£. 12*s.* 5*d.* advance Rent.
∴ 16£. + 11£. 12,5 = 27£. 12*s.* 5*d.* the Answer.

(7)

(7) First 44—20=24£. and 1,9487171=Amount of 1£. for 7 Years. = 250£. 0s. 0d.

Then 1,9487171 × 24 = 46,9292104 =	46	18	7
At 10£. per Annum for 7 yrs. =	70	0	0
Answer	366	18	7

(8) First 33,3̇ × 50 = 166,6,6̇ = 166£. 13s. 4d. Value of the Annuity.

Then per Table VI. 1£. for 12 yrs. at 4 per Cent. will purchase ,1065522, per Annum.

∴ 1666,6̇ × ,1065522 = 177,587 = 177£. 11s. 8¾d. the Answer.

(9) First 237—10=227£. the Annuity.

Then the present Worth for 1£. for 4, 12, and 14 Years, is 3,5459505, 8,8632516, and 9,8986409, per Table V.

∴

				£.	s.	d.
3,5459505	× 227 =	804,9307635	=	804	18	7¼
8,8632516		2011,9581132	=	2011	19	1¾
9,8986409		2246,9914843	=	2246	19	9¾
		Sum	=	5063	17	6¾
		Discount 4+12+14×5=30×5	=	150	0	0
		Answer, received		4913	17	6¾

87. SUPERFICIAL MEASURE.

EXAMPLES.

PROBLEM I.

	Ft.	′
(1) Multiply	17	7
by		6
Answer	105	6

	Ft.	′	″
(2) Multiply	47	8	0
by		8	4
	15	10	8
	381	4	0
Answer	397	2	8

(3)

(3) Multiply 7 10 0 by 8 6

	Ft.	′	″
Multiply	7	10	0
by		8	6
	3	11	0
	62	8	0
Answer	66	7	0

(4) Multiply 64 7 0 by 4 8

	Ft.	′	″
Multiply	64	7	0
	×4	8	
	43	0	8
	258	4	0
Answer	301	4	8

(5) Mult. 12 8 9 0 0 by 9 6 7

	Ft.	′	″	‴	⁗
Mult.	12	8	9	0	0
		×9	6	7	
		7	5	1	3
	6	4	4	6	0
	114	6	9	0	0
Answer	121	6	6	7	3

(6) Mult. 9 11 6 0 by 11 8

	Ft.	′	″	‴
Mult.	9	11	6	0
		×11	8	
	6	7	8	0
	109	6	6	0
Answer	116	2	2	0

(7)

		Ft.	′	″	‴
6	½	64	0	10	0
				14	9
3	½	32	0	5	0
		16	0	2	6
		896	11	8	0
Answer		945	0	3	6

(8)

		Ft.	′	″	‴
6	½	124	4	0	0
4	⅓		42	0	9
	½		5	2	2
			2	7	1
			14	0	0
			248	0	0
			496	0	0
Answer			5229	9	3

(9)

	Ft.	′	″	‴	⁗
	16	7	10	0	0
			6	5	7
	0	9	8	6	10
	6	11	3	2	0
	99	11	0	0	0
Ans.	107	7	11	8	10

(10)

		Ft.	′	″		
6′	½	474	6	8		
6	½	186	7	4		
1	⅙	237	3	4	0	0
4″	⅓	39	6	6	8	0
		13	2	2	2	8
		93	0	0	0	0
8	⅑	10	4	0	0	0
		2844	0	0	0	0
		8532	0	0	0	0
Answer		88557	4	0	10	8

		Ft.	′	″	‴	⁗	v
(11) 6′	½	24	11	8	6	7	0
				8	6	7	
		1	2	6	9	10	1
		12	5	10	3	3	6
		199	9	8	4	8	0
Answer		213	6	1	0	11	4

		Ft.	′	″	‴	⁗	v	vi
(12) 4	⅓	46	6	8	4	0	0	0
					6	4	8	6
″ 6	⅓	15	6	2	10	0	0	0
2	⅓	1	11	3	4	3	0	0
		0	7	9	1	5	9	0
		0	1	11	3	4	2	0
		279	4	2	0	0	0	0
Answer		297	7	4	7	0	11	0

PROBLEM II.

(1) Here A B=14,5, and A C=9,75 Chains, per Fig. 2. Then per Theo. 14,5×9,75=131,375=131 a. 1 r. 20 p. the Area, required.

(2) Here A B=A C=245 yds. per Fig. 1. Then 245×245=60025 Acres, or yds.

∵ 4840)60025(12,4=12 A. 1 r. 24 p.

(3) Here A B=64 ft. 6 inc. and A C=47 ft. 8 inc. see Fig. 3.

Then per Problem I. 64 ft. 6 inc. ×47 ft. 8 inc.=3074 ft. 6 inc.

∵ 9)3074 ft. 6 inc. (=341 yds. 5 ft. 6 inc. the Answer.

PROBLEM III.

(4) Here A B=28, and C D=20,5 Chains, see Fig. 4.

Then per Theo. $\frac{28\times 20,5}{2}=14\times 20,5=287$ Acres.

(5) Here A B=738, and C D=583 Links.

Then $\frac{738\times 583}{2}=\frac{430254}{2}=215127$ Links, or 2,15127 Acres.

∵ As 2,15127 : 12£. :: 1 : 5,5781=5£. 11*s.* 6¾*d.* the Answer.

(6) Here $\frac{a+b+c}{2}=\frac{400+348+312}{2}=\frac{1060}{2}=530$ $=s.$ Then per Rule.

$$530-\left\{\begin{array}{l}400=130=e.\\348=182=f.\\312=218=g.\end{array}\right\}\text{ the three Remainders.}$$

∵

$\because$ $530 \times 130 \times 182 \times 218 = 2733676400$.

And $\sqrt[2]{2733676400} = 52284,5$.

Then $4840)52284,5(10,8 = 10$ acr. 3 r. 8 p. the Answer.

PROBLEM IV.

(7) Here $AC = 45$, $BF = 17,25$, and $DE = 14$, see Fig. 5.

Then per Theo. $\frac{17,25 \times 14}{2} \times 45 = \frac{31,25}{2} \times 45 =$ $15,625 \times 45 = 703,125$ ft. these $\div 9 = 78$ yds. 1 ft. 18 inc. the Answer.

Or thus,

RULE.

If the Trapezium can be inscribed in a Circle, that is, if the Sum of any two opposite Angles be equal to two right Angles, or 180°; then multiply any two adjacent Sides together, and the other two Sides together, and multiply by the Sum of these Products, half the Sine of the Angle included by either of the Pairs of Sides which are multiplied together, so shall this last Product be the Area.

Thus $\frac{AD \times DC + AB \times BC \times s.\angle A, \text{ or } s.C}{2} =$ the Area

EXAMPLE.

If the Sides be $AD = 4$, $DC = 7,5$, $AB = 6$, $BC = 5,5$, and the Angle $C = 74^\circ\ 40\frac{1}{4}'$.

Therefore the Angle $A = 180^\circ - 74^\circ\ 40\frac{1}{4}' = 105^\circ\ 19\frac{3}{4}'$.

Then by Rule $\frac{4 \times 6 + 7,5 \times 5,5}{2} \times ,9644229$ ($s.$ 74° $40\frac{1}{4}'$) $= 31,46429+$ Square Feet, the Area.

Or thus,

RULE.

From half the Sum of the four Sides subtract each Side severally: multiply the four Remainders continually together, and the square Root of the last Product will be the Area.

Thus $\sqrt{\frac{a+b+c-d}{2} \times \frac{a+b+d-c}{2} \times \frac{a+d+c-b}{2} \times \frac{b+c+d-a}{2}} =$ the Area.

 EX-

EXAMPLE.

Suppose the Sides be $AB=a=15,6$, $BC=b=13,2$, $CD=c=10$, and $AD=d=26$, what is the Area?

Then per Rule, first $\frac{15,6+13,2+10+26}{2}=\frac{64,8}{2}=32,4$.

$\therefore \sqrt{\overline{32,4-15,6}\times\overline{32,4-13,2}\times\overline{32,4-10}\times\overline{32,4-26}}$ $=\sqrt{16,8\times19,2\times22,4\times6,4}=21,504$, the Area required. (See Quest. 25, in Algebra.)

PROBLEM V.

(8) Here $AB=48$, $CD=41,57$, and $n=5$.

Then per Theo. $\frac{48\times41,57\times5}{2}=\frac{9976,8}{2}=4988$, 4 ft. the Area required.

(9) First the Multiplier for an Hexagon is 2,598076 by the Table.

Then per Rule II. thus $2,598076\times30\times30=2338,2684$, the Area.

(10) Here the Multiplier for an Octagon is 4,828427 by the Table.

Then $4,828427\times24\times24=2781,173952$, the Area.

PROBLEM VI.

(11) Thus $3,1416\times7=21,9912$, or rather 22 the Circumference required. (per Rule I.)

(12) Thus $22\div3,1416=7$ (nearly) the Diameter, (per Rule II.)

(13) Thus $3,1416\times8000=25132,8$ Miles, the Diameter required.

PROBLEM VII.

(14) Here $C=6,2832$, $D=\frac{6,2832}{3,1416}=2$.

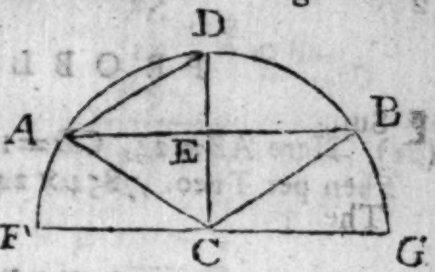
Then per Theo. $\frac{6,2832}{2}\times\frac{2}{2}=\frac{6,2832}{2}=3,1416$, the Area. (per Rule I.)

(15) First $12\times12=144$, Square of the Diameter.

Then $,7854\times144=113,0976$, the Area. (per Rule II.)

PRO-

PROBLEM VIII. Fig. 22.



(16) Here DE=9, ADB = 29,5, and b=,01745329.
Then per Theorem ,01745329 × 29,5 × 9 = 4,633848495, the Length of the Arch required.

PROBLEM IX.

(17) Here AC=55, and AB=59; see Fig. 9.

Then per Theo. $55 \times \frac{59}{2} = 55 \times 29{,}5 = 1622{,}5$, the Area required.

PROBLEM X.

(18) First ,7854 × 18 × 18=254,4696, the Area of the whole Circle. (per Prob. VII.)

$\because \frac{254{,}4696}{4} = 63{,}6174$, Area of the Quadrant CADB; See Fig. 22.

And $\frac{9 \times 9}{2} = \frac{81}{2} = 40{,}5$, Area of the Triangle ABC. (per Prob. III.)

Then from 63,6174 take 40,5, remains 23,1174, the Area of the Segment ABD.

PROBLEM XI.

(19) Here AB = 84, CD = 72,5, and AC = 3,5; see Fig. 10.

Then per Theo. $\frac{84+72{,}5}{2} \times 3{,}5 = \frac{156{,}5}{2} \times 3{,}5 = 78{,}25 \times 3{,}5 = 273{,}875$, the Area required.

(20) First, as 7 : 22 :: 24 : 75,4, which ÷2=37,7, the greatest Semicircle AB.

Again, as 7 : 22 :: 16 : 50,2, which ÷2=25,1, the lesser CD.

Also $\frac{24-16}{2} = \frac{8}{2} = 4$ = AB; see Fig. 10.

Then $\frac{37{,}7+25{,}1}{2} \times 4 = \frac{62{,}8}{2} \times 4 = 31{,}4 \times 4 = 125{,}6$, Area required.

PROBLEM XII.

(21) Here AB$=24$, CD$=18$, and $b=,7854$.
Then per Theo. $,7854\times24\times18=339,2928$, the Area.

88. Of ARTIFICERS WORK.

(1)

	Ft.	′	″
	3	10	0
		×2	8
	2	6	8
	7	8	0
	10	2	8
		×	12
	122	8	0 Content.

		Ft.	′	″	
6*d*	$\frac{1}{40}$	122	8	0	at $8\frac{1}{2}d.$
2	$\frac{1}{2}$	3	1	0	
$\frac{1}{2}$	$\frac{1}{4}$	1	0	4	
		0	5	1	
6′	$\frac{1}{2}$	0	0	$4\frac{1}{2}$	$=\frac{1}{8}$ of 8*d.*
2	$\frac{1}{3}$	0	0	$1\frac{1}{2}$	
£.		4	6	$10\frac{1}{2}$	Value.

(2)

	Ft.	′
	6	6
	5	3
	4	9
Sum	16	6
		×4
	66	0
	3	9
	247	6 Content.

		Ft.	′	
4*d*	$\frac{1}{3}$	247	6	at 1*s.* 4*d.*
		82	4	
6′	$\frac{1}{2}$	0	8	$=\frac{1}{2}$ of 16*d.*
	2,0)	33,0	0	
£.		16	10	Answer.

By Decimals, thus $6,5+5,25+4,75=16,5$, Sum of their Heights; which $\times4$, (the Number of Windows in a Tier) $=66$ Heights together.
Then $66\times3,75$ (the Breadth) $=247,5$ the Area.
Now 16*d.* $=,0\dot{6}$£. $\therefore$ $247,5\times,0\dot{6}=16,499$, or 16£. 10*s.* as before.

(3)

	Ft. ′ ″		Ft. ′ ″
(3)	6 6 0		21 1 6 at 8*s.*
	× 3 3		× 4
	1 7 6		8 8 0
	19 6 0	1′ $\frac{1}{12}$	0 0 8 = $\frac{1}{12}$ of 8*s.*
		6″ $\frac{1}{2}$	0 0 4
	21 1 6 Content.		8 9 0 £. Anſwer.

By Decimals, thus 3,25 × 6,5 = 21,125 ft.
∵ 21,125 × ,4 = 8,45 = 8£. 9*s.* as before.

(4) Firſt 16 × 2 = 32 the Length; and 9 × 3 = 27 the Breadth.
Then 32 × 27 = 864, ſquare Inches.
∵ 144)864(6 ft. the Content required.

	Ft. ′ ″	For the odd Parts, thus			
(5)	176 6 0				
	× 56 9 0	3 Ft.	$\frac{1}{3}$	6*d.*	
	88 3 0	1		2	
	44 1 6			2	
	28 0 0			$0\frac{1}{2}$	$\frac{2}{3}$
	1056 0 0	4′	$\frac{1}{3}$	$0\frac{1}{2}$	$\frac{2}{3}$
	880 0 0	6″	$\frac{1}{8}$		$\frac{2}{3}$
	9)10016 4 6			$5\frac{1}{2}$	
6*d.* $\frac{1}{40}$	1112 8 4 6 at 6*d.*				
	27 16 0				
	+ $5\frac{1}{2}$				
	27 16 $5\frac{1}{2}$ £. the Anſwer.				

By Decimals, thus 175,5 × 56,75 = 10016,375 Feet.
Then 9)10016,375(1112,93 yds. at 6*d.* per yd.
∵ 6*d.* $\frac{1}{40}$)1112,93(27,823 = 27£. 16*s.* $3\frac{1}{2}$*d.* the Anſwer, as before.

 (6)

	Ft.	′	″
(6)	9	6	0
	×8	3	
	2	4	6
	76	0	0
	9)78	4	6

yds. 8 6 4 6 at 6*s.*
×3

2	8	0
0	4	3

£. 2 12 3 the Anſwer.

For the odd Parts, thus

3 *Ft.*	$\frac{1}{3}$	6*s.*			
		2			
3	$\frac{1}{3}$	2			
4′	$\frac{1}{9}$	0	$2\frac{1}{2}$	$\frac{2}{3}$	
6″	$\frac{1}{8}$	0	$0\frac{1}{4}$	$\frac{1}{3}$	
		4	3		

By Decimals, thus 9,5 × 8,25 = 78,375 Feet.
Then 9)78,375(8,7083 yds. at 6*s.* or 3£.
∵ 8,7083 × ,3 = 2,6125 = 2£. 12*s.* 3*d.* the Anſwer.

			Ft.	In.	
(7)	Firſt 84 ft. × 9 ft. 6 inc.	=	798	0	Room
	And $\left\{\begin{array}{l} 6 \times 3{,}5 = 61{,}6 \\ 4 \times 4 - = 16 \end{array}\right\}$	=	77	6	Win. and Chim.
			9)720	6	Diff.
			80 yds.	6 inc.	

∵ 80 yds. 6 inc. × 2 = 160 yds. 1 ft. the Anſwer.

		Ft.	Inc.		
(8)	Firſt 47 ft. 7 inc. — 4 ft. =	43	7		Breadth.
		×47	7		Length.
		9)2073	10	1	

6*d.*	$\frac{1}{40}$	yds. 230	3	10	1	at 6*d.* per yd.	
3 ft.	$\frac{1}{3}$	5	15	0			
6 in.	$\frac{1}{6}$			2		$= \frac{1}{3}$ of 6*d.*	
4	$\frac{1}{9}$			$\frac{1}{4}$	$\frac{1}{3}$	$= \frac{1}{6}$ of 2*d.*	
					$\frac{2}{3}$	$\div \frac{1}{9}$ of dit.	
		£. 5	15	$2\frac{1}{2}$		the Anſwer.	

(9)

Ft. In. Ft. In. — *Ft. In. Ft.*

(9) 129 6 × 16 3 = 2104 4 6 Room.

7 × 3 9 = 13 1 6
2
7 3 × 4 6 = 32 7 6 } ½ Doors and Shutters.

8,6 + 46 × 2 = 26 × 1,2, × 2 = 60 8 0 Cheek-board.

14 + 3,9 = 17,9 × 1,4 = 23 8 0 Door-linings.

Ft. In. Ft. — 2234 5 6

To be Deducted 3 9 × 3 = 11 3 0 Chimney.

9)2223 2 6 Wood-work.

s. d.

2 6 | ⅛ | yds. 247—2 6″ at 3*s.* 6*d.*

1 0 | $\frac{1}{20}$ | 30 17 6
12 7 0 } = 43£.4*s*.6*d*. the Cost.

Yds. Ft. — *d.* *Yds. Ft.*

(10) 34 6 — 6 | $\frac{1}{40}$ | 76 6 at 10½*d.* per yd.
× 20
9)690 0 — 3 | ½ | 1 18
76 6 — 1½ | ½ | 19
ft. | | 9 6
3 | ⅓ | 0 3½ = ⅓ of 10½*d.*
3 | ⅓ | 3½

£. 3 7 1 the Answ.

Ft. In.

(11) 6 ½ 34 8
× 14 6
17 4
485 4
9)502 8
⅕ 55 yds. 7 ft. 8 inc. the whole Area.
11 1 6⅖

					£.	s.	d.
Plaistered	44	6	1⅗ at 8*d.* per yd. =		1	9	9¼
Whited	66	9	2⅕ at 2*d.* per yd. =			11	4½
				Answer £.	2	1	1¾

(12)

			Ft.	In.		
(12)	6'	$\frac{1}{2}$	120	6		=
				12	9	
	3	$\frac{1}{2}$	60	3	0	
			31	1	6	
			1446	0	0	
				Sq.Ft.	In.	''
		1,00)	15,36	4	6(15,36	4 6 the Anſwer.

(13) First $28 \times 20 = 560$
And $14 \times 10 \times 2 = 280$ } $= 280$ the Diff.

1,00)840 Sum.

8,4 ſq. at 2£. 5*s*. or 2,25£.

∵ $2,25 \times 8,4 = 18,9 =$ 18£. 18*s*. the Anſwer.

(14) $30,5 \times 20,5$ ⎫ ⎧ 2501 Area of the 4 Floors.

$5,25 \times 6$ ⎬ $\times 4 =$ ⎨ 126 = dit. of 4 Fire-places.
$4,25 \times 4$ ⎪ 68 = dit. of ditto.
$8,5 \times 10$ ⎭ ⎩ 340 = dit. of a Well-hole.

The whole Deductions = 534

1,00)19,67 ſq. at 8,5£.

∵ $19,67 \times 8,5 = 167,195 =$ 167£. 3*s*. 10$\frac{3}{4}$*d*. the Anſwer.

(15) Firſt $1,25 \times 15 = 18,75$ Area of one Plank.
And $33,5 \times 60,5 = 2026,75$ ditto of the Room.
∵ 18,75)2026,75(108 Planks, (nearly.)

(16) Firſt $36 + \frac{36}{2} = 36 + 18 = 54$ Width of the Roof.

Then $64 \times 54 = 34,56$ Feet, the Area.
And 1,00)34,56(34,56 ſqrs. at 12*s*. 6*d*. or ,625£.
Or $34,56 \times ,625 = 21,6 =$ 21£. 12*s*. the Anſwer.

(17)

(17) Firſt 30 × 2 = 60, and 70 × 60 = 4200 ft. the Area.
Then 1,00)42,00(42 ſqr. at 10*s.* 8*d.* per ſqr.

$$\begin{array}{rrrl} & \textit{s.} & \textit{d.} & \\ \because & 10 & 8 & \\ & & \times 7 & \times 6 = 42. \\ \hline & 3 & 14 & 8 \\ & & & 6 \\ \hline \text{Anſwer £.} & 22 & 8 & 0 \end{array}$$

(18)

$$\begin{array}{lrl} & 22{,}75 & \multirow{2}{*}{\text{Flat and Half.}} \\ \textit{In.} \quad \textit{Ft.} & 11{,}375 & \\ 10 = {,}83 \times 2 = & 1{,}666 & = \text{Eves Boards.} \\ \hline & 35{,}7916 & \text{Whole Width.} \\ & \times 32{,}75 & \text{Length.} \end{array}$$

1,00)1172,177081(11,72177081 ſqr. at ,75£.

∵ 11,72177081 × ,75 = 8,7913281075 = 8£. 15*s.* 10*d.* (nearly) the Anſwer.

(19) Firſt 470 ft. × 9 ft. 6 inc. = 4465 ft.
And 4465 × 6 half Bricks = 26790.
Then per Rule I. 816)26790(32 rds. $229\frac{1}{3}$ ft. the Anſw.

(20) Firſt 840 × 9 = 7560 Feet.
× 5 Half Bricks thick.

Per Rule II. 3)37800

$$\begin{array}{lll} & \textit{Rds.} & \textit{Ft.} \\ 272)12600(& 46 & 88 \end{array}$$

Rds. *Ft.*
46 88 at 4£. 19*s.* 6*d.* per Rod.
× 4

$$\begin{array}{lllrrl} \text{£.} & \text{£.} & \textit{s.} & 184 & 0 & \text{at } 4\text{£.} \\ 46 - & 1 & 3 = & 44 & 17 & \text{at } 19\textit{s.}\ 6\textit{d.} \\ 68 & \frac{1}{4} & & 1 & 4 & 10\frac{1}{2} = \frac{1}{4} \text{ of } 4\text{£. } 19\textit{s.}\ 6\textit{d.} \\ 17 & \frac{1}{4} & & & 6 & 2\frac{1}{2} \\ 3 & \frac{1}{6} & & & 1 & 0\frac{1}{2} \text{ nearly.} \\ \hline & & \text{£.} & 230 & 9 & 1\frac{1}{2}\text{£. the Anſwer.} \end{array}$$

(21)

(21) First $38 \div 4 = 9,5$ Height of the Gable.
And $40 \div 3 = 13,\dot{3}$

$$\frac{24,5 \times 13,\dot{3} \times}{24,5 \times 13,\dot{3} \div 2} \left\{ \begin{matrix} 4 = 1306,66\dot{6} & \text{at } 2 \\ 3 = 980 & \text{at } 1\frac{1}{2} \\ 2 = 653,33\dot{3} & \text{at } 1 \\ = 166,375 & \text{at } \frac{1}{2} \end{matrix} \right\} \text{Bricks thick.}$$

$816)3106,375(3,8068$ Rods.

∵ $3,8068 \times 5,5 = 20,6 = 20£.\ 18s.\ 9d.$ nearly, the Answer.

QUESTIONS.

(1) First 14 ft. 3 inc. $= 171$ inc. and 3 square yds. $= 36 \times 36 = 1296$ Inches.
∵ $171)1296(7\frac{11}{19}$ Inches, the Answer.

(2) First a Foot square $= 144$ Inches.
Then $144 \div 27 = 5,\dot{3}$ Inches, Breadth of the first Cut.
And $58 - 5,\dot{3} = 52,\dot{6}$ remaining Length.
Also $144 \div 52,\dot{6} = 2,734$ Breadth of 2d Cut.
Then $27 - 2,734 = 24,266$ remaining Breadth.
$144 \div 24,266 = 5,934$ Breadth of 3d Cut.
$52,\dot{6} - 5,934 = 46,732$ remaining Length.
$144 \div 46,732 = 3,0814$ Breadth of the 4th Cut.
$24,266 - 3,0814 = 21,1846$ remaining Breadth.
$144 \div 21,1846 = 6,7974$ Breadth of the 5th Cut.
$46,732 - 6,7974 = 39,9346$ remaining Length.
$144 \div 39,9346 = 3,6059$ Breadth of the 6th Cut.
$21,1846 - 3,6059 = 17,5787$, remaining Breadth.
$144 \div 17,5787 = 8,1917$ Breadth of the 7th Section.
$39,9346 - 8,1917 = 31,7429$ remaining Length.
$144 \div 31,7429 = 4,5364$ Breadth of the 8th Cut
$17,5787 - 4,5364 = 13,0423$ remaining Breadth.
$144 \div 13,0423 = 11,0411$ Breadth of the 9th Section.
Then $31,7429 - 11,0411 = 20,7018$ remaining Length.
Also $144 \div 20,7018 = 6,956$ Breadth of the 10th Section.
∵ $13,0423 - 6,956 = 6,0863$ Breadth remaining at the last. *Q. E. F.*

(3) First $10584 \div 6 = 1764$.
Then $\sqrt[2]{1764} = 42$ long Rows.
∵ $42 \times 6 = 252$ short Rows, 42 in a Row.

(4) First $7 \times 2,5 = 17,5$ Inches, Area of an End.
And $17,5 \times 2 = 35$, double Area.
∵ $35 \div 3 = 11\frac{2}{3}$ Inches deep, the Anſwer.

(5) First $19 \times 11 = 209$, Area of an End.
Then $209 \div 4 = 52,25$ Area of an End of the Piece wanting.
∵ $52,25 \div 9 = 5,805$ Width required.

(6) First a yd. $= 36 \times 36 = 1296$ Inches. And 3*s.* 2*d.* = 38*d.*
Then, as 6*d.* : 1296 inc. :: 38*d.* : 8208 inc. Area of the whole Trough.
Alſo $102 \times 21 \times 2 =$ 4284 Area of the two Sides.
3924 Area of the Bot. and Ends.
Then $\overline{102 + 21} \times 2 = 144$.
∵ $3924 \div 144 = 27\frac{1}{4}$ Inches, the Breadth required.

(7) First $26 + \overline{40 \times 2} = 106$ Breadth of the Bot. and Sides.
Then $106 \times 74 = 7844$ Area of Bottom and Sides.
And $40 \times 26 \times 2 = 2080$ ditto of both Ends.
Alſo $26 \times 16 \times 3 = 1248$ ditto of the Stays.
∵ 144)11172(77,583 ſq. Feet.

$28\frac{1}{4}$ lb.)77,583(19,3958$\dot{3}$ cwt. at 1,1£.
∵ $19,3958\dot{3} \times 1,1 = 21,33541\dot{6} =$ 21£. 6*s.* $8\frac{1}{2}$*d.* Value of the Ciſtern.
Now 3*s* 6*d.* = ,175 £. Therefore $21,33541\dot{6} - ,175 = 21,16041\dot{6}$ Maſon's Bill.
7*d.* = ,0291$\dot{6}$)21,16021$\dot{6}$(725,5 ſqr. Feet, Shop.
22 ft. 10 inc. = 22,83)725,5(31,778 = 31 ft. $9\frac{1}{4}$ Inches, the Anſwer.

(8) First 27 ft. 112 inc. = 4000 ſq. Inches.
Then $20 \times 16 \times 2 = 640$ Area of the two Ends.
∵ $\overline{20 \times 2} + 16 =$ 56)3360 ditto of Bot. and Sides.
12) 60 Inches.
Anſwer 5 Feet.

Yds.

(9) First $26\frac{1}{2}$ ft. = 5,5
Then $5,5 \times 5,5 = 30,25$
And $6 \times 6 = 36$
Alſo $7 \times 7 = 49$
} 1 Per. { Statue, Cheſhire, Yorkſhire } Meaſure

∵ As

∵ As 30,25 : 110 :: 36 : 92 1 28 Cheſh. } Meaſure,
30,25 : 110 :: 49 : 67 3 25 Yorkſh. } the Anſ.

(10) Firſt $7 \times 4 = 28$ ſqr. Feet, Area that each Plant takes.
∵ $3584 \times 28 = 100352$ ſqr. Feet, in all.
Square Feet in an Acre $= 43560)100352(2,303764 =$ 2 acr. 1 rd. $8\frac{1}{2}$ pol. the Anſwer.

(11) Firſt 18*d.*$=,075)100(133,\dot{3}$ Feet, the Area.
Now ſuppoſe Fig. 4, in the Guide, to repreſent the Court.
Then $1333,\dot{3} \div \frac{88}{2} = 1333,\dot{3} \div 44 = 30,0\dot{3} =$ C D, the Perpendicular.
Then $44 \times 44 \quad = 1936$ Square of A D, or B D.
Alſo $30,0\dot{3} \times 30,0\dot{3} = 918,\dot{2}\dot{7}$ Square of D C.

∴ $\sqrt[2]{2854,\dot{2}\dot{7}} = 53,425 =$ A C, or B C.

And $53,425 \times 2 = 106,85$ the Sum required.

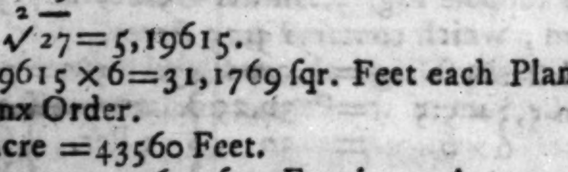
(12) Here the laſt mentioned Fig. may repreſent the form of the Bath.
Theu $3)125(41,\dot{6} =$ A B, or A C, or B C.
And $2)41,6(20,8\dot{3} =$ A D, or B D.
Then $41,\dot{6} \times 41,\dot{6} \quad = 1736,11\dot{1}$
Alſo $20,8\dot{3} \times 20,8\dot{3} = 434,028$

$\sqrt[2]{1302,083\dot{1}} = 36,084 =$ D C.

∴ $\frac{36,084}{2} \times 41,\dot{6} = 751,75 = 75\frac{3}{4}$ the Area per Prob. III.

(13) Square Order. Quincunx Order.

Fig. 23.



Firſt $6 \times 6 = 36$ ſqr. Feet each in the Square Order.
And $3 \times 3 = \underline{9}$
$\sqrt[2]{27} = 5,19615.$
Then $5,19615 \times 6 = 31,1769$ ſqr. Feet each Plant in the Quincunx Order.
Now an Acre $= 43560$ Feet.
∴ $43560 \times 10 = 435600$ ſqr. Feet in 10 Acres.

There-

Plants.

Therefore 31,1769)435600(13972 Quincunx. } Order.
36)435600(12100 Square. }

Answer 1872 Difference.

(14) First $10+2\frac{1}{2}+1=13\frac{1}{2}=\frac{27}{2}$ whole Breadth.
Then $\frac{27}{2}\times\frac{2}{9}=3$ Feet, Rise of the Roof.
And $13,5\div2=6,75$ Half the Width.
Also $6,75\times6,75=45,5625$
$3\times3 \quad =9$

$\sqrt[2]{54,5625}=7,38664$ Length.

Then $13,5\times2\times7,38664=199,4393$ Feet, the Area, at $3\frac{1}{2}d.$ or ,0146£.
∵ $199,4393\times,0146=2,9251=2£.$ 18*s.* 6*d.* Answer.
(15) See Example XXV. Sect. LIV.
(16) First, as 4,5 : 8 :: 9 A to 16 B.
And 4 : 8,5 :: 16 : 34 = A's + C's.
Then 34 − 9 = 25 = C's.
∵ 9 + 16 + 25 = 50 = A's + B's + C's, the Sum of the Sides.
Now $\sqrt[2]{9}=3$ A's, $\sqrt[2]{16}=4$ B's, and $\sqrt[2]{25}=5$ C's, Area.
Also $\frac{3+4+5}{2}=\frac{12}{2}=6$ Poles, the Area of Triangle.
Therefore $272,25\times144=39204$ Inches in a Pole.
×6 Poles.

295224 Inches in 6 Poles.
6 Feet = ×72 Inches, the Depth.

A gal. = 231 inc.)16936128(73316,57 galls.

∵ 63)73316,57(1163,755 = 1163 Hhds. $47\frac{1}{2}$ galls. the Answer.
(17) Here suppose Fig. 5, in the Guide, to represent the Orchard; which contains 3,75 Acres.
Then AC = the Diagonal; and 430 + 360 = 790 = BF + DE, the Sum of the Perpendiculars; half of which is 395 Links, and 3,75 Acres = 3,75000 Links.
∵ 395)3,75000($949\frac{29}{79}$ Links = AC, the Diagonal.

(18) First $3,1416 \times 16 = 50,2656$ Poles, the Circumference, (per Prob. VI. Rule I.)

Then $\frac{50,2656}{2} \times \frac{16}{2} = 25,1328 \times 8 = 201,0624$ Poles, the Area, (per Prob. VII.)

$\because$ 160)201,0624(1 Acre, 41 Poles, the Answer.

(19) First $16,5 \div 2 = 8,25$ one Round.

$\because$ 3,1416)8,2500(2,626 ft. = 2 ft. $7\frac{1}{2}$ Inches, the Answer.

(20) First 3,1416)130,0000(41,38 ft. the Diameter.

Then $\frac{41,38}{2} \times \frac{130}{2} = 20,68 \times 65 = 1344,85$.

9)1344,85(149,428 yds. at 4*d.* or ,016.

$\because$ $149,428 \times ,016 = 2,490466 = 2$£. 9*s.* $9\frac{1}{2}$*d.* ,8, Answer.

(21) First $4840 \div 2 = 2420$ yds. half an Acre.

Then ,7854)2420(3081,23 yds. Square of the Diameter, (per Prob. VII. Rule II.)

$\because$ $\sqrt[2]{3081,23} = 55,5$ Diameter.

Therefore $55,5 \div 2 = 27,75 = 27\frac{3}{4}$ yds. the Answer.

(22) First $42 + 14,5 \times 2 = 71$, greater Diameter.

Then $71 \times 71 = 5041$; also $5041 \times ,7854 = 3959,2014$

And $42 \times 42 = 1764$; also $1764 \times ,7854 = 1385,4456$

Diff. of their Areas, are 2573,7578

Then 144)2573,7578(17,8733 ft. at 8*d.* or 6*s.*

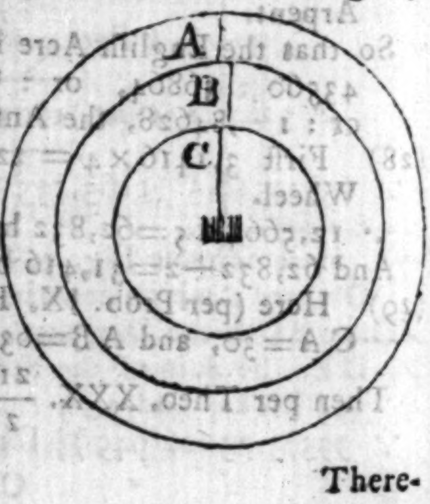
$\because$ $17,8733 \times 6 = 11,9155 = 11$*s.* 11*d.* (nearly) the Answ.

(23) First, as $3,25 : 5 :: 60 : 92,307$ Circumference.

$\because$ $92,307 \div 3,1416 = 29,38$ Diameter.

And $29,38 \div 2 = 14,69$ Inches, the Answer.

Fig. 24.



(24) First $36 \times 36 \times ,7854 = 1017,8784$. Area of the whole Stone and Spindle-Hole. (per Prob. VII. Rule II.)

And $5 \times 5 \times ,7854 = 39,27$, Area of the Circle circumscribing the Spindle-Hole.

$\because$ $1017,8784 - 39,27 = 978,6084$, Area of the whole Stone.

There-

Therefore, s.

$$\text{As } 20s. : 978{,}6084 :: \left\{ \begin{array}{l} 7 : 324{,}5129 \text{ A's} \\ 8 : 391{,}4433 \text{ B's} \\ 5 : 244{,}6521 \text{ C's} \end{array} \right\} \text{Area.}$$

Then $244{,}6521 + 39{,}27 = 283{,}9221$, this $\div ,7854 = 361{,}5$, whose Square Root $= 19{,}03$ inc. the Diameter, where C begins to grind.

And $283{,}9221 + 391{,}4433 = 675{,}3654$, this $\div ,7854 = 859{,}9$, whose Square Root $= 29{,}324$ inc. the Diameter, where B begins to grind.

(25) First $10 \times 10 = 100$; also $100 \times 2 = 200$, Area of the inside Circle.

Then $\sqrt[2]{200} = 14{,}142135$ Side of the inscribed Square.

Also $14{,}142135 \times 14{,}142135 = 200$, its Area.

Again $20 \times 20 = 400$ Area of the circumscribed Square.

Lastly $,7854 \times 400 = 314{,}16$ Area of the circular Section.

Hence $314{,}16 - 200 = 114{,}16$ Inches, too little.

And $400 - 314{,}16 = 85{,}84$ ditto, too much.

(26) First 2*s.* 4*d.* $= ,116$£. : 1 ft. :: 10£. : 85,7143 ft. Area of the Semi-Circle, which $\times 2 = 171{,}4286$, Area of the whole Circle.

Then $,7854)171{,}4286(218{,}269$ Square of the Diameter, (per Prob. VII. Rule II.)

$\because \sqrt[2]{218{,}269} = 14{,}7739 = 14$ ft. $9\frac{1}{4}$ inc. the Answer.

(27) First $,18 \times 18 \times 100 = 32400$ French Feet, the Arpent.

And $16{,}5 \times 16{,}5 \times 160 = 43560$ English Feet, in an Arp.

Then $16 \times 16 = 256$; also $15 \times 15 = 225$.

$\because$ Recipe 256 : 32400 :: 225 : 36864 English Feet, in an Arpent.

So that the English Acre is to the Arpent of France, as 43560 : 36864, or : 605 : 512, nearly 13 : 11, or : 1 : ,84628, the Answer.

(28) First $3{,}1416 \times 4 = 12{,}5664$ Circumference of the Wheel.

$\because 12{,}5664 \times 5 = 62{,}832$ by the greater.

And $62{,}832 \div 2 = 31{,}416$ by the less.

(29) Here (per Prob. IX. Fig. 9. in the Guide) is given C A = 30, and A B = 63 ft. = 21 yds.

Then per Theo. XXX. $\frac{21}{2} = 30 \times 10{,}5 = 315$ yds. Answ.

(30) First 15 Inches $=,416$ Yards.
Then $21-,416=20,583=AB$, (see Fig. 22.)
And $20,583 \div 2=10,2915=AE$.
Also $30 \times 30 = 900 = {}^2AC$.
$10,2915 \times 10,2915 = \underline{105,9184} = {}^2AE$.
Diff. $794,0814 = {}^2CE$.

$\therefore \sqrt[2]{794,0814}=28,18=CE$, the Perpendicular.

Then per Prob. X. $30 \times \frac{21}{2} = 315$, Area of the Sector ABCD.

And $10,2915 \times 28,18 = 290,01916$, Area of the Triangle ABC.

$\therefore 315-290,01916=24,98083$, Area of the Segment ABD.

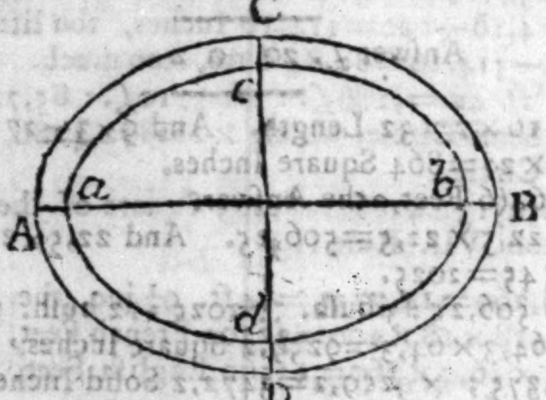


Fig. 25.

(31) First $14 \times 2 = 28$ Inches, or 2,3 Feet.

Lin. Ft. Lin. Ft.
Then, as $100 : 66 :: 840 : 554,4 = ab$.
$100 : 66 :: 612 : 403,92 = cd$.
$\therefore 544,4 + 2,3 = 556,73 = AB$.
$403,92 + 2,3 = 406,253 = CD$; then per Prob. XXVI.
$556,73 \times 406,253 \times ,7854 = 177637,66 =$ Area of ABCD.
$554,4 \times 403,92 \times ,7854 = 175877,17 =$ ditto $abcd$.

Covered by the Wall 1760,49 Diff.

Now an Acre $=4840$ yds. $\times 9 = 43560$ ft.
$\therefore 43560)1760,49(4,0375=4$ Acres, 6 Perches, the Answer, (see Prob. XII.)

(32)

(32) First $4\times10=40$ Feet, and $7\times7=49$ Square Side.
Then, as $4:49::40:490$ Square of the Diameter.

$\because \sqrt[2]{490}=22,136$ Inches, the Diameter required.

(33) First $6\times6=36$. And $3\times3\times4\times2=72$.
Then, as $36:3::72:6$ Hours, the Answer.

(34) First $9\times9=81$. And $12\times12\times2=288$.
Then, as 81 : 22 lb. :: 288 : $78\frac{2}{7}$ lb. the Answer.

(35) First $1,25\times1,25\times20=31,25$ Feet.
And $\frac{7}{8}=,875$; $\times,875\times50=38,28125$ Feet.
$\because$ as $31,25:\frac{1}{2}$ Ton, or 1120 lb. :: 38,28125 : 1372 lb. at $3\frac{1}{2}d.$ per lb.

		d.	*lb.*		
3	$\frac{1}{80}$	1372			
$\frac{1}{2}$	$\frac{1}{6}$	17	3		
		2	17	2	
Answer £.		20	0	2	

(36) First $16\times2=32$ Length. And $9\times3=27$ Breadth,
Then $32\times27=864$ Square Inches.
$\because$ 144)864(6 Feet, the Answer.

(37) First $22,5\times22,5=506,25$. And $22,5\times2=45$.
Also $45\times45=2025$.
Then, as 506,25 : 3 bush. :: 2025 : 12 bush. the Answer.

(38) First $64,3\times64,3=9259,2$ Square Inches.
And $\frac{3}{8}=,375$; $\times9259,2=3472,2$ Solid Inches.
Then $3472,2\times4,36=15151,418$ Ounces, which $\div1792$ (oz. in 1 Cwt.) $=8,455$ Cwt.
$\because$ as 19,5 : 21£. :: 8,455 : 9,10538=9£. 2*s.* $1\frac{1}{4}d.$ the Answer.

89. MEASUREMENT OF SOLIDS.

PROBLEM XIII.

(1) Thus $2,5\times2,5\times2,5=15,625$ Feet, the Solidity.

(2) First $57\times42\times34=81396$ cubic Inches.
A gall. of Ale $=282$)81396(288,638 gall. the Answer.

(3) First $45\times45\times,7854=15,90435$ Area of the Base, (per. Prob. VII. Rule II.)
Then $15,90435\times8=127,2248$ Feet, the Content req.

PROBLEM XIV.

(4) Thus $10,5 \times 7,75 \times = 81,375$, the Surface required.

PROBLEM XV.

(5) First $12 \times 12 = 144$ Area of the Base.

Then $144 \times \frac{25}{3} = 144 \times 8,3 = 1200$ Feet, the Answer.

(6) Now (per Prob. III. Rule III.) $5,5 \times 3 = 16,5$, Sum of the three Sides, half of which is $= 8,25$.

Then $8,25 - 5,5 = 2,75$ Difference.

And $8,25 \times 2,75 \times 2,75 \times 2,75 = 171,57421875$.

Then $\sqrt{171,57421875} = 130,986 =$ Area of the Base.

$\because 130,986 \times \frac{30}{3} = 130,986 \times 10 = 1309,86$, the solid Content required.

(7) First $3,5 \times 3,5 \times ,7854 = 96,2115$ Area of the Base, (per Prob. VII. Rule II.)

Then $96,2115 \times \frac{6}{3} = 96,2115 \times 2 = 192,423$, the Solidity required.

PROBLEM XVI.

(8) Thus $\frac{3,25 \times 3,25 \times 20}{2} = \frac{21125}{2} = 105,625$, the Surface required.

(9) Thus $\frac{45 \times 20}{2} = \frac{900}{2} = 450$ Feet, the convex Surface required.

PROBLEM XVII.

(10) First $15 \times 15 = 225 = A$. And $6 \times 6 = 36 = a$. Also 24.

Then per Theo. $225 \times 36 = 8100$.

And $\sqrt{8100} = 90$ the mean Prop.

$\because 225 + 36 + 90 \times \frac{24}{3} = 351 \times 8 = 280$ Inches.

Therefore $144)280(= 19,5$ Feet, the Solidity.

Or thus, to the Squares of the $\left\{ \begin{matrix} \text{Diameters} \\ \text{Circumference} \end{matrix} \right\}$ of the Ends, add their Product: multiply by the Altitude of the Frustum, and the Product which arises by $\left\{ \begin{matrix} ,2618 \\ ,0265 \end{matrix} \right\}$ and this last Product will be the Content.

(11)

(11) Now by the preceding Rule $20\times20=400$.

And $3\times3=9$. Also $20\times3=60$, which are the Squares of the Diameters and the Product.

Then $400+9+60=469$ their Sum.

∵ $469\times60=28140$, which $\times,2618=7366,052$, the Content.

(12) First $66\times66=4356$. And $56\times56=3136$.

Also $66\times56=3696$, which are the Square of the Circumferences and their Product.

Then $4356+3136+3696=11188$, their Sum.

∵ $11188\times4\times,0265=1185,928$ the Content.

PROBLEM XVIII.

(13) First 3 ft. 4 inc. + 2 ft. 2 inc. $\times4=40$ inc. $+26\times4=66\times4=264$ Inches, Sum of the Perimeters.

Then $\frac{264\times10}{2}=\frac{2640}{2}=1320$ Inches.

∵ 12)1320(=110 Feet, the Surface.

(14) First $32+8=40$, Sum of the Circumference.

Then $\frac{40\times7}{2}=\frac{280}{2}=140$ Feet, the Content.

(15) First, as 30 : 6 :: 10 : 2, Circumference of the Top of the Frustum.

And $30-6=24$ the slant Height.

Then $\frac{10+2\times24}{2}=\frac{288}{2}=144$ Feet, the Surface req.

PROBLEM XIX.

(17) First $30\times16\times\frac{12}{2}=480\times6=2880$ ft. the Content.

PROBLEM XX.

(18) First $16+\overline{30\times2}=16+72=88$, Sum.

Then $88\times20=1760$.

Also $1760\times\frac{12}{6}=1760\times2=3520$ cubic Feet.

PROBLEM XXI.

(19) First $16\times16=256$. And $13\times13=169$.

Also $16\times13=208$, which are the Areas of the Ends, and the Product of their Sides,

Then

Then $256+169+208=633$ Sum.

$\therefore 633\times\frac{60}{3}=633\times20=12660$, the Content required.

PROBLEM XXII.

(20) First $304\times20=6080$. And $300\times16=4800$.

Also $\overline{304+300}\times\overline{20+16}=604\times36=21744$.

Now $6080+4800+21744=32644$, their Sum.

Then $32644\times\frac{5}{6}=32644\times,83=27186$, the Content required.

PROBLEM XXIII.

(21) First (per Prob. VI.) $3,1416\times7=21,9912$ the Circumference.

Then $21,9912\times7=153,9384$, the Surface required.

(22) Thus $3,1416\times12\times12=452,3904$, the Surface.

(23) First $7957,75\times3,1416=25000$ Miles, (nearly) the Circumference, (per Prob. VI. Rule I.)

Then $7857,75\times25000=198943750$ square Miles, the whole Surface required.

(24) First $3,1416\times42=113,0976$ the Circumference.

Then $113,0976\times9=1017,8784$, the Surface required.

PROBLEM XXIV.

(25) Thus $2,7\times7\times7\times,5236=179,5948$, the Solidity required, (per Rule II.)

(26) First, the Surface is 198943750 Miles, (per Prob. XXIII. See Ex. 30.)

Then $19843750\times\frac{7957,75}{6}=263857437760$ Miles, the Solidity required, (per Rule I.)

By Rule II. thus $\overline{7957,75}|^{3}\times,5236=263857624944$, the Solidity by this Rule.

The Difference arises by taking the Number ,5236 a little too great.

PROBLEM XXV.

(27) First $\overline{18\times3}-\overline{4\times2}=54-8=46$.

Then $46\times4\times4\times,5236=385,3696$, the Content.

PRO-

PROBLEM XXVI.

(28) For an Oblate, thus
$,5235 \times 33 \times 55 \times 55 = 52258,3875$ solid Inches.
Then $1728)52258,3875(30,242+$ solid Feet.
For an Oblong, thus $,5235 \times 55 \times 33 \times 33 = 31355,0325$ solid Inches.
$\because 1728)31355,0325(=18,1452$ solid Feet.

PROBLEM XXVII.

(29) First $,41888 \times 36 \times 36 \times 99 = 53743,97952$ solid Inc.
Then $1728)53743,97952(31,10184$ Feet, the Content required.
By Rule II. Thus $,7854 \times 36 \times 36 = 1017,8784$, Area of the greatest Circle, (per Prob. VII. Rule II.)
$\because 1017,8784 \times 99 = 100769,9616$.
Which $\div 15 = 6717,99744 = \frac{1}{15}$.
Therefore $6717,99744 \times 8 = 53743,97952$ solid Inches, as before.

PROBLEM XXVIII.

(30) By Rule I. Thus, first $42 \div 4 = 10,5$ Quarter Girth.
And $10,5 \times 10,5 \times 16 = 1764$ Inches.
$\because 144)1764(12\frac{1}{4}$ Feet, the Content by this Rule.
By Rule II. Thus, first $42 \div 5 = 8,4 = \frac{1}{5}$ of the Girth.
And $16 \times 2 = 32 =$ twice the Length.
Then $8,4 \times 8,4 \times 32 = 2257,92$ Inches.
$\because 144)2257,92(15,68$ Feet, the true Content.
(31) By Rule I. Thus $64 \div 4 = 16$.
Then $16 \times 16 \times 30,5 = 7808$ Inches.
These $\div 144$, gives $44,2$ Feet, the Content by this Rule.
By Rule II. Thus $64 \div 5 = 12,8$. And $30,5 \times 2 = 61$.
Then $12,8 \times 61 = 9994,24$ Inches.
$\because 144)9994,24(69,335$ Feet, (very nearly) the true Content.

Note, the first Rule differs from the Truth about $\frac{1}{4}$ of its Contents; that is, when it produces 4 for the Content, it should be above 5. The second Rule is about 50 times nearer to the Truth than the other, for it differs from the Truth only 1 Foot in 190; and it is full as easy in Practice, besides it hath in every Respect the Advantage of it. Therefore, I think it ought to be brought into general Use among the Measurers of Timber, who should certainly prefer Truth to such gross Errors as are always introduced by the other Method.

(31)

(32) First $42 \times 30 \times 40 = 50400$.
Then 144)50400(350 Feet, the Content.
∴ 5,0)35,0(7 Tons, the Answer.

PROBLEM XXXIX.

By RULE I.

(33) Thus $,7854 \times 32 \times 32 \times 2 = 1608,4992$
$,7854 \times 26 \times 26 = 530,9304$

their Sum = 2139,4296
which Multiply by $\frac{40}{3}$ = 13,3̇

gives 28525,728 cubic Inc.

∴ 282)28525,728(101,155 Ale } Gallons.
231)28525,728(123,488 Wine }

By RULE II.

(34) First $36 \times 36 = 1296$ = Square of the Head.
And $40 \times 40 \times 2 = 3200$ = 2 ditto of the Bung.

their Sum = 4496

Now $40 - 36 = 4$. And $4 \times 4 = 16$, $\frac{2}{5}$ of which $= 6,4$.
Then $4496 - 6,4 = 4489,6$.
∵ $4489,6 \times 64 = 287334,4$.
Therefore

$287334,4 \times \begin{cases} ,00092837 = 266,75 & \text{Ale} \\ ,00113333 = 325,64+ & \text{Wine} \end{cases}$ Gallons.

Or 1077,157)287334,4(266,75 Ale } Gallons, as be-
882,35)287334,4(325,64 Wine } fore.

QUESTIONS.

(1) First $6 \times 6 \times 6 = 216$ Solid Inches in $\frac{1}{8}$ a Foot solid.
And 2)1728

∵ 216) 864(solid Inches in $\frac{1}{2}$ a solid Foot.

Answer 4 Times as much as the first; or one is $\frac{1}{4}$ of the other.

(2)

	Ft. In.	Ft. In.	Ft.		Ft.	In.	″
(2) First	25 6 ×	20 2 ×	14	=	7199	6	0
$\frac{1}{2}$ =	12 9 ×	10 1 ×	7 × 2	=	1799	10	6 which is just $\frac{1}{4}$

of the first; or as 4 is to 1, the Answer.

	Ft. Inc.	Ft. Inc.	Ft. Inc.		Ft.	Inc.	″
(3) First	17 7 ×	13 10 ×	9 6	=	2310	8	11
And	5 6 ×	2 1 ×	9 6	=	108	10	3
			Remains		2201	10	8
Fire-Place, and Windows			27 × 4 =		108	0	0
			Answer		2309	10	8

Sol. Ft.

(4) First 112,5 × 32 × 5,5 = 19800 in the whole Hold.
Also 112,5 × 4,5 × 5,5 = 2784,375 Gang-Way.

Remaining Capacity 17015,625

Then 3,3̇ × 2,16̇ × 3 = 21,626̇, Content of 1 Bale.

∵ 21,626̇)17015,625(787,2 + Bales, the Answer.

(5) First a Fodder of Lead = 2184 lb.
Then 16)2184(136,4 Square Feet.
∵ 4,25 × 8 = 34 Area of the Bottom.
Also 136,5 — 34 = 102,5 = Area of the Sides and Ends.
8 × 2 + 4,25 × 2 = 16 + 8,5 = 24,5 round.
Then 24,5)102,5(4,183673 Feet, = 50,204 Inches.
∵ 50,204 — ,75 = 49,454 Depth taken at.
Now 8 ft. = 96 inc. Length, and $4\frac{1}{2}$ ft. = 54 inc. Breadth.
Therefore 96 × 54 × 49,454 = 242126,784 cubic Inches.
∵ 282)242126,784(858 galls. = 16 hhds. 42 galls. the Answer.

(6) First 18 inc. = 1,5 ft. Also 14 = 1,16̇.
Then 1,16̇ × 1,5 = 1,75 Breadth and Depth.
∵ 1,75)2,5(1,42857 Length of the Piece cut off.
Therefore 18,5 — 1,42857 = 17,07143 Feet, the Answer.

(7) First 8,5)126,25(14,853 ft. = 2138,8234 Inches, Area of an End.
∵ 38,5)2138,8234(55,55 Inches deep, the Answer.

(8)

(8) First $18,5\times 18,5\times ,7854\times 8=2150,4252$ cubic Inches in a Bushel.

Now 7 ft. 10 inc. $=94$ inc. Length; 3 ft. 10 inc. $=$ 46 inc. Breadth; and 4 ft. 2 inc. $=50$ inc. Depth.

Then $94\times 46\times 50=216200$ cubic Inches in the Bin.

$\because$ 2150,4252)216200,0000(100,5 bush. $=12$ qrs. $4\frac{1}{2}$ bush. the Answer.

(9) By the last Question a Bushel $=2150,4252$ Inches.

Then $2150,4252\times 9=19353,8268$ Inches, the Content.

$\because 19353,8268\div 12=1612,8189$ Area of the Circle.

Which $\div ,7854=2053,49$ Square of the Diameter.

And $\sqrt[2]{2053,49}=45,3$ Inches, the Answer.

(10) First $28\div 2=14$ Radius. And $14\times 14=196$.

Also $7\times 7=49$. Then $196-49=147$.

Then $\sqrt[2]{147}=12,1243557$ Perpendicular.

$\because 12,1243557\times 7=84,8705$, Area of one Triangle.

And $84,8705\times 6=$ 509,223, Area of the Base.

Also $14\times 6\times 134=11256$, ditto Sides.

Other Base 509,223

	Yds.	Ft.	Inc.	
A yd. $=$ inc. 1296)12274,446(	9	4	34	Superf.

Again $509,223\times 134=68235,88$ solid Inches.

$\because$ 1728)68235,88(39 Solid Feet, 84 inc. the Answer.

(11) First 3 cubic Feet $=5184$ cubic Inches.

As the Circumference is 44, the Diameter is 14, (per Prob. VI.)

Then $\frac{44}{2}\times\frac{14}{2}=22\times 7=154$, Area of an End, (per Prob. VII. Rule I.)

$\because$ 154)5184(33,66 Inches, the Answer.

(12) First $1,25\times 1,25\times ,7854=1,2271875$, Area of the Circle, (per Prob. VII. Rule II.)

And a Pint contains 28,875 cubic Inches.

$\because$ 1,227187)28,875(23,5294 Inches, the Answer.

(13) A Bushel contains 2150,4252 cubic Inches, (per Quest. VIII.)

And $13,5\times 8=108$ Bushels, in $13\frac{1}{2}$ qrs.

Then $2150,4252\times 108=232245,921$ cubic Inches.

$\because \sqrt[3]{232245,9216}=61,468+$, the Answer.

(14)

(14) First $20 \times 15 \times 8 = 2400$ Cubic Inches in the Stone.
And 290 Tons $= 649600$ lb.
∵ as 220 : 2400 :: 649600 : 7086545 Inches.
Then 1728)7086545(4101 Feet, the Anſwer.

(15) A piece of Timber a Foot long, and 4 Feet round, is a ſolid Foot, cuſtomary Meaſure.
Alſo if a Circle be 4 Feet round, its Diameter will be $4 \div 3{,}1416 = 1{,}2732$. (per Prob. VI.)
Then a circular Piece of Timber 1 Foot in length, will contain 1,2732 Feet.
Therefore $1{,}2732 \times 50$ (Feet in a Load) $= 63{,}66$ ft.
∵ $63{,}66 - 50 = 13{,}66$ Feet in a Load, the Anſwer.

(16) *Sol. Inc.*

$$20 \times 20 \times 20 \times \left\{ \begin{array}{l} ,7854 = 6283{,}2 \text{ Cyl.} \\ ,5236 = 4188{,}8 \text{ Globe.} \\ ,2618 = 2094{,}4 \text{ Cone.} \end{array} \right\} \text{See Prob. XIII. XV. and XXIV.}$$

Now if the Diameter of a Circle be 20, the Circumference will be $3{,}1416 \times 20 = 62{,}832$. (per Prob. VI.)
Alſo if the Height (C D) of a Cone $= 20$, and the Diameter of the Baſe A B $= 20$.

Then $AD = \frac{20}{2} = 10$. (See Fig. 16. in the Guide.)

Then per Sect. LIV. Caſe IV. $\overline{20 \times 20} + \overline{10 \times 10} = 400 + 100 = 500$.
∵ $\sqrt{500} = 22{,}31614 = AC$, the Slope Side.
Then $62{,}832 \times 10 =$ 628,32 Area of the two Baſes.
And $62{,}832 \times 20 =$ 1256,64

$$\left.\begin{array}{lll} \text{Cylinders} & 1884{,}96 & \text{Superficial Cont.} \\ 62{,}832 \times 20 = & 1256{,}64 & \text{Globe's ditto.} \\ \frac{62{,}832}{2} \times 22{,}31614 & = 1015{,}24 & \text{Cone's ditto.} \end{array}\right\} \text{See Pr. XVI. and XXIII.}$$

their Sum $= 4156{,}84$ Inches, their Area.

A Yard $= 1296 : 8 :: 4156{,}84 : 25{,}6 = 2s.\ 1\frac{1}{2}d.$ the Anſwer.

(17) First $2170 \times 2170 \times 2170 \times ,5236 = 5350308686,8$ solid Miles, (per Prob. XXIV.)

Then a Mile $= 1760 \times 1760 \times 1760 = 5451776000$ solid Yards.

$\therefore 5350308686,8 \times 5451776000 = 29168684491287756800$ solid Yards, in the Moon.

Now in a solid Yard are $36 \times 36 \times 36 = 46656$ sol. Inches.

Therefore $29168684491287756800 \times 46656 = 1360894143625521581260800$ solid Inc. in the Moon.

Then per Quest. VIII. a Quarter $= 2150,425 \times 8 = 17203,4) 1360894143625521581260800 (79107034948470144000$ Quarters of Wheat, Moon would hold, if hollow. *Q. E. F.*

Again $2170 \times 3.1416 = 6817,272$ Circumference of the Moon, (per Prob. VI.)

Then per Prob. XXIII. $6817,272 \times 2170 = 14793480,25$ square Miles, the Surface of the Moon.

A Mile $= 1760 \times 1760 = 3097600$ square Yards.

$\therefore 14793480,25 \times 3097600 = 45824284391424$ square Yards of Stuff. *Q. E. F.*

(18) First $7970 + \overline{60 \times 2} = 7970 + 120 = 8090$ Diameter of the Earth and Atmosphere.

Then $7970 \times 7970 \times 7970 \times ,5236 = 265078559622,8$ sol. Miles in the Globe of the Earth, (per Prob. XXIII.)

Also $8090 \times 8090 \times 8090 \times ,5236 = 277233177544,4$ solid Miles in the Earth and Atmosphere.

$\therefore 277233177544,4 - 265078559622,8 = 12154617921,6$ solid Miles in the Atmosphere.

And per the last Quest. a Mile $= 5451776000$ solid Yards.

Then

$12154617921,6 \times 5451776000 = 66264254274148761600$ solid Yards in the Atmosphere. *Q. E. F.*

(19) First 30 In. $= 2,5$ Feet.

Then $2,5 \times 2,5 = 6,25$ Area of the Base.

$\therefore 6,35 \times \dfrac{21}{3} = 6,25 \times 7 = 43,75$ solid Feet, (per Prob. XV.)

Then $21 \times \dfrac{2,5}{2} = 21 \times 1,25 = 26,25$ Area of one triangular Side, (per Prob. III.)

$\therefore 26,25 \times 4 = 105$ Area of the four Sides.

Now

		£.	s.	d.
Now 43,75 at 7s.	=	15	5	8
And 105 at 8*d*.	=	3	10	0
Anſwer Coſt		18	15	8

(20) In order to complete the Cone, uſe this Analogy; as half the Difference of the Top and Bottom are to the Depth, ſo is half greater Diameter to the Altitude of the whole Cone.

Thus $\frac{72-54}{2}=\frac{18}{2}=9$. Alſo $\frac{72}{2}=36$.

Then as 9 : 42 :: 36 : 168 Altitude.

$\because$ $72\times72\times,7854\times\frac{168}{3}=2288004,7616$ Area of the whole Pyramid, (per Prob. XV.)

Again $\frac{168-42}{3}=\frac{126}{3}=42$ Altitude of the Piece wanting.

Then $54\times54\times,7854\times42=96189,5088$ Area of the Piece wanting.

Therefore $2288004,7616-96189,5088=131815,2528$ cubic Inches.

$\because$ 282)131815,2528(467 Galls. $3\frac{61}{141}$ pts. the Anſwer.

(21) Firſt $16+\frac{16}{5}=16+3,2=19,2$ Bottom Diameter.

And $19,2\times8=153,6$ the Height.

Alſo $153,6\div3=51,2$ cylindrical.

$153,6-51,2=102,4$ a conical Fruſtum.

Now $19,2\times19,2\times,7854=289,529856$ Area of the gr.

Alſo $16\times16\times,7854=201,0624$ ditto leſſer.

Sum of their Areas 490,592256

Again $289,52985\times201,0624=58213,567719$, whoſe ſquare-Root $=241,274$; which added to $490,592256$ $=731,866256$.

$\because$ $731,866256 \times \frac{102,4}{3} = 24981,034874546$ Conical.
And $201,0624 \times 51,2 \quad = \quad 10294,39488$ Cylinder.

Solid Content of the Pillar 35275,42975 Inches.

$\because$ 1728)35275,42975(20,41988 solid Feet, at 3*s.* 6*d.* or ,175 £.
Then 20,41988 × ,175 = 3,573479 = 3 £. 11*s.* $5\frac{1}{2}$*d.* Answ.
Farther
16 × 3,1416 = 50,2656 Circumference of the Cylind.
19,2 × 3,1416 = 60,3187 ditto Base.

2)110,5843(55,29215

Then $\frac{19,2}{2} - \frac{16}{2} = 1,6$, and $1,6 \times 1,6 \; = \; 2,56$
Also $102,4 \times 102,4 \quad = \quad 10485,76$

their Sum 10488,32

$\because$ $\sqrt{10488,32} = 102,4125$ Slope Height.
Then 102,412 × 55,29215 = 5662,6534 Con. Superf.
Also 51,2 × 50,2656 = 2573,5987 Cylinder.
289,526 Bottom Area.
201,0624 Top ditto.

their Sum 8726,8405 Inches.

$\because$ 144)8726,8405(60,8 Feet, the Superficial Content.

(22) First 19,5 − 13,5 = 6 Diff. of the Sides.
And 6 × 6 = 36, ÷ 3 = 12 the $\frac{1}{3}$ of the sq. of their Dif.
Now 16 ft. 6 inc. = 198 Inches, the Length.
Then 19,5 × 13,5 × 12 × 198 = 5499,5 cubic Inches.
1728)5499,5(31,539 solid Feet, at 2*s.* 6*d.* or ,125 £.
$\because$ 31,539 × ,125 = 3,942375 = 3 £. 18*s.* 10*d.* the Answer.

(23) First 3,1416 × 26 × 26 × 2 = 4247,4432
Also 3,1416 × 20 × 20 = 1256 64

Sum 5504,0832

Then

Then $5504,0832 \times \frac{100}{3} = 183469,44$ cubic Inches, (per Prob. XXIX.)

$\because$ 289)183469,44(650½ Ale=20 Bar. London Beer.
231)183469,44(794 Brandy. *Q. E. F.*

(24) First 6 Feet=72 Inches.
Then $3,1416 \times 72 = 226,19568$, the Circumference.
And $226,19568 \times 72 = 16286,0544$ Inches, superficial Content, (per Prob. XXIII.) at 3½*d.* per Inc.=237*£.* 10*s.* 1*d.* 2, nearly the Answer.

(25) First $21 \times 21 \times ,7854 = 346,3614$, Area of the Base.
Then $346,3614 \times 31 = 10737,2034$ solid Feet.
$\because$ $10737,2034 \times 1728 = 18553887,4752$ cubic Inches.
231)18553887,4752(80319,8 Gallons.
A Tun = Galls. 252)80319,8(318 Tuns, 183,8 Gallons, the Answer.

90. SPECIFIC GRAVITY OF METALS.

CASE I.

(1) Thus $,0330946 \times 12096 = 400,31228\,16$ lb.
Adv. $\div 112 = 3,5742+$ Cwt. the Answer.

(2) First $\frac{20}{5} = 4$. Also $4 \times 4 = 16$ sqr. of $\frac{1}{5}$ Girth.
And 40 Feet =480 Inches.
Then $480 \times 2 \times 16 = 960 \times 16 = 15360$ cubic Inches, (per Prob. XX. in Mensuration.)
By Rule II. $,023763 \times 15360 = 345$ lb. (nearly) = $3\frac{9}{112}$ Cwt. the Weight required.

(3) First $7 \times 7 \times 7 \times ,5236 = 178,8748$ solid Inches, (per Prob. XXIV.)
Then $178,8748 \times ,2580647 = 46,387+$ lb. the Answer.

(4) First 2580647)42,0000000(162,7499 the Solidity.
Then ,5236)162,7499(=310,84778457 Cube of the Diameter, (by Prob. XXIV.)
$\because$ $\sqrt[3]{310,84778457} = 6,7743$, the Diameter required.

(5) First $16-6=10$, Diameter of the Concavity.

Then $16\times16\times16\times,5236=2144,6656$ Cont. of the whole.

And $10\times10\times10\times,5236=\ 523,6$ Cont. of the Concavity, (per Prob. XXIV.)

Solidity of the Shell. $=1621,0656$ Inches.

$\because$ $1621,0656\times,2580647=418,3398$ lb. the Weight req.

(6) First $76+5=81$, the greater Diameter.

Also 3 ft. 4 inc. $=3,3$ the greater Breadth.

$\because$ as $81 : 3,3 :: 76 : 3,127572$, its least Breadth.

Here the Chords and their Arches being nearly equal, in so small a Part of so large a Circle, differs very little from a Right-line: the Figure of the Key-stone may be reckoned a Prismoid, and measured accordingly, (see Prob. XXII.)

Now $3,3\times4=13,3$; $3,127572\times4=12,510288$.

And $6,5519053\times8=52,4152426$, which are the Numbers to be added,

Then $13,3+12,510288+52,4152426=78,259864$, Sum.

$\because$ $78,259864\times\frac{5}{6}=\frac{391,29932}{6}=65,216553$ solid Feet.

Then $65,216553\times1728=112694$ cubic Inches, (nearly.)

$\because$ $112694=10475,3918842$ lb.

Therefore a Ton $=$ lb. $2240)10475,3918842(4,6765+$ $=4$ Tons 13 cwt. 2 qrs. 3,3 lb. the Answer.

(7) First $63\times12\times12=9072$ solid Feet.

Then $9072\times1728=15676416$ cubic Inches.

$\because$ $15676416\times,0977286=1532034,1887$ lb.

And $2240)1532034,1887(683$ Tons 18 Cwt. 98 lb. the Answer.

CASE II.

(8) First 8 Tons 14 cwt. $=19488$ lb.

Then $,0977286)19488,0000000(199409,4$ Inches.

Now $1728)199409,4(115,4$ cubic Feet, at 6*s.* or ,3£.

Then $115,4\times,3=34,6,2=34$£. 12*s.* $4\frac{1}{2}$*d.* the Answer.

(9) First $1300\times4=5200$ hhds. and a hhd. $=282\times54=15228$ cubic Inches.

$\because$ $15228\times5200=79185600$ cubic Inches, displaced.

Then $79185600\times,037253=2949901$ lb. Averd.

$\because$ $2240)2949901(1316$ Tons 18 cwt. 1 qr. 17 lb. the Weight required.

(10)

(10) First $3\times3=9$ solid Inches, in the Gold Chain.
Then $9{,}962625\times9=89{,}663625$ its Weight in Air.
And $0{,}527458\times9=\ \ 4{,}747122$ wt. of its Bulk of Water.

Weight of the Gold 84,916503 in Water, *Q. E. F.*

Now a solid Inch of Silver $=5{,}556769$ oz. Troy.
Then as $5{,}556769 : 1 :: 14{,}5 : 2{,}6094$ inc. of Silver.
$\because$ $89{,}663625-14{,}5=75{,}163625$ oz. of Gold.
And $9{,}962625)75{,}163625(7{,}5546$ Space taken up by the Gold,
2,6094 dit. of Silver.

Sum 10,164 by both.
9, ditto of Gold.

1,164 ditto of Silver.

$\because$ $9)1{,}164(,1293$ inc. higher. *Q. E. F.*

Sol. Inc.

(11) First $10{,}36)63{,}00(\ 6{,}08108$, had it been Gold.
Also $5{,}85)63{,}00(10{,}76923$, if all Silver.
Then by Sect. XXIX.

$$8{,}2245\left\{\begin{matrix}6{,}08108\\10{,}76923\end{matrix}\right\}\left\{\begin{matrix}2{,}54473\\2{,}14342\end{matrix}\right.$$

Sum 4,68815

4,68815)2,54473(,5428, Part Gold, per oz.
4,68815)2,14342(,4572, Part Silver.

Then *oz. dwts. grs.*

$$\left\{\begin{matrix},5428\\,4572\end{matrix}\right\}\times63=\left\{\begin{matrix}34{,}1884=34 & 3 & 22\tfrac{1}{4} \text{ Gold.}\\28{,}8036=28 & 16 & 1\tfrac{3}{4} \text{ Silver.}\end{matrix}\right.$$

91. CHRONOLOGY.

(1) Thus $1779\div4=444$, and 3 remains, so it is the 3d Year after Leap Year.

(2). Thus $1776\div4=444$, and 0 remains, so it is Leap Year.

(3)

(3) First 4)1779(444+1779=2223, Sum.
And 7)2223(317, and 4 remains.
Then 7—4=3=C, the Dominical Letter.

(4) First 4)1776(444+1776=2220, Sum.
And 7)2220(317, and 1 remains.
Then 7—1=6=F, for the Dominical Letter.

(5) The Dominical Letter for this Year is C, (per Prob. III. Ex. III.)
Now per the Verse, the 1st of May is B, viz. on Saturday.
Then B 1, B 8, B 15, C 16, D 17, E 18, F 19; so that the 19th of May happens on Wednesday.

(6) The Dominical Letter is F, per Ex. IV. and per the Verse, the first of June is E.
Then E 1, F 2, G 3, A 4; consequently the 4th of June must be on a Tuesday.

(8)

	1779	1779	1779
	9	1	3
	28)1788(63	19)1780(93	15)1782(118
Rem.	sol. Cyc. =24	lun. Cyc. =13	Indiction =12
	1776	1776	1776
	9	1	3
	28)1785(63	19)1777(93	15)1779(118
Rem.	sol. Cyc. =21	lun. Cyc. =10	Indiction =9.

So that 24 and 21 are the solar Cycles, 13 and 16 lunar Cycles, or Golden Numbers; 12 and 9 the Indic. Cycles.

(8) The Golden Number is 10, (per Prob. IV.)
Then 10×11=110, which ÷30=3, and 20 remains.
∵ 20—11=9, the Epact.

(9) The Golden Number is 13.
Then 13×11=143, which ÷30=4, and 23 remains.
∵ 23—11=12, the Epact.

(10) The Epact is 9, (per Prob. V.) Number of the Month 3, and Day of the Month 21.
Now 9+3+21=33.

Then

Then $33-30=3$ Days, the Moon's Age.

Alſo $30-3=27$ Days after the 21ſt of May, or June the 17th, the Day of the next new Moon.

(11) The Epact is 12, Number of the Month 1, and Day of the Month 24.

Now $12+1+24=37$.

Then $37-30=7$ Days, the Moon's Age.

Alſo $30-7=23$ Days, after the 24th of March, or the 16th of April, is the Day of the next new Moon.

(12) The Dominical Letter is F, (by Prob. II.)

On March 20th the Moon's Age is 4, (by Prob. VI.) ſo that the neareſt new Moon to March 20th falls on March 16th, and the 14th Day of that Moon is March 26th.

Now March 1ſt is D, or Friday, (by the two Lines in Prob. III.) then the 26th is Tueſday, and the next Sunday or Eaſter-day, is on the 31ſt of March.

(13) The Dominical Letter is C, (by Prob. II.

On March 21ſt the Moon's Age is 4, (by Prob. VI.) ſo that the neareſt new Moon to March 21ſt falls on March 17th, and the 14th of that Moon is March 31ſt.

Now March 1ſt or Monday is D, (by the two Lines in Prob. III.) then the 31ſt will be on Wedneſday.

∵ The next Sunday after will be on April 4th, which is Eaſter Sunday: from which the other moveable Feaſts may be found.

Or by knowing what Day of the Month Shrove Sunday is on, all the reſt may be found.

Shrove Sunday is always the firſt Sunday after the ſecond Change of the Moon, which happeneth after New-Years-Day: and if that Day of the ſecond Change be Sunday, then that Sunday is Shrove-Sunday.

EXAMPLE.

What Day of the Month is Shrove-Sunday in the Year 1779, being Feb. 12th.

Firſt the Moon's Age is 4 Days old*, (per Prob. VI.)

Then by running back, I find the Day of her ſecond Change will be Feb. 8th.

* In this caſe the Epact muſt always be uſed for the Year before; that is, for 1779 uſe the Epact for 1778, which is 20, (per Prob. V.)

Now

Now by the Lines in Prob. III. the first of Feb. is D, or Monday, as the Dominical Letter is C, (per Prob. II.)

Therefore D being the first, D is 8, E 9, F 10, G 11, A 12, B 13, and C 14, so that Shrove-Sunday will be on Feb. 14th, being the first Sunday after the second Change.

Now Shrove-Sunday being found as above, Febr. 14th.

Quadragesima, or the first Sunday in Lent, must be	Febr. 21.
Easter-Day, being 7 Weeks after Shrove-Sunday, must be	April 4.
Easter-Term, beginning a Fortnight after Easter-Wednesday, must be	April 21.
Rogation Sunday, being 5 Weeks after Easter-Wednesday, must be	May 9.
Ascension-Day, being 39 Days after Easter, must be	May 13.
Easter-Term, ends the Monday after Ascension-day,	May 16.
Whit-Sunday, is 7 Weeks after Easter, must be	May 23.
Trinity-Sunday, the next after Whit-Sunday, must be	May 30.
Trinity-Term, begins Friday after Trinity-Sunday,	June 4.
Trinity-Term, ends Wednesday Fortnight afterwards	June 23.

And the Beginning and End of Hilary, and Michaelmas Terms, are fixed.

Note, the Septuagesima is a Fortnight, and Sexagesima a Week, before Shrove-Sunday.

Advent Sunday, is always that Sunday next the 30th of Nov. whether before or after it.

Epiphany is reckoned among the moveable Feasts; but it is always the 6th of January.

(14) 3 Moon's Age.
$\times 4$

5)12

Answ. 2 h. 24 m. in the Aftern.

(15) 7 Moon's Age.
$\times 4$

5)28

Ans. 5 h. 36 m. in the Aft.

(16)

		h.	m.	
(16)	The Moon south's at	2	24	p. m.
	At Lond. ☽ bears N.E. or S.E.	3	00	
	Sum	5	24	

Answ. 24 min. past 5 in the Afternoon.

		h.	m.	
(17)	The Moon South's at	5	36	p. m.
	At Bristol Key ☽ bears E. by S. and W. by N.	6	45	
	Sum	12	21	

Answer 21 min. past 12 at Night.

QUESTIONS.

(1) In the reign of William I. were 5 intercalary Days, (by Prob. I.)
Between Sept. 9, and Oct. 4, are 25 Days.
And 1087—1066=21 yrs. wanting 25 Days.
Then $\overline{21 \times 365} + 5 - 25 = 1645$ Days, William I. reigned.
In the reign of William Rufus were 4 intercalary Days.
Between August 9, and Sept. 9, are 38 Days.
And 1100 - 1087=13 yrs. all but 38 Days.
Then $\overline{365 \times 13} - 38 + 4 = 4711$ Days, William II. reigned.
Lastly, in the reign of William III. were 3 intercalary Days.
From Feb. 3, to March 8, are 33 Days.
And 1701—1689=12 Years. and 33 Days.
Then $\overline{365 \times 12} + 33 + 3 = 4416$ Days, Will. III. reigned.

(2) In the reign of Richard I. were 2 intercalary Days.
Betwixt April 6, and July 7, are 92 Days.
And 1199—1189=10 yrs. wanting 92 Days.
Then $\overline{365 \times 10} + 2 - 92 = 3560$ Days, Richard I. reigned.
In the reign of Richard II. were 5 intercalary Days.
From June 21, to Sept. 30, are 101 Days.
And 1399—1377=22 Years and 101 Days he reigned.
Then $\overline{365 \times 22} + 101 + 5 = 8136$ Days, Rich. II. reigned.

In

In the reign of Richard III. were 1 intercalary Day.
From June 18, to August 22, are 65 Days.
And $1485-1483=2$ yrs. and 65 d. he reigned.
Then $365\times2+65+1=796$ Days, Richard III reigned.
Therefore $3560+8136+796=12492$ Days, the Anfwer.

(3) In the reign of Queen Mary, were only 1 intercalary Day.
But from 1553 till 1602, were 12 intercalary Days.
So that in the reign of Queen Elizab. were 11 intercal. Days.
From July 8, to Nov. being 4 mo. 9 d. are 132 Days.
Then $365\times5+132+1=1958$ Days, Mary reigned.
Now $1602-1553=49$ Years.
And from March 14, to July 8, are 115 Days.
Alfo $365-115=250$ Days.
Then $365\times49+250+12=18147$ Days, to the Beginning of James's reign.
$\because$ $18147-1958=16189$ Days, Elizabeth reigned.
In the reign of James I. were 6 intercalary Days.
From March 14, to the 27th is 13 Days.
And $1625-1602=23$ Years, but the Date altering at our Lady-Day, fo that the intercal. was only 22 Years.
Therefore $365\times22+13+6=8049$ Days James I. reigned.
In the reign of Charles were 6 intercalary Days.
From March 27, to January 30, are 309 Days.
And $1648-1625=23$ Years.
Then $365\times23+309+6=8710$ Days Charles I. reigned.
$\because$ $16189+8049+8710=32948$ Days, the Anfwer.

(4) The Grant begun Decemb. 14th. 1109, and refumed Nov. 19th. 1219.
From Nov. 19, to Dec. 14, are 25 Days.
And $1219-1109=110$ yrs. wanting 25 Days.
In which Time are 27 Intercalary Days.
Then $365\times110+27-25=40152$ Days, the firft Continuance.
The fecond Grant revived July 16th 1497, and ended May 10th 1524.
Then from May 10, to July 16, are 67 Days.
And $1524-1497=27$ yrs. wanting 67 Days, in which Time there are 7 intercalary Days.

There-

Therefore $\overline{365} \times 27 + 7 - 67 = 9795$ Days, laſt in Force.
∵ $40152 + 9795 = 49947$ Days, the Grant was in Force.
The firſt Grant reſumed Nov. 19th 1219, and revived July 16th 1498.
From Nov. 19th, to July 16, are 239 Days.
And $365 - 239 = 126$ Days.
Alſo $1498 - 1219 = 279$ Years, wanting 126 Days; in which Period are 69 intercalary Days.
Therefore $\overline{365} \times 279 + 69 - 126 = 101778$ Days, ſuperſeded. *Q. E. F.*

92. GEOGRAPHY.

(1) Lat. 39° 45′ N. 57 S. and Long. 111° E. 80° W.
(2) Jamaica and Cape of Good Hope.
(3) Diff. 10° 30′ and 48°.
(4) 4691½ Miles, Buda, in Hungary, Madrid, &c.
(5) Decl. 19° 15′ N. Right Aſc. 60° 45′, Mer. Alt. 57° 15′.
(6) R. 30′ P. 4, ſets 37′ P. 7, Amp. 30° N. Twil. beg. 21′ P. 1, ends 30′ P. 10.
(7) Azim. 78° from the N. Alt. 12° 30′.
(8) Port Royal bears W. from London, and London bears N. E. from Port Royal.
(9) 30′ P. 7, P. M. at Peking, Breakfaſt at Port Royal, Dinner at Rome, and Supper at the Ladrone Iſles.
(10) Antigua, Pagan, one of the Ladrone Iſles, &c.
(11) May 10th, and Sept. 2d.
(12) All thoſe Places whoſe Lat. is equal to 72° 45′ N.
(13) Begins May 4th, and leaves them Aug. 9th, following.
(14) Riſing to N. Mexico, Ter. Fuego, &c. ſetting to Canady in the iſle of Ceylon, and Part of Aſia, &c. Noon at Porto, St. Vincent, &c.
(16) Beg. in N. and S. America, the S. and Pacific Seas, Part of Tartary, the Land of Jeſſa, &c. Mid. all the Great S. Sea, the E. and W. Indies, Part of Aſia, &c. End all Aſia, and the Eaſt Indies, New Holland, the S. Sea, Part of N. America, &c.
(17) 63°, (nearly.)

(18) Antoeci, Part of the S. Ocean, Peri. Part of the great Pacific Ocean, Antip. Part of New Zealand, or Part of the S. Ocean.

CELESTIAL GLOBE.

(1) At ¼ p. 3, sets 20′ p. 4, no Night.

(2, 3, 4,) For these Examples, the best will be to answer them for the Year you are in; as you will then (by Parker's Ephemerides) have it in your Power to find the Node of the Moon, and her Place at Noon, likewise the Places of any of the Planets.

(5) Decl. 29° N. Right Asc. 112, Lat. 6° 30′, Long. 19° 45 Leo.

(6) Aldabaran ris. at 1 A. M. sets at 4 P. M. comes to the Mer. ½ p. 8, Amp. 26° N.

(7) R. cosmically Sept. 24th, sets July 6th.

(8) R. acronically Jan. 25th, sets Decem. 18th.

(9) Jan. 13th.

(10) Nov. 16th.

(11) Obl. Decl. 120°, continues above the Horiz. 9 Hrs.

(12) Azim. 10°, from the N. Alt. 8°.

(13) ¾ p. 8.

(14) 14° 30′ N.

(15) 7′ p. 12.

(16) ½ p. 3, A. M.

(17) Lat. 44° 30′ N.

(18) Lat. 22° 30′ N.

(19) 41° 30′ S.

(20) Arcturus on the Mer. nearly, Canis Minor setting, and the Stars Lyra, Altayr, Cor Hydræ, Cor Leonis, Castr. Cappella, &c. will be visible.

(21) Lat. 3° 30′ Long. 26° 36′ of ♒, and Lat. 25° S. Long. 12° 30′ ♍.

(22) R. 3 h. 45 m. A. M. Amp. 14°, sets 2 h. 25 m. P. M. Decl 9° 15′ S. Right Asc. 227° 30′.

(23) Through the Tail of Capricorn, the Head of Indus, the Neck and Body of Pavo, the Neck of Apus, Tail of Centaurus, between the two Stars in the Back of Hydra, and then to the Ecliptic near Cor Leonis: Velocity 4° ⅙ per Day.

ALGE-

ALGEBRA.

93. ADDITION.

EXAMPLES in CASE I.

	(2)	(3)
To	$6a+7b-3c$	$ab-6b+4x+10y-15z+6$
Add	$10a+b-7c$	$6ab-b+x+4y-9z+3$
Sum	$16a+8b-10c$	$7ab-7b+5x+14y-24z+9$

CASE II.

	(5)	(6)	(7)
To	$-4b-6c$	$-6b-7c-8x$	$6a-6x+7y-10$
Add	$2b+9c$	$4b+9c+5x$	$-6a+6x-4y+13$
Sum	$-2b+3c$	$-2b+2c-3x$	$*\quad *+3y+3$

CASE III.

	(9)	(10)
To	$6b-3x$	$4a+6b+4c-6$
Add	$3c-4y$	$3x-7y+4z$
Sum	$6b+3c-3x-4y$	$4a+6b+4c-6-3x-7y+4z$

(11)

To	$2a-6be$
Add	$6x+10$
Sum	$2a-6be+6x-10$

94. SUBTRACTION.

EXAMPLES.

	(2)	(3)	(4)
From	$3a$	$2a-4x+7y-7$	$6b-4c+4x$
Take	$-3a$	$6a+4x+7y+4$	$7b+7c-9x$
Rem.	$6a$	$-4a-8x\ *\ -11$	$-b-11c+13x$

95. MULTIPLICATION.

EXAMPLES in CASE I.

	(2)	(3)	(4)	(5)
Mul.	$-a$	$a+c$	$-a-b-c$	$x+y+z$
by	$-a$	b	$-d$	a
Prod.	aa	$ab+bc$	$ad+bd+dc$	$ax+ay+az$

CASE II.

	(7)	(8)	(9)
Mul.	$-8x$	$3a+7b$	$12x+6y$
by	$-4a$	$5b$	$4a$
Prod.	$32ax$	$15ab+35bb$	$48ax+24ay$

CASE III.

	(11)	(12)	(13)
Mul.	$-6a$	$6a-7c$	$4x-5y-z$
by	$7b$	$4d$	$6f$
Prod.	$-42ba$	$24ad-28cd$	$24fx-30fy-6fz$

CASE IV.

(15)

$$\begin{array}{rl} \text{Mul.} & 2a-4b \\ \text{by} & 2a-4b \\ \hline & 4aa-8ab \\ & \quad\;\; -8ab+16bb \\ \hline \text{Prod.} & 4aa-16ab+16bb \end{array}$$

(16)

$$\begin{array}{rl} \text{Mul.} & aa+ab+bb \\ \text{by} & a-b \\ \hline & aaa+aab+abb \\ & \quad\; -aab-abb-bbb \\ \hline \text{Prod.} & aaa \quad * \quad * \; -bbb \end{array}$$

(17)

$$\begin{array}{rl} \text{Mul.} & xx-2x \\ \text{by} & a+x \\ \hline & axx-2ax \\ & \qquad\quad xxx-2xx \\ \hline \text{Prod.} & axx-2ax+xxx-2xx \end{array}$$

(18)

(18)

$$
\begin{array}{lllllllll}
\text{Mul.} & 7b+5d-4x+6 \\
\text{by} & 2b-6x+4y-4 \\
\hline
 & 14bb+10bd-8bx+12b \\
 & \quad -42bx-30dx+24xx-36x \\
 & \qquad\qquad 28by+20dy-16xy-24y \\
 & \qquad\qquad\quad -28b-20d+16x-24 \\
\hline
\text{Prod.} & 14bb+10bd-50bx-16b-30dx+24xx-20x+28by+20dy-16xy+24y-20d-24 \\
\hline
\end{array}
$$

(19) Mul. $xx+xy+yy$
by $xx-xy+yy$

$$\begin{array}{l} x^4+x^3y+x^2y^2 \\ \quad -x^3y-x^2y^2-xy^3 \\ \qquad\quad x^2y^2+xy^3+y^4 \end{array}$$

Prod. $x^4 \quad * \quad +x^2y^2 \quad * \quad +y^4$

(20)

M. $aaa-3aab+3ab-bbb$
by $aa-2ab+bb$

$$\begin{array}{l} a^5-3a^4b+3a^3b-a^2b^3 \\ \quad -2a^4b+6a^3b-6a^2b^2+2a\,b^4 \\ \qquad a^3b^2-3a^2b^3+3a\,b^3-b^5 \end{array}$$

P. $a^5-5a^4b+9a^3b-4a^2b^3-6a^2b^2+2ab^4-a^3b^2+3ab^3-b^5$

96. DIVISION.

EXAMPLES in CASE I.

(2) $d)ad+6d(a+6$ (3) $-d)-ad-bd(a+b$

(4) $a)aa+ab(a+b$

CASE II.

(5) $-a)ab(-b$ (6) $b)-ab-bd(-a-d$

(7) $-bc)abc+bcd+bcf(-a-d-f$

CASE III.

(8) $6a)24ab(4b$ (9) $7b)42db(6d$

(10) $26x)8abx-18bx(4a-9$

CASE IV.

(12) $2b)ab+bb(=\dfrac{ab+bb}{2b}=\dfrac{a+b}{2}$, here Unity, or $1b$ is the common Meaſure. (See Sect. XXXVIII. Caſe I.)

(13) $20a)10ab+15ac(=\dfrac{10ab+15ac}{20a}=\dfrac{2b+3c}{4a}$, here 5 is the common Meaſure.

(14)

$$(14)\quad a+b)\,aa+2ab+bb\,(a+b$$
$$aa+ab$$
$$*\quad ab+bb$$
$$ab+bb$$
$$*\quad *$$

$$(15)\quad a+b)\,aa-bb\,(a-b$$
$$aa+ab$$
$$*-ab-bb$$
$$ab-bb$$
$$*\quad *$$

$$(16)\quad 3a-6)\,6a^4-96\,(2a^3+4a^2+8a+16$$
$$6a^4-12a^3$$
$$*\quad 12a^3-96$$
$$12a^3-24a^2$$
$$*\quad 24a^2-96$$
$$24a^2-48a$$
$$*\quad 48a-96$$
$$48a-96$$
$$*\quad *$$

$$(17)\quad 1-a)\,1****(1+a+a^2+a^3+a^4,\ \&c.$$
$$1-a$$
$$*a$$
$$a-a^2$$
$$*-a^2$$
$$a^2-a^3$$
$$a^3$$
$$a^3-a^4$$
$$a^4$$
$$a^4-a^5$$
$$a^5$$

It

It often happens that the Operation may be continued without End, as in the last Example, and then you have an infinite Series for the Quotient; but by comparing the first three or four Terms you may find what Law the Terms observe: by which Means, without any more Division, you may continue the Quotient as far as you please.

Thus, the last Example may be continued as far as you please, by adding the Power of a.

(18)

$3x^2-4x+5)18x^4-45x^3+82x^2-67x+40(6x^2-7x+8$

$18x^4-24x^3+30x^2$

$* \quad -21x^3-52x^2-67x$

$-21x^3-28x^2-35x$

$* \quad -24x^2-32x+40$

$-24x^2-32x+40$

$* \quad * \quad *$

(19)

$4x-5a)48x^3-76ax^2-64a^2x+105a^3(12x^2-4ax-21a^2$

$48x^3-60ax^2$

$* \quad -16ax^2-64a^2x$

$-16ax^2+20a^2x$

$* \quad -84a^2x+105a^3$

$-84a^2x+105a^3$

$* \quad *$

(20)

$3x+4a)81x^4 * * * -256a^4(27x^3-36ax^2+48a^2x-64a^3$

$81x^4+108ax^3$

$* \quad -108ax^3$

$108ax^3-144a^2x$

$* \quad 144a^2x$

$144a^2x+192a^3x$

$-192a^3x-256a^4$

$192a^3x-256a^4$

$* \quad *$

(21)

(21) $1+x)1 \ * \ * \ * \ * \ (1-x+x^2-x^3+x^4$, &c.

$$
\begin{array}{l}
1+x \\
\hline
* -x \\
\quad -x-x^2 \\
\hline
\quad * \quad x^2 \\
\qquad\quad x^2+x^3 \\
\hline
\qquad\quad * -x^3 \\
\qquad\qquad -x^3-x^4 \\
\hline
\qquad\qquad * \quad x^4 \\
\qquad\qquad\qquad x^4+x^5 \\
\hline
\qquad\qquad\qquad\quad -x^5 \text{ \&c.}
\end{array}
$$

(22)

$2x-3a)16x^4 * -72a^2x^2 * +81a^4(8x^3+12ax^2-18a^2x-27a^3$

$$
\begin{array}{l}
16x^4-24ax^3 \\
\hline
* \quad 24ax^3-72a^2x^2 \\
\quad\;\; 24ax^3-36a^2x^2 \\
\hline
\quad * \quad -36a^2x \quad * \\
\qquad\quad -36a^2x+54a^3x \\
\hline
\qquad\quad * \quad -54a^3x+81a^4 \\
\qquad\qquad\quad -54a^3x+81a^4 \\
\hline
\qquad\qquad\quad * \qquad *
\end{array}
$$

97. FRACTIONS.

(2) Thus $a - x + \frac{a^2 - ax}{x} = \frac{ax - xx + a^2 + ax}{x} =$ $\frac{a^2-x^2}{x}$, (by Sect. XXXVIII. Case III.)

(3) Thus $a+b+\frac{x}{z}=\frac{az+bz+x}{z}$.

(4)

(4) Thus $a-x+\frac{aa-ax}{x}=\frac{ax-xx+aa-ax}{x}=\frac{aa-xx}{x}$.

(6) Thus $\frac{a^2-x^2}{x}=-x+\frac{a^2}{x}$.

(7) Thus $\frac{az+bz+x}{z}=a+b+\frac{x}{z}$.

(8) Thus $\frac{ax-xx+aa-ax}{x}=-x+\frac{aa}{x}$.

(10) Thus $\frac{a}{b},\frac{c}{d},\frac{e}{f},=\frac{adf}{bdf},\frac{cbf}{bdf},\frac{edb}{bdf}$.

(11) Thus $\frac{b+c}{a+b}$ and $\frac{d-c}{b-d}=\frac{bb-bd+cb-cd}{ab-ad+bb-bd}$ and $\frac{ad-ac+bd-bc}{ab-ad+bb-bd}$.

(13) First

$$
\begin{array}{l}
a^2+2ab+b^2)\,a^3-\ *\ \ ab^2\,(\\
\qquad\qquad\quad a^3+2a^2b+\ ab^2 \\
\hline
\end{array}
$$

$$
\begin{array}{l}
\text{Rem.}\ -2a^2b-2ab^2)\,a^2+2ab+b^2\,(-\frac{1}{2b}-\frac{1}{2a} \\
\qquad\qquad\qquad\qquad\qquad a^2+\ ab \\
\qquad\qquad\qquad\qquad\qquad \overline{\qquad\quad ab+b^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad\ ab+b^2 \\
\qquad\qquad\qquad\qquad\qquad\qquad \overline{\ \ *\ \ *\ }
\end{array}
$$

Hence it appears that $-2a^2b-2ab^2$, is the common Measure; by which a^3-ab^2 being divided,

$$
\begin{array}{l}
\text{viz. } -2a^2-2b^2)\,a^3-\ *\ -ab^2\,(-\frac{a}{2b}+\frac{1}{2} \\
\qquad\qquad\qquad\quad a^3+a^2b \\
\qquad\qquad\qquad\quad \overline{\quad -a^2b-ab^2} \\
\qquad\qquad\qquad\qquad\ a^2b-ab^2 \\
\qquad\qquad\qquad\qquad \overline{\ \ *\quad *\ }
\end{array}
$$

(t) Then

Then $\frac{a}{2b}+\frac{1}{2}=\frac{-a+b}{2ba}$, the new Numerator.

And $\frac{1}{2b}-\frac{1}{2a}=\frac{-a-b}{2ba}$, the Denominator.

Let both be $\times 2ba$, and we ſhall have $\frac{-a^2+ab}{-a-b}$ new N. Deno.

Or change the Signs of all the Quantities, it will be $\frac{a^2-ab}{a+b}$, the new Fraction required.

That is $\frac{a^2-ab}{a+b}=\frac{a^3-ab^2}{a^2+2ab+b^2}$.

(14) Thus $\frac{25za}{5xz+15az}=\frac{5a}{x+3a}$, by expunging z, and dividing by 5.

Alſo $\frac{a^3+b^3}{a^2-b^2}=\frac{a^2-ab+b^2}{a-b}$, by dividing by $a+b$, the common Meaſure.

ADDITION.

EXAMPLES.

(2) Thus $\frac{a}{b}+\frac{c}{a}+\frac{d}{e}=\frac{ade+bce+ddb}{bde}$, the Sum.

(3) Thus $\frac{a-b+d}{b+a}+\frac{a+b-d}{d+a}=\frac{2a}{a+d}$, their Sum.

(4) Thus $\frac{2a-b}{d+c}+\frac{2b-a}{a+c}=\frac{a+b}{c+a}$, the Sum.

(5) Thus $\frac{a+b}{d}+\frac{2a+c}{d}=\frac{3a+b+c}{d}$, the Sum.

SUBTRACTION.

EXAMPLES.

(2) Thus $\frac{x}{2}-\frac{x}{3}=\frac{3x}{6}-\frac{2x}{6}=\frac{x}{6}$, the Difference.

(3) Thus $\frac{a+x}{b} - \frac{a-x}{c} = \frac{ac+cx}{bc} - \frac{ab-bx}{bc} =$ $\frac{ac+cx-ab+bx}{bc}$, the Difference.

(4) Thus $\frac{b^2+a^2}{c} - \frac{b^2}{c} = \frac{a^2}{c}$, the Difference.

(5) Thus $\frac{2b}{a+d} - \frac{a+b-d}{a+d} = \frac{b-a+d}{a+d}$, the Diff.

MULTIPLICATION.

EXAMPLES.

(2) Thus $\frac{a+b}{c} \times \frac{a-b}{d} = \frac{a^2-b^2}{cd}$, the Product.

(3) Thus $a+\frac{b}{c} \times \frac{d}{e} = \frac{ca+b}{c} \times \frac{d}{e} = \frac{acd+bd}{ce}$, the Product.

(4) Thus $\frac{3a-2b}{2d+c} \times \frac{4a+2b}{d} = \frac{12a^2-2ab-4b^2}{2ad-dc}$, the Product.

(5) Thus $2a+\frac{b}{c}-25 \times 36+4c = \frac{2ac+b-25c}{c} \times$ $\frac{36+4c}{1} = \frac{6bac+3b^2-75bc+8ac^2+4bc-100c^2}{c} =$ $6ba-71b-8ac-100c+\frac{3b^2}{c}$, the Product.

DIVISION.

EXAMPLES.

(2) Thus $\frac{acd+bd}{cd} \div \frac{d}{e} = \frac{acde+bde}{cdd}$, or $\frac{ace+be}{cd}$, the Quotient.

(3) Thus $\frac{a-b}{a} \div \frac{a+b}{a-b} = \frac{a^2-2ab+b^2}{a^2+ba}$, the Quot.

(4)

(4) Thus $a+\frac{b}{c}\div d+\frac{e}{f}=\frac{ac-b}{c}\div\frac{fd+e}{f}=\frac{fac-bf}{fdc+ce}$, the Quotient.

(5) Thus $\frac{ab}{c}\div\frac{1}{c}=ab$, the Quotient.

SURDS.

98. ADDITION.

EXAMPLES in CASE I.

	(2)	(3)	(4)	(5)
To	$7\sqrt{ab}$	$7b\sqrt{ax}$	$a\sqrt{bb+cc}$	$b+\sqrt[3]{aa-cc}$
Add	$4\sqrt{ab}$	$7b\sqrt{ax}$	$4a\sqrt{bb+cc}$	$b-\sqrt[3]{aa-cc}$
Sum	$11\sqrt{ab}$	$14b\sqrt{ax}$	$5a\sqrt{bb+cc}$	$2b$ *

SUBTRACTION.

EXAMPLES in CASE I.

	(2)	(3)	(4)
From	$10a\sqrt{bc}$	$6b\sqrt{aa+xx}$	$b+a$
Take	$6a\sqrt{bc}$	$4b\sqrt{aa+xx}$	$\sqrt{cc-aa}$
Diff.	$4a\sqrt{bc}$	$2b\sqrt{aa+xx}$	$b+\sqrt{cc-aa}$

ADDITION.

EXAMPLES in CASE II.

	(2)	(3)
To	$6d\sqrt{a}$	$\sqrt[3]{ac-ba}$
Add	$2b\sqrt{ac}$	$\sqrt{ac+ba}$
Sum	$6d\sqrt{a}:+2b\sqrt{ac}$	$\sqrt[3]{ac-ba}:+\sqrt{ac+ba}$

(4) To $6x\sqrt{aa-ba}$
Add $3y\sqrt{zx}$

Sum $6x\sqrt{aa-ba}:+3y\sqrt{zx}$

SUBTRACTION.

EXAMPLES in Case II.

(2)
From $a-b\ \sqrt{ccc+dc}$
Take $b-2c\sqrt{ab+bb}$

Diff. $a-b\ \sqrt{ccc+dc}-b+2c\sqrt{ab+bb}$

(3)
$\sqrt{bc}$
$\sqrt{ba}$

$\sqrt{bc}-\sqrt{ba}$

MULTIPLICATION.

EXAMPLES.

(2) Mul. $\sqrt{bc+dc}$
By $\sqrt{ac}$

Prod. $\sqrt{abcc+aacc}$

(3) $\sqrt{xx+zz}$
$\sqrt{xx-zz}$

$\sqrt{xxxx-zzzz}$

(5) Mul. $6cd\sqrt{b+da}$
By $3a\ \sqrt{ca}$

Prod. $18acd\sqrt{bca+cdaa}$

(6) $15\sqrt{az}$
$5\sqrt{z}$

$25\sqrt{azx}$

DIVISION.

EXAMPLES.

(2) $\sqrt{cx})\overline{bcxxx+dcxx}(\sqrt{bx+dx}$, the Quotient.

(3) $\sqrt{zz-xx})\sqrt{zzzz-xxxx}(=\sqrt{zz+xx}$, the Quot.

(5)

(5) $7\sqrt{yxx}$) $14zya\sqrt{yyxx+axx}$ ($2zya\sqrt{y+a}$, the Quot.

(6) $20\sqrt{2cy}$) $60ab\sqrt{10acxy}$ ($3ab\sqrt{5ax}$, the Quotient.

70. INVOLUTION.

(1) Thus the fourth Power of $x=x^4$.

(2) The fifth Power of $\overline{ax+z}|^2=\overline{ax+z}|^{10}$.

(3) The third Power of $3x^3z^2=27x^9z^6$.

(4) The sixth Power of $\frac{5ab}{2c}=\frac{15625a^6b^6}{64c}$.

(5) Thus

$$\begin{array}{l}
a+b \\
a+b \\
\hline
aa+ab \\
\quad\quad ab+bb \\
\hline
a^2+2ab+b^2\text{, the Square, or second Power.} \\
a+b \\
\hline
a^3+2a^2b+ab^2 \\
\quad\quad a^2b+2ab^2+b^3 \\
\hline
a^3+3a^2b+3ab^2+b^3=\text{ the Cube, or third Power.} \\
a+b \\
\hline
a^4+3a^3b+3a^2b^2+ab^3 \\
\quad\quad a^3b+3a^2b^2+3ab^3+b^4 \\
\hline
a^4+4a^3b+6a^2b^2+4ab^3+b^4\text{, the fourth Power.} \\
a+b \\
\hline
a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4 \\
\quad\quad a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5 \\
\hline
a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\text{, the fifth Power.} \\
a+b \\
\hline
a^6+5a^5b+10a^4b^2+10a^3b^3+5a^2b^4+ab^5 \\
\quad\quad a^5b+5a^4b^2+10a^3b^3+10a^2b^4+5ab^5+b^6 \\
\hline
a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6=\text{ 6th P.}
\end{array}$$

(6) Thus

$$\begin{array}{l} a-b \\ a-b \\ \hline a^2-ab \\ \quad ab+b^2 \\ \hline a^2-2ab+b^2, \text{ second Power.} \\ a-b \\ \hline a^3-2a^2b+ab^2 \\ \quad -a^2b+2ab^2-b^3 \\ \hline a^3-3a^2b+3ab^2-b^3, \text{ third Term.} \\ a-b \\ \hline a^4-3a^3b+3a^2b^2-ab^3 \\ \quad -a^3b+3a^2b^2-3ab^3+b^4 \\ \hline a^4-4a^3b+6a^2b^2-4ab^3+b^4, \text{ fourth Power.} \\ a-b \\ \hline a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4 \\ \quad -a^4b+4a^3b^2-6a^2b^3+4ab^4-b^5 \\ \hline a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5, \text{ fifth Power.} \\ a-b \\ \hline a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5 \\ \quad -a^5b+5a^4b^2-10a^3b^3+10a^2b^4-5ab^5+b^6 \\ \hline a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6=\text{6th P.} \\ \hline \end{array}$$

These Examples are performed by the Theorem as follows:

Here m, Index of the proposed Power, being 6; the first Term a^m, of the general Expression is equal to a^6; the second $ma^{m-1}b=6a^5b$; the third $\frac{m\times m-1}{1\quad 2}\times a^{m-2}b$ $=15a^4b^2$; the fourth $\frac{m\times m-1\times m-2}{1\quad 2\quad 3}\times a^{m-3}b=$ $20a^3b^3$;

$20a^3b^3$; the fifth $\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times a^{m-4}b$ $=15a^2b^4$; the sixth $\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}$ $\times a^{m-5}b=6ab^5$; the seventh $\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}$ $\times\frac{m-4}{5}\times\frac{m-5}{6}\times a^{m-6}b=b^6$; and the eighth $\frac{m}{1}\times\frac{m-1}{2}$ $\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6}\times\frac{m-6}{7}\times a^{m-7}b=$ nothing.

Therefore the sixth Power of $a+b=a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$, as before.

71. EVOLUTION.

(2) $x^2-2xy+y^2\,(x-y$, the Root.

$$\begin{array}{rl}
 & x^2-2xy+y^2\,(x-y \\
 & x^2 \\
\hline
2x-y) & *\;-2xy+y^2 \\
 & \;\;\,-2xy+y^2 \\
\hline
 & *\quad *
\end{array}$$

(3) $x^4-2x^3y+3x^2y^2-2xy^3+y^4\,(x^2-xy+y^2$, the Rt.

$$\begin{array}{rl}
 & x^4-2x^3y+3x^2y^2-2xy^3+y^4\,(x^2-xy+y^2 \\
 & x^4 \\
\hline
2x^2-xy) & *\;-2x^3y+3x^2y^2 \\
 & \;\;\,-2x^3y+\;\,x^2y^2 \\
\hline
2x^2-2xy+y^2 & *\quad 2x^2y^2-2xy^3+y^4 \\
 & \qquad 2x^2y^2-2xy^3+y^4 \\
\hline
 & \qquad *\quad *\quad *
\end{array}$$

(4) $x^3-6x^2y+12xy^2+8y^3\,(x-2y$, the Root.

$$\begin{array}{rl}
3x^2) & 6x^2y \\
\hline
 & x^3-6x^2y+12xy^2+8y^3 \\
\hline
 & *\quad *\quad *\quad *
\end{array}$$

(5)

$$16x^4-96x^3y+216x^2y^2-216xy^3+81y^4\,(2x-3y, \text{ Rt.}$$

$$32x^3)\ *\quad 96x^3y$$

$$16x^4-96x^3y+216x^2y^2-216xy^3-81y^4$$

* * * * *

72. REDUCTION OF SIMPLE EQUATIONS.

(1) Given $20-3x-8=60-7x$.

Then by tranſpoſing $7x$, we ſhall have $4x+12=60$.

Therefore $x=\frac{60-12}{4}=\frac{48}{4}=12$.

For $\overline{20-12}\times3-8=60-\overline{12\times7}=-24$.

(2) Given $5x-16=3x+12$.

Here by tranſpoſing the $3x$ and 16, we get $2x=12+16$.

Therefore $x=\frac{12+16}{2}=\frac{28}{2}=14$.

For $\overline{14\times5}-16=\overline{14\times3}+12=54$.

(3) Given $\frac{3x}{4}+5=\frac{5x}{6}+2$.

Then (per Rule III.) $18x+120=20x+48$.

Therefore $2x=72$, whence $x=72\div2=36$.

For $\frac{36\times3}{4}+5=\frac{36\times5}{6}+2=32$.

(4) Given $\frac{7x}{8}-5=\frac{9x}{10}-8$.

Then $70x-400=72x-640$, whence $2x=240$, and $x=120$.

For $\frac{120\times7}{8}-5=\frac{120\times9}{10}-8=100$.

(5) Given $\frac{5x}{9}-8=74-\frac{7x}{12}$.

Then by multiplying both Sides of the Equation by 9, and 12, we ſhall have $60x-864=7992-63x$.

Therefore $123x=7992+864=8856$, and $x=8856\div123=72$.

(6)

(6) Given $56-\frac{3x}{4}=48-\frac{5x}{8}$.

Then $1792-24x=1536-20x$.

Therefore $4x=256$, and $x=256\div4=64$, the Proof.

(7) Given $36-\frac{4x}{9}=8$.

Then by multiplying both Sides of the Equation by 9, we have $324-4x=72$.

Therefore $4x=324-72=252$.

Therefore $x=252\div4=63$.

(8) Given $\frac{2x}{3}=\frac{176-4x}{5}$.

Then $10x=528-12x$.

And $22x=528$.

Therefore $x=528\div22=24$.

(9) Given $\frac{45}{2x+3}=\frac{57}{4x-5}$.

Then multiplying by $2x+3$, we ſhall have $45=\frac{114x+171}{4x-5}$; alſo multiplying by $4x-5$, we have $180x-225=114x+171$.

Therefore $180x-114x-225=171$, that is $66x-225=171$.

Therefore $66x=171+225$, that is $66x=396$, and $x=396\div66=6$.

(10) Given $\frac{42x}{x-2}=\frac{35x}{x-3}$.

Then by multiplying by $x-2$, and $x-3$, we get $42x^2-126x=35xx-70x$.

Therefore $42x^2-35x^2=126x-70x$, that is $7x^2=56x$.

Therefore by dividing by $7x$, we have $x=8$.

(11) Given $\frac{xx-12}{3}=\frac{xx-4}{4}$.

Then by multiplying both Sides of the Equation by 3, and 4, we have $4x^2-48=3x^2-12$.

Therefore $4x-3x=48-12$, that is $x^2=36$.

Whence $x=\sqrt[2]{36}=6$.

(12)

(12) Given $\frac{5xx}{6}=8+12$.

Then by multiplying by 16, we get $5x^2=20\times16=320$. Therefore $x^2=320\div5=64$, and $x=\sqrt{64}=8$.

(13) Given $\frac{x+1}{2}+\frac{x+2}{3}=16-\frac{x+3}{4}$.

Then by multiplying by 2, 3, and 4, we get $12x+12+8x+16=384-6x-18$, that is $20x+28=366-6x$. Therefore $20x+6x=366-28$, that is $26x=338$. Whence $x=338\div26=13$.

(14) Given $ax+b^2=\frac{ax^2+ac^2}{a+x}$.

Then $\overline{ax+b^2}\times\overline{a+x}=ax^2+ac^2$, that is $a^2x+ab^2+ax^2+b^2x=ax^2+ac^2$.

Therefore $a^2x+b^2x=ac^2-ab^2$, and $x=\frac{ac^2-ab^2}{a+b}$.

(15) Given $\sqrt{\frac{5x}{3}}+12=17$.

Then $\sqrt{\frac{5x}{3}}=17-12=5$, and $\sqrt{5x}=5\times3=15$. Therefore (by Rule IV.) $5x=225$, and $x=225\div5=45$.

(16) Given $\sqrt{12+x}=2+\sqrt{x}$, then (by Rule IV.) $12+x=4+4\sqrt{x}+x$.

Whence by tranſpoſition $4\sqrt{x}=8$, and by diviſion $\sqrt{x}=2$, conſequently $x=4$.

(17) Given $\sqrt{x}+\sqrt{a+x}=\sqrt{\frac{2a}{a+x}}$.

Then $\sqrt{ax+xx}+a+x=2a$, or $\sqrt{ax+xx}=a-x$. Whence $ax+x^2=a^2-2ax+x^2$. Therefore $3ax=a^2$.

$\therefore x=\frac{a^2}{3}$.

(18)

(18) Given $615x-7x^3=48x$.

Then by tranſpoſition $615x-48x=7x^3$, that is $7x^3=567x$.

Whence divided by $7x$, we get $x^2=81$.

$\because\ x=y$.

(19) Given $\sqrt{a^2+x^2}=\sqrt{b^4+x^4}$.

Then by raiſing both Sides to the fourth Power, we have $\overline{aa^2+xx}|^2=b^4+x^4$, that is $a^4+2a^2x^2+x^4=b^4+x^4$.

Therefore $2a^2x^2=b^4-a^4$.

$\because\ x^2=\dfrac{b^4-a^4}{2a^2}=\dfrac{b^4}{a^2}-a^2$.

73. Of the EXTERMINATION of UNKNOWN QUANTITIES; or, the REDUCTION of two or more EQUATIONS, to a single one.

(1) Given $\begin{cases}5x+8y=106\\4x-5y=\ \ 5\end{cases}$

Here by multiplying the firſt Equation by 4, and the ſecond by 5, in order that the Coeficient of x may be the ſame in both, there ariſes

$$20x+32y=424$$
$$20x-25y=\ 25$$

By ſubtracting the latter from the former we have $57y=399$.

Hence $y=\dfrac{399}{57}=7$.

And ſo by the firſt Equation $x=\dfrac{5+\overline{5\times7}}{4}=\dfrac{40}{4}=10$.

(2) Given $\begin{cases}5x-\ \ 3y=150\\10x+15y=825\end{cases}$

Here the firſt Equation being multiplied by 2, (in order that the Coeficients of x, in both Equations, may be the ſame) we have $10x-6y=300$. Let this Equation be ſubtracted from the ſecond, and we ſhall have $21y=525$.

Hence

Hence $y=\frac{525}{21}=25$.

Therefore by the first Equation $x=\frac{150+25\times 3}{5}=\frac{225}{5}=45$.

(3) Given $\begin{cases} \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62 \\ \frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47 \\ \frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38 \end{cases}$

Here the given Equations cleared of the Fractions, become

$$12x+8y+6z=1488$$
$$20x+15y+12z=2820$$
$$30x+24y+20z=4560$$

Now, to exterminate z, let the second of these Equations be subtracted from the double of the first. And also, three Times the third, from four Times the second. Whence is had

$$4x+y=156$$
$$10x+3y=420$$

Again, to exterminate y, from three Times the first take the second, and we shall then have $12x-10x=468-420$. From which $x=\frac{48}{2}=24$.

Therefore $y=156-24\times 4=60$.

And $z=\left(\frac{1480-8y-12x}{6}\right)=120$.

(4) Given $\begin{cases} \frac{x}{4}+\frac{y}{5}=15 \\ \frac{x}{6}+\frac{y}{9}=9 \end{cases}$

Here our Equations cleared of Fractions will be

$$5x+4y=300$$
$$9x+6y=486$$

And

And if from double of the latter we take triple of the former, we ſhall have $18x-15x=972-900$, that is $3x=72$.

Whence $x=\frac{72}{3}=24$.

And $y=\frac{300-5x}{4}=45$.

(5) Given $\begin{cases} \frac{x}{2}-12=\frac{y}{4}+8 \\ \frac{x+y}{5}+\frac{x}{3}-8=\frac{2y-x}{4}+27 \end{cases}$

Here the Equations cleared of the Fractions we ſhall have

$$4x-96=2y+64$$

And $12x+12y+20x-480=30y-15x+1620$

Which contracted, become

$$4x-\ 2y=64+96=160$$
$$47x-18y=2100$$

From the laſt of which ſubtract nine Times the former, and we have $41x-36x=2100-1440$, that is $11x=660$.

Hence $x=\frac{660}{11}=60$.

And $y=\frac{4x-160}{2}=\frac{240-160}{2}=\frac{80}{2}=40$.

(6) Given $\begin{cases} x+y=80 \\ x+z=70 \\ y+z=60 \end{cases}$

Here by ſubtracting the ſecond Equation from the firſt (in order to exterminate x) we have $y-z=10$, to which the third Equation being added, z will likewiſe be exterminated, there coming out $2y=70$, or $y=35$.

Whence $z=y-10=25$, and $x=80-y=45$.

(7) Given $\begin{cases} x+100=\ y+\ z \\ y+100=2x+2z \\ z+100=3x+3y \end{cases}$

To the double of the firſt, let the ſecond Equation be added: ſo ſhall the x's, on the contrary Sides deſtroy each other, and we ſhall have $300+y=2y+4z$, or $y+4z=300$.

More-

Moreover, to the triple of the first let the third Equation be added, whence will be had $z+400=6y+3z$, or $2z+6y=400$.

Now if from the double of this last Equation (which $4z+12y=800$) the former (viz. $y+4z=300$) be subtracted, then will come out $11y=500$.

Therefore $y=\frac{500}{11}=45\frac{5}{11}$, and $z=\frac{300-y}{4}=75-\frac{y}{4}$ $=75-11\frac{4}{11}=63\frac{7}{11}$.

And $x=y+z-100=109\frac{1}{11}-100=9\frac{1}{11}$.

(8) Given $x-y=2$, and $xy+5x-6y=120$.
Then to exterminate x.

By the first Equation $x=2+y$; which Value being substituted in the last (according to Rule II.) it becomes $\overline{y+2}\times y+5\times\overline{y+2}-6y=120$, that is $y^2+2y+5y+10-6y=120$, or $y^2+y=110$.

(9) Given $\left\{\begin{array}{l} x+y=s \\ x^2-y^2=d \end{array}\right\}$ quere x and y.

Then $x=s-y$; the Square of which is $x^2=s^2-2sy+y^2$.
Also ${}^2x=d+y^2$.
Therefore $d+y^2=s^2-2sy+y^2$.
Hence $d=s^2-2sy$.
And $2sy=s^2-d$.
So $y=\frac{s^2-d}{2s}$.
And $x=\frac{s^2+d}{2s}$.

(10) Given $\left\{\begin{array}{l} x+y+z=12 \\ x+2y+3z=20 \\ \frac{x}{y}+\frac{y}{2}+z=6 \end{array}\right\}$ To find x, y, and z.

Then we have $\left\{\begin{array}{l} 1.\ x=12-y-z \\ 2.\ x=20-2y-3z \\ 3.\ x=68-\frac{3}{2}y-3z \end{array}\right.$

From whence we have these two Equations. $\left\{\begin{array}{l} 12-y-z=20-2y-3z \\ 12-y-z=18-\frac{3}{2}y-3z \end{array}\right.$

Therefore we have (by Rule I.) $y=4-z=2$, and $x=6$.

Some-

Sometimes the Equations are such, that the same Quantities in different Equations may have contrary Signs, and destroy each other; or to be otherwise affected, so as to shorten the common Progress very much.

Thus $\begin{cases} x+y+z=26 \\ x-y=4 \\ x-z=6 \end{cases}$

Then by Addition only, $3x=36$.
Hence $x=12$, $y=x-4=8$, and $z=x-6=6$.

104. QUADRATIC EQUATIONS.

(1) Given $x^2-4x=32$, to find the Value of x.
Here $x^2-4x+4=32+4=36$, (by Rule III.)
And $x-2=\sqrt{36}=6$, (by Rule IV.)
Therefore $x=6+2=8$.

(2) Given $12x^2-420x=-1200$, to find x.
Here $x^2-35x=-100$, (by Rule II.)
And $x^2-35x+306,25=206,25$, (by Rule III.)
Also $x-17,5=\sqrt{206,25}\pm 14,361406$, (by Rule IV.)
Therefore $x=31,861406$, or $3,138594$.

(3) Given $x^2+60x=216$, to find x.
Here $x^2+15x=54$, (by Rule II.)
And $x^2+15x+56,25=110,25$.
Also $x+7,5=\sqrt{110,25}=10,5$.
Therefore $x=10,5-7,5=3$.

The fifteen following Equations, or Questions, are from Mr. WARD's MATHEMATICIAN's GUIDE.

(1) Given $\left.\begin{cases} a+e=s=240 \\ a-e=d=192 \end{cases}\right\}$ to find.
Here by Addition we get $2a=s+d$.

Therefore $a=\frac{s+d}{2}=\frac{240+192}{2}=\frac{432}{2}=216$.

Then by subtracting the second Equation from the first we get $2e=s-d$.

And $e=\frac{s-d}{2}=\frac{240-192}{2}=\frac{48}{2}=24$.

 Now

Now as a and e are found, the rest may be very easily found as follows

$ae = \frac{ss-dd}{4}$, or $216\times24=5184=p$.

And $\frac{a}{e}=\frac{s+d}{s-d}$, or $216\div24=9=q$.

Also $a^2+e^2=\frac{s^2+^2d}{2}$, or $\overline{216\times216}+\overline{24\times24}=47232$ $=z$.

Likewise $a^2-e^2=sd$, or $\overline{216\times216}-\overline{24\times24}=46080$ $=x$.

(2) Given $\left\{\begin{array}{l} a+e=s=240 \\ ae\quad =p=5184 \end{array}\right\}$ to find a, e, d, q, z, x.

Here by subtracting 4 Times the second Equation from the Square of the first, we have $a^2-2ae+e^2=s^2-4p$.

Therefore $a-e=\sqrt{s^2-4p}=d$. Now by adding this Equation to the first we get $2a=s+\sqrt{\frac{s^2-4p}{2}}$.

Hence $a=\frac{s+\sqrt{ss-4}}{2}=216$.

And by subtracting we have $2e=s-\sqrt{ss-4p}$.

Hence $e=\frac{s\sqrt{s^2-4p}}{2}=24$.

Therefore the rest may be very easily found by proceeding as in the last.

(3) Given $\left\{\begin{array}{l} a+e=s=240 \\ \frac{a}{e}\quad =q=\ 9 \end{array}\right\}$ to find the rest.

Here by subtracting the second Equation from the first, we have $e=s-qe$.

Therefore $e=\frac{s}{9+1}$, $=\frac{240}{10}=24$. Which taken from the first Equation gives $a=s-\frac{s}{q+1}=240-\frac{240}{9+1}=$ $240-24=216$.

From hence the rest may be very easily found.

(4)

(4) Given $\left\{\begin{array}{l} a+e=s=240 \\ a^2+e^2=z=47232 \end{array}\right\}$ to find the rest.

Here from the Square of the first Equation take the second, and we have $2ae=ss-z$ Which taken from the second Equation gives $a^2-2ae+e^2=2z-s^2$.

Therefore $a-e=\sqrt{2z-s^2}=d$.

This added to the first Equation gives $2a=s+\sqrt{2z-s^2}$.

Hence $a=\dfrac{s+\sqrt{2z-s^2}}{2}=216$.

And subtracted gives $2e=s-\sqrt{2z-s^2}$.

Whence $e=\dfrac{s-\sqrt{2z-s^2}}{2}=24$.

(5) Given $\left\{\begin{array}{l} a+e=s=240 \\ a^2-e^2=x=46080 \end{array}\right\}$ to find the rest.

Here the second Equation divided by the first gives

$a-e=\dfrac{x}{s}=d$.

Which added to the first, we have $2a=s+\dfrac{x}{s}=\dfrac{s^2+x}{s}$.

Hence $a=\dfrac{s^2+x}{2s}=216$.

And the third subtracted from the first gives

$2e=s-\dfrac{x}{s}=\dfrac{s^2-x}{s}$.

Hence $e=\dfrac{s^2-x}{2s}=24$.

(6) Given $\left\{\begin{array}{l} a-e=d=192 \\ ae=p=5184 \end{array}\right\}$ to find the rest.

Here to the Square of the first Equation add 4 Times the second, and we shall have $a^2+2ae+e^2={}^2d+4p$.

Therefore $a+e=\sqrt{dd+4p}=s$.

This Equation added to the first gives $2a=d+\sqrt{dd+4p}$.

Therefore $a=\dfrac{d+\sqrt{dd+4p}}{2}=216$.

 Then

Then by taking the first Equation from the fourth, we have $2e=\sqrt{dd+4p}-d$.

Hence $e=\sqrt{\dfrac{dd+4p-d}{2}}=24$.

(7) Given $\left\{\begin{array}{l} a-e=d=192 \\ \dfrac{a}{e}\quad=q=\ \ 9 \end{array}\right\}$ to find the rest.

Here by transposing e in the first Equation, and multiplying the second by e, we get $qe=d+e$.

Then $e=\dfrac{d}{q-1}=24$.

For $\overline{q-1}\times e=qe-e$. And by adding this last Equation to the first we have $a=d+\dfrac{d}{q+1}=216$.

(8) Given $\left\{\begin{array}{l} a-e=d=\quad 192 \\ a^2+e^2=z=47232 \end{array}\right\}$ to find the rest.

Here by taking the square of the first from the second, we have $2ae=z-dd$. This added to the second Equation gives $a^2+2ae+e^2=2z-dd$. Therefore $a+e=\sqrt{2z-dd}$. Which added to the first Equation gives $2a=d+\sqrt{2z-dd}$.

Therefore $a=\dfrac{d+\sqrt{2z-dd}}{2}=216$.

Then by taking the first Equation from the fifth we get $2e=\sqrt{2z-dd}-d$. Hence $e=\sqrt{\dfrac{2z-dd-d}{2}}$.

(9) Given $\left\{\begin{array}{l} a-e=d=\quad 192 \\ a^2-{}^2e=x=46080 \end{array}\right\}$ to find the rest.

Here by dividing the second Equation by the first, we have $a+e=\dfrac{x}{d}$. Then by adding this Equation to the first, we get $2a=d+\dfrac{x}{d}$. Hence $a=\dfrac{dd+x}{2d}$.

And

And by taking this Equation from the third, we have $e=\frac{x-dd}{2d}$.

(10) Given $\left\{\begin{array}{l} ae=p=5184 \\ \frac{a}{e}=q=\quad 9 \end{array}\right\}$ to find the reſt.

Here by multiplying the two Equations together, we have $aa=qp$. For $\frac{ae}{1}\times\frac{a}{e}=\frac{aae}{e}=aa$. Then $a=\sqrt{qp}$, $=216$.

And by dividing the firſt Equation by the ſecond, we ſhall have $ee=\frac{p}{q}$.

For $\frac{a}{1}\Big)\frac{ae}{1}\Big(\frac{aee}{a}=ee$. Hence $e=\sqrt{\frac{p}{q}}=24$.

(11) Given $\left\{\begin{array}{l} ae \qquad =p=\ \ 5184 \\ a^2+e^2=z=47232 \end{array}\right\}$ to find the reſt.

Here to the ſecond add twice the firſt, and we have $a^2+2ae+^2e=z+2p$.

Therefore $a+e=\sqrt{z+2p}=s$.

Then by taking twice the firſt Equation from the ſecond, we have $a^2-2ae+^2e=x-2p$.

Hence $a-e=\sqrt{z-2p}$. This Equation added to the fourth gives $2a=\sqrt{z+2p}+\sqrt{z-2p}$.

Therefore $a=\frac{\sqrt{z+2p}+\sqrt{z-2p}}{2}=216$.

Then by taking the ſixth Equation from the fourth, we have $2e=\sqrt{z+2p}-\sqrt{z-2p}$.

Hence $e=\frac{\sqrt{z+2p}-\sqrt{z-2p}}{2}=24$.

(12) Given $\left\{\begin{array}{l} ae \qquad =p=5184 \\ a^2-^2e=x=46380 \end{array}\right\}$ to find the reſt.

Here to the Square of the second Equation add 4 Times the Square of the first, and we shall have $a^4+2a^2e^2+e^4=xx+4p^2$. Hence $a^2+e^2=\sqrt{x^2+4p}=z$.

This Equation added to the second will give $2a^2=x+\sqrt{x^2+4p^2}$. Whence $a=\sqrt{x+\sqrt{x^2+4p^2}}=216$.

Then by subtracting the second Equation from the fourth, we have $2e^2=\sqrt{x^2+4p^2}-x$.

Hence $e=\sqrt{:\frac{\sqrt{x^2+4p^2}-x}{2}}=24$.

(13) Given $\left\{\begin{array}{l}\frac{a}{e}=q=9\\ a^2+e^2=z=47232\end{array}\right\}$ to find the rest.

Here from the second Equation take the Square of the first, and we shall have $ee=z-qqee$, or $qqee+ee=z$.

Hence $ee=\frac{z}{qq+1}$. For $\overline{qq+1}\times ee=qqee+ee$. Therefore $e=\sqrt{\frac{z}{qq+1}}=24$.

Then by taking this last Equation from the second, we have $aa=z-\frac{z}{qq+1}=\frac{qq}{qq+1}$. Hence $a=\sqrt{\frac{qqz}{qq+1}}=216$.

(14) Given $\left\{\begin{array}{l}\frac{a}{e}=q=9\\ aa-ee=x=46080\end{array}\right\}$ to find the rest.

Here by comparing the first when multiplied by e, and squared with the second more ee, we have $qqee=x+ee$, or $qqee-ee=x$.

Hence $ee=\frac{x}{qq-1}$.

Therefore $e=\sqrt{\frac{x}{qq-1}}=24$.

Then by adding the fourth Equation to the second, we get $aa=x+\frac{x}{qq-1}=\frac{qqx}{qq-1}$. Hence $a=\sqrt{\frac{qqx}{qq-1}}=216$.

(15)

(15) Given $\left\{\begin{array}{l} aa+ee=z=47232 \\ aa-ee=x=46080 \end{array}\right\}$ to find the rest.

Here by addition (only) we have $2aa=x+z$.

Hence $a=\sqrt{\frac{x+z}{2}}$.

Then by subtracting the second Equation from the first, we get $2ee=z-x$.

Therefore $e=\sqrt{\frac{z-x}{2}}$.

The Reason of my proceeding (only) with the first Question throughout, is being confined, not having Room; but I hope this will not be the Pupil's Case.

PROBLEMS.

(1) Here let $x=$ the greater Number.
Then $x-20=$ the lesser.
Which added together gives $2x-20=70$.
Therefore $2x=70+20=90$.
And $x=90\div 2=45$, the greater Number.
$\therefore$ $45-20=25$, the lesser.
For $45-25=20$.
And $45+25=70$, the Proof.

(2) Here let $x=$ the greater Number.
Then $x-14=$ the lesser.

Therefore by the Quest. $\frac{x}{x-14}=3$.

Hence $x=3x-42$, and $2x=42$.
$\therefore$ $x=42\div 2=21$, the greater Number.
Likewise $21-14=7$, the lesser.
For $21-7=14$.
Also $21\div 7=3$, the Proof.

(3) Suppose x to be the Number.

Then by the Quest. $\frac{x}{3}+\frac{x}{4}=21$.

Therefore $7x=252$.
Hence $x=36$, the Number required.

For $\frac{36}{3}+\frac{36}{4}=12+9=21$, the Proof.

(4)

(4) Let x be the Number.

Then per Quest. $\frac{x}{3}-\frac{x}{4}=4$.

Hence $x=48$, the Number required.

For $\frac{48}{3}-\frac{48}{4}=16-12=4$, the Proof.

(5) Suppose $x=$ the Number required.

Then per Quest. $\frac{x}{3}-4=\frac{x}{4}$,25.

Hence $4x-48=3x-3$.

Therefore by Transposition $x=45$.

For $\frac{45}{3}-4=\frac{45}{4}$—,25$=\overline{15-4}=\overline{11,25-}$,25, the Proof.

(6) Let x represent the greater, and $x-8$ the lesser Number.

Then per Quest. $16x-64=208$.

Hence $16x=272$, and $x=17$, the greater.

Also $17-8=9$, the lesser Number.

For $17-9=8$, the Difference.

And $\overline{17\times17}-\overline{9\times9}=289-81=208$, the Proof.

The above Question may be solved by making use of two Letters. See Mr. WARD's 9th Question.

(7) Let $x=$ the greater Number, and y the lesser.

Then by the Quest. we shall have $x+y=60$.

And as $x:y::9:3$.

Therefore $9y=3x$, or $9y-3x=0$.

To which add the first Equation multiplied by 3, and we shall have $12y=180$.

$\because y=15$, the lesser Number.

And $60-15=45$, the greater.

For $45+15=60$.

And as $45:15::9:3$, the Proof.

(8) Here let $x=$ the greater Number, and y the lesser.

Then per Quest. $xy=108$.

And $\frac{3x}{y}=4$.

Therefore by destroying x, we shall have $\frac{108}{y}=\frac{4y}{3}$.

Hence

Hence $4y^2=324$, and $y^2=81$.
$\because y=9$, the leffer Number.

And $\frac{108}{9}=12$, the greater.

For $12\times9=108$.

Alfo $\frac{12\times3}{9}=\frac{36}{9}=4$, the Proof.

(9) Let $x=$ the greater Number, and y the leffer.
Then by the Queftion we fhall have

$$1.\quad x+y+8=2x$$

$$2.\quad x-y-4,5=\frac{y}{2}$$

Then by multiplying the fecond Equation by 2, we get $2x-2y-9=y$.
And by the firft Equation $x=y+8$.
Which fubftituted for x in the laft Equation, we fhall have $y=7$, the leffer Number.
Then $x=7+8=15$, the greater.
For $15+7+8=15\times2=30$.

Alfo $15-7-4,5=\frac{7}{2}=3,5$, the Proof.

(10) For the Numbers fought, put x, y, and z.
Then by the Queftion we fhall have

$$\left.\begin{array}{ll}1. & x+\frac{y+z}{2}= \\ 2. & y+\frac{x+z}{3}= \\ 3. & z+\frac{x+y}{4}=\end{array}\right\}34=a.$$

Then by clearing the Equations of the Fractions, we get as follows

$$4.\quad 2x+y+z=2a$$
$$5.\quad 3y+x+z=3a$$
$$6.\quad 4z+x+y=4a$$

Now (to exterminate z) let the fourth Equation be fubtracted from the fifth; alfo the fixth from four Times the fifth, whence is had

$$7.\quad 2y-x=a$$
$$8.\quad 11y+3x=8a$$

From which we have $2y-a=\frac{8a-11y}{3}$.

There-

Therefore $17y = 11a = 374$.

Hence $y = \frac{374}{17} = 22$.

Then by the seventh, $x = 44 - 34 = 10$.

Also by the fourth, $z = 68 - 42 = 26$.

For $10 + \frac{22+26}{2} = 34$.

And $22 + \frac{10+26}{3} = 34$.

Also $26 + \frac{10+22}{4} = 34$, the Proof.

(11) Let $3x =$ the major Part of the first Division.

Then $100 - 3x =$ the minor Part.

And by the Question we shall have $x =$ the minor Part of the second Division.

Also $100 - x =$ the major Part.

Whence by the Question $100 - x = 200 - 6x$.

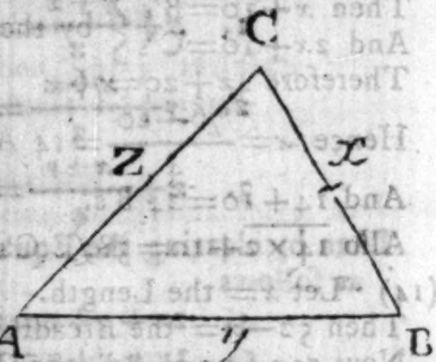
Hence $x = \frac{100}{5} = 20$, the minor Part of the 2d Division.

And $100 - 20 = 80$, the major Part.

Again $20 \times 3 = 60$, major } Part of first Division.
And $100 - 60 = 40$, minor }

For $60 = 20 \times 3$, and $80 = 40 \times 2$, the Proof.

(12) Here put s and c for the natural Sine and Cosine of the Angle ABC, Radius $= 1$, a for the Area of the Triangle, m for the Sum of the Cubes of the Sides, and x, y, and z, for the Sides, as in the Figure.



Then we shall have $sxy = 2a$, and $x^2 + y^2 = z^2 + 2cxy$ (per Trigonometry.)

Hence $xy \frac{2a}{s}$ which call p, and $x^2 + y^2 = z^2 + 2cp$, for $2cp$ put b; then $x^2 + y^2 = z + b$.

Now $x = \frac{p}{y}$, $\therefore x^3 = \frac{p^3}{y^3}$, and therefore the Equation

tion $x^3+y^3+z^3=m$, as given by the Queſt. becomes $\frac{p^3}{y^3}+y^3+z^3=m$, whence $z^3=m-y^3-\frac{p^3}{y^3}$.

Again $z^2=x^2+y^2-b$, whence $z^2=\frac{p^2}{y^2}+y^2-b$, conſequently $z^6=\overline{m-y^3-\frac{p^3}{y^3}}\big|^2$, and $z^6=\overline{\frac{p^4}{y^2}+y^2-b}\big|^3$, hence $\overline{my^3-y^6-p^3}\big|^2=\overline{p^2+y^4-by^2}\big|^3$.

This Equation properly reduced will give $y=8$, from whence x is found to $=10$, and $z=6$, the Sides required.

Suppoſe the Sum of the Squares of the Sides had been given, inſtead of the Sum of the Cubes; then we ſhould have had $x^2+y^2+z^2=m$, and the other Equations as before, viz. $xy=p$, and $x^2+y^2=z+b$, hence $z^2+b+z^2=m$; or $z=\sqrt{\frac{m-b}{2}}$, which being now known, put $z^2+b=n$, then $x^2+y^2=n$; but $xy=p$, therefore $x^2+2xy+y^2=n+2p$, and $x^2-2xy+y^2=n-2p$. Conſequently $x+y=\sqrt{a+2p}$, and $x-=y\sqrt{n-2p}$; therefore $x=\sqrt{n+2p}+\sqrt{n-2p}$.

(13) Let x repreſent A's Contribution.

Then $x+10=$ B's } by the Queſtion.
And $2x+10=$ C's }

Therefore $4x+20=76$.

Hence $x=\frac{76-20}{4}=14$ A's.

And $14+10=24$ B's.

Alſo $\overline{14\times2}+10=38$ £. C's Contribution.

(14) Let $x=$ the Length.

Then $52-x=$ the Breadth.

Now $52x-xx=480$, per Queſtion.

Therefore $xx-52x=-480$.

And $xx-52x+676=676-480=196$.

$\therefore x-26=14$.

Hence $x=40$ in Length.

And $52-40=12$, in Breadth.

This Queſtion may be ſolved by making uſe of two Letters. See Mr. WARD's ſecond Queſtion.

(15)

(15) Suppose $x=$ the Quantity sold at 12*d.* per lb.

Then as $\frac{3}{4}:\frac{2}{3}::x:\frac{8x}{9}=$ the other at 15*d* per lb.

Therefore by multiplying each Quantity by its Price, we have $12x+\frac{40x}{3}=19\times 12=228$.

Hence $36x+40x=684$.

From which $x=\frac{684}{76}=9$ lb. at 1*s.* and $\frac{8x}{9}=8$ lb. at 1 *s.* 3*d* per lb.

For as $9:8::\frac{3}{4}$ to $\frac{2}{3}$, the Proof.

(16) Suppose he had x Guineas.

Then by the Quest. $x-\frac{x}{4}-\frac{x}{5}=66$.

$\therefore\ 20x-9x=1320$.

Hence $11x=1320$.

And $x=120$ Guineas, had at first.

(17) Let $x=$ the greater, and y the lesser Number.

$\left.\begin{array}{l}\text{Then } xx+xy=77\\ \text{And } xy-yy=12\end{array}\right\}$ by the Question.

Now by destroying xy, we get the following Equation $77-xx=12+yy$.

Hence $xx=65-yy$.

$\therefore\ x=\sqrt{65-yy}$.

Then by substituting this for x, in the second Equation, we have $y\sqrt{65-yy}-yy=12$.

Or $\sqrt{65-yy}=\frac{12+yy}{y}$.

Then by squaring each Side, $65-yy=\frac{144+24y^2+y^4}{y^2}$.

Or $2y^4-41y^2=-144$.

Therefore by $\div 2$, we have $y^4-20{,}5y^2=-72$.

And by completing the Square, we get $y^4-20{,}5y^2+105{,}0625=33{,}0625$.

Or $y^2-10{,}25=\sqrt{33{,}0625}=5{,}75$.

Then $y^2=16$.

Hence $y=4$, and $65-16=49=x^2$.

Therefore $x=7$.

(18)

(18) Let $x=$ the Number required.

Then by the Quest. $4xx-40x+100-1=7x$.

Or $4x^2-47x=-99$.

Therefore by $\div 4$, we have $x^2-11{,}75x=-24{,}75$.

Or $x^2-11{,}75x+34{,}515625=9{,}765625$.

$\therefore x-5{,}875=\sqrt{9{,}765625}=3{,}125$.

Hence $x=3{,}125+5{,}875=9$, the Number required.

For $\overline{9\times 2}-10=8$.

Also $\overline{8\times 8}-1=9\times 7=63$, the Proof.

(19) Here let $x=$ his part, and y her's.

Then $\left\{\begin{array}{l}\frac{x}{5}=\frac{y}{4}+10\\ x+y=1000\end{array}\right\}$ by the Question.

Therefore by destroying the x's we get $\frac{5y+200}{4}=1000-y$.

Or $5y+200=4000-4y$.

Then $y=3800\div 9=422\frac{2}{9}$ her $\}$ Share.
And $1000-422\frac{2}{9}=577\frac{7}{9}$ his $\}$

(20) Let $x=$ the least Number.

Then we shall have x, $x+2$, $x+4$, and $x+6$, for the Numbers.

But $x\times\overline{x+2}\times\overline{x+4}\times\overline{x+6}=x^4+12x^3+44x^2+48x$.

Therefore $x^4+12x^3+44x^2+48x=945$, (by the Quest.)

Wherefore $105x^2=945$. Hence $x^2=9$. And $x=3$, the least.

Therefore the Numbers are 3, 5, 7, and 9.

(21) Put $x=$ his Age in Months.

Then $x+\frac{x}{2}+\frac{x}{8}-1=21\times 21=441$, (by the Quest.)

Or $16x+8x+2x-16=7056$. Or $26x=7072$.

Hence $x=\frac{7072}{26}=272$ m. or 22 yrs. 8 m. his Age.

For $272+\frac{272}{2}+\frac{272}{8}-1=21\times 21=441$, the Proof.

(22) Let x, y, and z, be the Numbers required.

Then per Quest. $x-y=6$; or $y-6=x$, and $z-y=15$; or $z=15+y$.

But $x:y::y:z$; that is $y-6:y::y:15+y$.

 There-

Therefore $yy=90-9y+yy$. Hence $9y=90$. And $y=10$, the second.

Then $10-6=4$, the first.

And $15+10=25$, the third Number.

(23) Suppose x to be the Number.

Then $x^6-19x^3=216$, (by the Quest.)

Put $x^3=z$, $x^6=z^2$, and it will be $z^2-19z+\frac{361}{4}=216+\frac{361}{4}=\frac{1225}{4}$, (per Rule p 332. in the Guide.)

And $z-\frac{19}{2}=\pm\frac{35}{2}$. Whence $z=\frac{19\pm35}{2}=27$, or $=-8$. But $x=\sqrt{z}$. Therefore $x=+3$, or -2.

(24) Let $x=$ the Length.

Then per Quest. $3:2::x:\frac{2x}{3}=$ the Breadth.

Therefore $\frac{x\times2x}{3}=60000$, or $2xx=180000$.

Hence $xx=90000$. And $x=300$ Men in Rank.

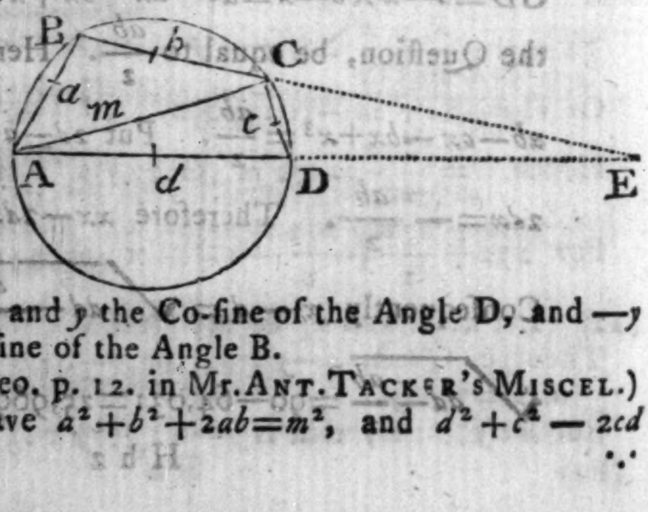
Also $\frac{300\times2}{3}=200$ Men in File.

Now $\overline{300-1}\times2{,}75=822{,}25$ yds. Length.

And $\overline{200-1}\times2{,}75=547{,}25$ ditto Breadth.

4840)449976312 5(92 acr. 3 rds. 35,2 p. the Ground they stand on.

(25) Put $a=AB=15{,}60$, $b=BC=13{,}2$, $c=CD=10$, and $d=AD=26$.



Also $m=AC$, and y the Co-sine of the Angle D, and $-y$ = the Co-sine of the Angle B.

Then (per Theo. p. 12. in Mr. ANT. TACKER's MISCEL.) we shall have $a^2+b^2+2ab=m^2$, and $d^2+c^2-2cd=m^2$.

∴

∴ $a^2+b^2+2ab=d^2+c^2-2dc$. Hence $2aby+2dcy=\overline{a^2+c^2}-\overline{a^2-b^2}$. And $y=\frac{d^2+c^2-a^2-b^2}{2ab+2ac}$, whose Sine call s.

Then $\frac{abs+dcs}{2}=21,504$, the Area of the Trapezium ABCD.

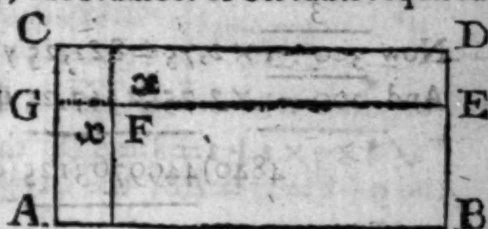
Or thus, suppose DE$=x$. Then by sim. Triangles $a : \overline{d+x} :: c : \frac{\overline{d+x}\times c}{a}=50=$CE. ∴ BE$=\frac{\overline{ba+d+x}\times c}{a}$.

Again $a : \frac{\overline{ba+d+x}\times c}{a} :: c : x$. Hence $x=\frac{\overline{ab+cd}\times c}{a^2-c^2}=$ 32,5. And the Area of the Trapezium ABCD $=$ 21,504 Acres, as before (by Sect. 70. Prob. III. Rule III.)

(26) Suppose $x=$ the Number of Servants.

Then by the Appendix we have $\frac{x\overline{x+1}-x}{x-1}=960799$, all the possible Variations of the Servants; which Equation solved, will give $x=7$, the Number of Servants required.

(27) Put AB$=100=a$, BC$=80=b$, AF$=$ CD, the Breadth of the Walk $=x$.



Then FG$=a-x$, GD $=b-x$. And FG $\times$ GD$=\overline{a-x}\times\overline{b-x}=ab-ax-bx+x^2$, which must by the Question, be equal to $\frac{ab}{2}$. Hence this Equation $ab-ax-bx+x^2=\frac{ab}{2}$. Put $2d=a+b$. Then $xx-2dx=-\frac{ab}{2}$. Therefore $xx-2dx+dd=dd-\frac{ab}{2}$. Consequently $x-d=\sqrt{dd-\frac{ab}{2}}$. Hence $x=d$ $\sqrt{dd-\frac{ab}{2}}=90-64,031=25,969$, the Breadth req.

 (28)

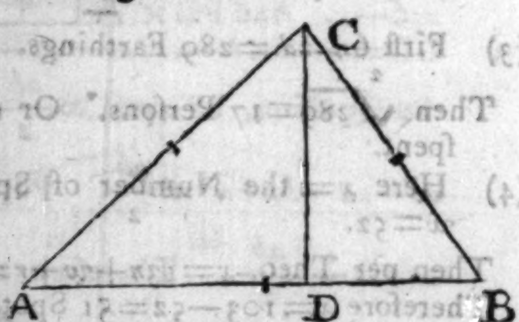
(28) Put x the side of the Square.
Then $xx+284=$ the whole Army.
And $x+1=$ the Number of Men designed for the Side of the Square.
Consequently $xx+2x+1-25=xx+284$. And by Transposition $2x=308$. Hence $x=154$ Men, the Side of the sq.
Wherefore $154\times154+284=24000$ Men, the whole Army required.

(29) Suppose $x=$ what he received.
Then per Quest. $\frac{x-94}{2}+\frac{x-94}{5}+\frac{x}{10}+94=x$.
This Equation cleared of the Fractions, we shall have $50x-4700+20x-1880+10x+9400=100x$, or $20x=2820$.
Hence $x=141$ £. the Sum required.

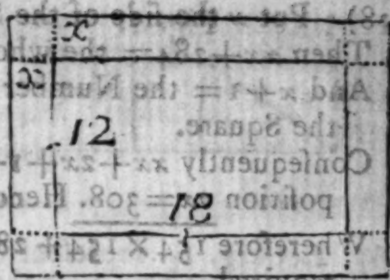
(30) Put BF$=9=a$, DE$=2=b$, and $x=$AD$=$DE.
Then DB$=a+x$, and AB$=$EB$=a+x-b$.
Therefore by the 47th. of EUCLID's first Book, we have DB$^2=$AB$^2+$AD^2, that is $a^2+2ax+x^2=x^2+a^2+2ax-2ab+x^2-2bx+b^2$, or $x^2-2bx=2ab-b^2$.

Then by Comp. the Square we get $x^2-2bx+b^2=2ab$.
Therefore $x-b=\sqrt{2ab}$. Hence $x=\sqrt{2ab}+b=\sqrt{2\times9\times2}+2=8=$AD, the Breadth.
Also $9+8=17=$AB, the Length.

(31) Here AB$=13=a$, AC$=14=b$, BC$=15=c$, and $x=$BD.
Then by the 47th. of Book I. of EUCLID. we shall have AB$^2-$BD$^2=$AC$^2-$CD^2. Or $a^2-x^2=b^2-c^2+2cx-x^2$, that is $a^2=b^2-c^2+2cx$.



And by Transposition we get $2cx=a-b^2+c^2$. Hence $x=\frac{a^2-b^2+c^2}{2c}=\frac{169-296+225}{30}=\frac{198}{30}=6{,}6=$ BD.
And $15-6{,}6=8{,}4=$DC. *Q. E. F.*



(32) Let $x=$ the Breadth sought.

$18+2x=$ Length AB or DC.

$12+2x=$AD or BC.

Hence $4x^2+60x+216=18\times12\times2=432$. Or $4x^2+60x=432-216=216$.

Which $\div$ 4 gives $x^2+15x=54$.

Now by Comp. the Square, we have $x^2+15x+56,25=110,25$. Or $x+7,5=10,5$.

Therefore $x=10,5-7,5=3$, the Breadth required.

QUESTIONS, ANSWERED BY COMMON ARITHMETIC, ALGEBRA, AND GEOMETRY.

(1) First $76\times56=4256$.

And $\sqrt[2]{4256}=65,238+$lb. the true Weight required.

(2) First 1 lb. = the first, or least Weight.

		lb.		
		1	first.	Weight.
Then	$\overline{2+1}$	$=3$	second.	
And	$\overline{3+1}\times2+1$	$=9$	third.	
Also	$\overline{9+3+1}\times2+1$	$=27$	fourth.	

Sum of which $=40$ the whole Weight.

(3) First 6*s.* ¼*d.*$=289$ Farthings.

Then $\sqrt[2]{289}=17$ Persons. Or 17 qrs.$=4\frac{1}{4}d.$ what each spent.

(4) Here $x=$ the Number of Spots. $n=7$, $r=12$, and $w=52$.

Then per Theo. $x=13n+w+r=\overline{13\times7-52}+12$.

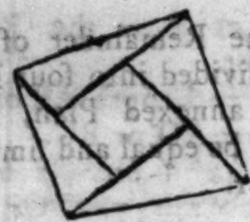
Therefore $x=103-52=51$ Spots.

(5) Supposing the Oranges to be laid in Rows, and upon each other; then the Solution will be thus $2,5\times2,5\times2,5\times2000=31250$.

∴ $\sqrt[3]{31250}=31,498$ Inches, Inside of the Box.

Fig. 1.

Fig. 2.



(6) Let Fig. 1. repreſent the Plank as it is to be cut, and the ſecond Fig. as they are placed to make the Square.

(7) The Hour-hand goes only $\frac{1}{12}$ the Circumference in an Hour, the other goes the whole, or $\frac{12}{12}=1$.

Then $\frac{12}{12}-\frac{1}{12}=\frac{11}{12}$, Minute-hand gains in an Hour.

Therefore if $\frac{11}{12}$ cir. : 1 h. : : 1 cir. : $\frac{12}{11}$h. = 1h. $5\frac{5}{11}$ min.

$\because$ 1h. $5\frac{5}{11}\times 5=27\frac{3}{11}$ min. paſt 5, the Time required.

For when the Hands are in Oppoſition, the Minute muſt be $\frac{6}{11}$ a head of the Hour-hand.

Hence as 11 : 1h. or 60 min. : : 6 : $32\frac{8}{11}$ min. paſt 12.

And for the next Conjunction it will be as 11 : 1h. : : $\overline{6\times 2}$ (12) : $=\frac{12}{11}=$ 1h. $5\frac{5}{11}$ min. as above.

As 11 : 1 : : $\overline{12+12}$ (24) : $\frac{24}{11}=$ 2 h. $11\frac{9}{11}$, the next Conjunction.

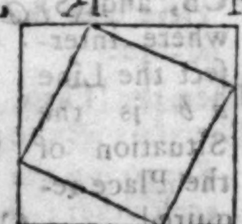
Alſo 11 : 1 : : (12 × 5260) : $\frac{60}{11}=$ 5 h. $27\frac{3}{11}=27\frac{3}{11}$ min. paſt 5, the Anſwer as above.

(8) Let ABCD, Fig. 2. repreſent the given Table = 27 Inches, and the Parallelogram EFGH Fig 1. repreſent the Plank = 4 ft. which as (per Queſt.) muſt be cut into four equal Parts, viz. EGI, KIG, IKF, and FHK.

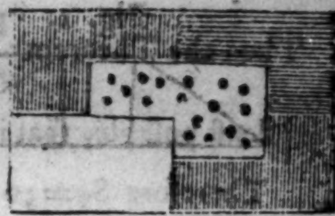
Fig. 1.

Then by properly applying the four equal Parts in the Parallelogram EFGH, to the given Table ABCD, then will EGI be = to NBD, KIG = LAB, IKF = MCA, and HFK will be = to ODC, which makes LMNO, a complete Square, each Side thereof = 36 Inches, which was required.

Fig. 2.



(9) If the Remainder of the Land be divided into four Parts, as in the annexed Plan, these parts will be equal and similar to each other.



(10) Let ABCD be the given Trapezium, and let A C and B D be produced to L; then upon L, make the Parallelogram DPIL= the Triangle DCL+ half the given Trapezium, and making the Angle LDO= LOD, produce IP to meet AL in G; then take EL, a mean Proportional between PI, and 2GL, and draw EF parallel to PD, so will EF be the Line required.

Demonstration, from the Similarity of Triangles, we have $\frac{1}{2}GL \times LI : \overline{GL}|^2 :: PI \times LI : \overline{EL}|^2$, consequently $\overline{EL}|^2 = 2GL \times PI$.

Therefore when the Triangle LEF= the Parallelogram PDLI, then EL will be a mean Proportional between 2GL, and PI, and EF is the shortest Line possible. *Q. E. D.* (See Theo. X. p. 112 of Simpson's Geom.)

(11) Thus let the Line A*a* be continued to C, so that C *a* = A *a*. Then draw CB, and (S) where intersect the Line *a b* is the Situation of the Place required.

∵ AS+CS=the whole Distance travelled.

For example thus, suppose $ab=50$, $Aa=40$, and $Bb=30$ required the Situation of a Place S, so that SA+SB, may be the least Distance possible.

Then the Theorem $\frac{Bb \times ab}{Aa+Bb}=Sb$, when AS+SB is the least possible, that is $\frac{30\times50}{40+30}=\frac{1500}{70}=21,42857=Sb$.

Therefore $Sa=50-21,42857=28,57143$.

GENEALOGICAL PARADOXES.

(1) Lot committing Inceſt with his two Daughters, and having by each of them a Son, produced the Paradoxical Relation of the Sons to each other; for by that means, Lot became, at the ſame Time, their Father, and Grand Father; and they were Brothers, and firſt Couſins to each other; alſo each Mother was Aunt to the other's Son. See Gen. xix. ver. 31. to the End.

(2) The Lord God formed the firſt Man, Adam, of the Duſt of the Ground; Gen. ii. ver. 7. and from Adam made he a Woman, and called her Name Eve. Gen. ii. ver. 22. Now Abel (who was the ſecond Son of Adam, and Eve his Wife) was murdered by his Brother Cain; therefore he got the Maidenhead of his Grandmother (the Earth) and was begot before his Father (Adam) who was made of the Earth, therefore was not begotten: and was born before his Mother (Eve) who was made of Man, therefore was not born.

APPENDIX.

1. CONCERNING DIVISORS.

IT being often neceſſary in arithmetical Calculations, to find ſuch Multipliers, or Numbers, which may be divided by any Number of given Diviſors, without any Remainder, or Remainders; by which Means many pleaſant Queſtions, not reducible to any other Rule in common Arithmetic, may be ſolved.

To find the leaſt Number that can be divided by any Number of Diviſors with a Remainder.

RULE.

RULE.

Multiply all the Prime * Numbers, and the Root of ſuch as are Square or Cube Numbers, continually; the Product will be the Number required.

EXAMPLES.

(1) Required the three leaſt Numbers, which divided by 20 ſhall leave 19 for a remainder; but, if divided by 19, ſhall leave 18, if divided by 18, ſhall leave 17; and ſo on, always leaving one leſs than the Diviſor, to Unity.

G. Diary, 1747.

By Algebra thus, ſuppoſe $x=$ the leaſt Number poſſible. And let a, b, c, d, &c. repreſent the Quotients reſpectively produced by dividing x, by 2, 3, 4, 5, &c. Then will the Remainders be 1, 2, 3, 4, &c. Hence $x=2a+1=3b+2=4c+3=5d+4$, &c. to $20A+19$.

Now $a=b+\frac{b+1}{2}$. Hence $\frac{b+1}{2}=m$, a whole Number.

$\because\ b=2m-1$, and $a=3m-1$.

Again $c=m-1+\frac{m}{2}$. Hence $\frac{m}{2}=n$, a whole Numb.

And therefore $m=2n$, $c=3n-1$, and proceeding in this Manner we get $A=11639628\,B-1$; hence the Number required $x=232792560\,B-1$, where B repreſents any whole poſitive Number; and when $B=1$, then the Number required $x=23279$.

The Number required $x=232792560-1 = 232792559$

2d. $\overline{232792560}\times 2-1 = 465585119$

3d. $232792560\times 3-1 = 698377679$

And thus may be performed other Numbers ad infinitum.

Or thus by the Rule,

Firſt 1, 2, 3, 5, 7, 11, 13, 17, and 19, are Prime Numbs.

Alſo $\sqrt[2]{4}=2$, $\sqrt[3]{8}=2$, $\sqrt[2]{9}=3$, and $\sqrt[3]{16}=2$, and all the reſt are compoſite Numbers.

* A Prime Number, is ſuch a Number as cannot be produced by the Multiplication of two, or more Integers.

∴ $1\times2\times3\times2\times5\times7\times2\times3\times11\times13\times2\times17\times19=$ 232792560, the least Number that can be divided by the given Divisors without a Remainder, as before.

(2) What's the least Number that can be divided by the Nine Digits, without a Remainder.

LADIES DIARY, 1719.

Divisors 1, 2, 3, 4, 5, 6, 7, 8, 9.

Now $\sqrt{4}=2$: 6 may be cancelled, being composed 2×3; and 3, 5, and 7 are Prime Numbers.

And $\sqrt[3]{8}=2$. Also $\sqrt[2]{9}=3$.

Then per Rule $1\times2\times3\times2\times5\times7\times2\times3=2520$.

(3) A country Girl to Town did go,
Some Walnuts there to sell;
A Gentleman she chanc'd to meet,
And thus it her befell:
My pretty Maid, says he to her,
What Number have you here?
I can't tell, Sir, said she to him,
But this I'll make appear;
I told them o'er, ere I came out,
By six's, five's, four's, three's, two's;
And every time I number'd them,
One remained overplus:
I told them o'er by seven's at last,
And there were no remains:
If you can find the Number out,
Pray take it for your pains.

First the least Number that can be divided by 1, 2, 3, 4, 5, 6, without a Remainder, will be $1\times2\times3\times2\times5=60$, per Rule.

Then $60+1=61$, will leave 1, when $\div$ 2, 3, 4, 5, 6.

But 7)61(8, and 5 Remains.

Also $\left.\begin{array}{l}\overline{60\times2}+1=121\\ \overline{60\times3}+1=181\\ \overline{60\times4}+1=241\end{array}\right\}$ None of which are divisible by 7, without a Remainder.

But $\overline{60\times5}+1=301$. is the least Number which admits of the Conditions of the Question.

Then to find the next least Number which admits of the same Conditions, we shall find (by proceeding as above) to be $\overline{60\times12}+1=721$

Also

Alſo 723—301=420, the common Difference of all the Numbers anſwering the Conditions of the Queſtion.

∵ 301, 721, 1141, 1561, &c. ad infinitum, will anſwer the Conditions of this Queſtion. (See Queſt. 18. p. 60.)

(4) To find the leaſt Number of Guineas, which being divided by 6, 5, 4, 3, and 2, ſhall leave 5, 4, 3, 2, and 1, reſpectively remaining.

LADIES DIARY, 1748.

Firſt $1\times2\times3\times2\times5=60$, the leaſt Number which will divide by 2, 3, 4, 5, and 6, as by the laſt Queſtion.

∵ $60-1=59$. *Q. E. F.*

2. COMBINATIONS.

Combination of Quantities, is the Manner of finding how many different Ways they may be varied, being taken 1 and 1, 2 and 2, 3 and 3, &c. as the Number of Combination of three Quantities, viz. *a*, *b*, and *c*, viz. *ab*, *ac*, *bc*. If three Quantities are to be combined, and their Number is only three, as *a*, *b*, and *c*, then the Number of Combinations will be only one, viz. *abc*; and if there are four Quantities *a*, *b*, *c*, and *d*, and three to be taken, then the Combinations will be four, viz. *abc*, *abd*, *bcd*, *acd*; and if the Number of Quantities to be combined be called *q*, and *u*, the Number of them to be taken, then the Number of Combinations will be $\frac{q-u+1}{1}\times\frac{q-u+2}{2}\times\frac{q-u+3}{3}\times\frac{q-u+4}{4}\times\frac{q-u+5}{5}$, &c. For ſuppoſe the Number of Quantities to be combined be 6, and the Number of them taken be 4, then the Number of Combinations will be $\frac{6-4+1}{1}\times\frac{6-4+2}{2}\times\frac{6-4+3}{3}\times\frac{6-4+4}{4}=\frac{1}{1}\times\frac{4}{2}\times\frac{5}{3}\times\frac{6}{4}=15$. The Number of all the poſſible Combinations beginning from the Combinations of every two, will be 2^q-q-1; as when the Number of Quanties be 5, then the Number of poſſible Combinations will be $2^5-6=26$. So that if *u* repreſents any Number of Quanties, then will $\frac{u^{u+1}-u}{u-1}$ expreſs the poſſible Number of all the Variation, as it

if $u = 7$, then $\frac{7^{8} - 7}{7 - 1} = \frac{5764801 - 7}{6} = \frac{5764794}{6} =$ 960799. (See Quest. 26. in Algebra.)

To find the different Combinations in any Number, or Quantities.

RULE.

Having placed the given Quantity by itself, decrease it gradually by an Unit, so often as there are Quantities in the Combination; placing one above another, with a Sign of Multiplication between them, which Numbers must be multiplied into one another for a Dividend: then placing an Unit with the like Number of Places, increasing by Unity till you arrive at the Number to be continued; which multiply continually for a Divisor, and the Quotient will be the Number of Combinations sought.

EXAMPLES.

(1) How many different Ways may 11 Halfpence husled in a Hat turn up?

First, as a Halfpenny has two Faces. $\therefore 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, or $2^{11} = 2048$, the different Combinations required.

Now to find the different Chances for any Number of Heads, or Tails, let a represent the Heads, and b the Tails; then by the Theorem in Page 328, we shall have $a^{11} + 11a^{10}b + 55a^{9}b^{2} + 165a^{8}b^{3} + 330a^{7}b^{4} + 462a^{6}b^{5} + 462a^{5}b^{6} + 330a^{4}b^{7} + 165a^{3}b^{8} + 55a^{2}b^{9} + 11ab^{10} + b^{11}$ are all the different Combinations, or Ways 11 Halfpence can turn up, viz. $1 + 11 + 55 + 165 + 330 + 462 + 462 + 330 + 165 + 55 + 11 + 1 = 2048$, as before.

Here it is to be observed that a^{11}, or all Heads, hath but one Way of turning up; the same for b^{11} Tails: but 10 Heads and 1 Tail, and the contrary, may come up 11 different Ways each. Also nine Heads and two Tails, or the contrary may each come up $\frac{11 \times 10}{2} = 53$ different Ways. Likewise eight Heads and three Tails, or the contrary may come up $\frac{55 \times 9}{2} = 165$ different ways, &c.

It

It may be also observed, that, by this Theorem, the Unciæ, or Coeficients, do only increase until the Indices of the two Letters become equal, or change Places, and then the rest decrease in the same Order.

(2) A famous General having serv'd his King,
Long Time in Wars, and had victorious been;
For which his Service, with a pleasant Smile,
Ask'd of his King, one Farthing for each File
Of ten Men in a File, which he could then
Make with a Body of one Hundred Men.
The King, considering his brave Actions past,
And seeming modesty of his Request;
Gives his Consent. To what will it amount
In sterling Money? Take your Pen and count.

Thus $\frac{100}{1} \times \frac{99}{2} \times \frac{98}{3} \times \frac{97}{4} \times \frac{96}{5} \times \frac{95}{6} \times \frac{94}{7} \times \frac{93}{8} \times \frac{92}{9} \times \frac{91}{10} = \frac{62815650955520472000}{3628800} =$ 17310309456440 Farthings, or 18031572350£. 9s. 2d. *Q. E. F.*

(3) Two Gamesters one Day, at Dice they did play;
And being full merry with Wine;
Said B unto A, what Odds will you lay,
I cast not the six Aces this Time?
Says A then to B, ten to one I'll lay thee,
With six Dice, the six Aces you cast not.
Pray Youths, shew, and here let them know,
For the Odds on the Cast, Sirs, they do not.

First $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46656 = 6^6$ different Combinations.

And $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ Variations.

Then $46656 - 720 = 45936$ Chances against A.

But as A laid 10 to 1. ∴ 7200 Chances to B.

Therefore A's Chance to that of B, :: 45936 : 7200, or : 6,38 : 1. *Q. E. F.*

(4). Two gamesters met the other Day,
The one call'd B, the other A;
But having neither Cards, nor Dice,
They got to hotch-cap in a trice;
With sixteen Halfpence fair and flat,
All which they hufled in a Hat:

Says A, to B, all these are mine,
And I will lay a Pint of Wine,
That in two Trials there will be
Nine Heads or Tails, as here you see.—
No Matter which, but on they play'd,
'Till Silver, Brass, and Gold were laid:
But as to B, his Chance was bad,
For he got broke of all he had.
What were the Odds? I pray declare.
Ye ingenious Youths, and place it here.

First $2^{16}=65536$, Number of different Chances on 16 Halfpence.

Now let a represent the Heads, and b the Tails.

Then (by Theo. in p. 328) we shall have $a^{16}+16ab^{15}+120a^{14}b^2+560a^{13}b^3+1820a^{12}b^4+4368a^{11}b^5+8008a^{10}b^6+11440a^9b^7$, &c.

Then $11440\times2=22880$ Chances for Nine Heads, or Tails to come.

$\because$ $65536-22880=42656$ Chances, not come up the first Time: viz. 22880 to 42656, that they come up nine Heads or Tails the first Toss.

Therefore as $65536 : 22880 :: 42656 : 14892\frac{7}{64}$.

Then $22880+14892\frac{7}{64}3=7772\frac{7}{64}$ for $\Big\}$ Nine Heads
Also $65536-37772\frac{7}{64}=36763\frac{57}{64}$ against $\Big\}$ or 9 Tails.

MAGICAL SQUARES.

By a Magical we understand a Square divided into several other small equal Squares, filled with Terms of an Arithmetical Progression, so transplanted, that all the same Line, or Rank, whether taken perpendicularly, horizontally, or diagonally, make the same Sum.

EXAMPLE.

(1) The Numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, being given, to form them in a Magic Square; viz. counting each Rank perpendicularly, horizontally, or diagonally, that these Ranks may be equal to each other.

Sup-

Suppose it done, and represented in its proper form, by the following symbols, placed as follows,

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & m \end{array}$$

Then by the Quest. we have $\left\{\begin{array}{l} a+e+m=15 \\ b+e+h=15 \\ c+e+g=15 \end{array}\right\}$ required e the middle Num.

Their Sum $=a+b+c+3e+m+h+g=45$.

Again $\left.\begin{array}{l} a+b+c=15 \\ \text{Also } m+h+g=15 \end{array}\right\}$ added together we get $a+b+c+m+h+g=30$; this taken from the fourth Equation gives $3e=15$.

Then $e=\dfrac{15}{3}=5$, the middle Number.

Or by Numbers, thus

First the Sum of the progressional Numbers are $1+2+3+4+5+6+7+8+9=45$.

Then $3=$ the Number of Rows.

$\because 45\div 3=15=$ the Sum of each Side or Rank.

And $15\div 3=5=e$, the middle Number, as before.

Again, to find the corner Figures; and first to find the Figure represented by a. Beginning with 1, I find the corner Letter a, or any other corner Letter cannot be 1; for if a was $=1$, then m must $=9$, $b+c=15-1=14$; as also $d+g=15-1=14$. But there remains no two Numbers after 5, 1, and 9, whose Sum is 14, but 6, and 8. Then if any of these Figures were b, the other would be c; and then no Figures would remain for the Value of either d or g; wherefore a is not equal to 1, nor any corner Letter $=$ to 1, or 9.

Now 3 cannot be $=a$; for if it were, then m should be $=7$; and $b+c=15-3=12$; as also $d+g=12$: but there remains no two Numbers after 5, 3, and 7, whose Sum is 12, but $8+4$, which cannot answer to b and c, and d and g; wherefore a, or any other corner Letter, is not $=3$; neither is m, nor another corner Letter $=7$; from what has been said, it is plain, that (if the Question proposed is capable of being solved) the corner Letters are all even Numbers; wherefore if $a=2$, m will be $=8$, and c must be either 4, or 6. Let $c=4$,

$c=4$, then $g=6$, $b=9$, $d=7$, $f=3$, and $h=1$; and so the Square will be completed as was required.

2	9	4
7	5	3
6	1	8

But if $c=6$ (a being $=2$;) then $g=4$, $b=7$, $d=9$, $f=1$, and $h=3$, and then the Squares will stand thus

2	7	6
9	5	1
4	3	8

Or they may be found mechanically: thus, set them all down progressively, about which draw a Square corner-ways; thus

1	2	3
4	5	6
7	8	9

Then set the four angular Figures at the Corners, and put the outermost alternately: that is, place 1 between 8 and 6, 9 between 4 and 2, 3 between 4 and 8, and 7 between 2 and 6; thus

2	—	6
—	5	—
5	—	8

2	7	6
9	5	1
4	3	8

There are many Ways of constructing Magic Squares, as also many surprising properties thereto relating, and is as itself very curious and entertaining; yet it cannot be denied, but that it is of very little or no Use in any other Parts of the Mathematics: therefore, whoever would see more of these Matters, may consult the DIARIAN REPOSITORY, Page 103 to 108, also from p. 223 to 225, where they will find this Subject more largely treated of.

F I N I S.